

Describing Trends in Time Series Data

URBPL 5/6010: Urban Research
University of Utah
Pam Perlich
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Readings – On Syllabus

Simon, Chapter 11: Using the Analysis Toolpack Add-ins
Mechan, Anita M. and Warner, C. Bruce. (2000) "Simple Linear Regression," Chapter 13, pages 91-100 from *Elementary Data Analysis Using Microsoft Excel*, (New York: McGraw-Hill). ERESERVE:

<http://ereserve.lib.utah.edu/ereserve/trms/annual/URBPL/5010/Perlich/simple.pdf>

Gottfried, Byron S. (2000) "Fitting Equations to Data," Chapter 5, pages 110-157 from *Spreadsheet Tools for Engineers: Excel 2000 Version*, (Boston: McGraw-Hill). ERESERVE:

<http://ereserve.lib.utah.edu/ereserve/trms/annual/URBPL/5010/Perlich/fitting.pdf>

Robert F. Nau (2005) The Logarithm Transformation,
<http://www.duke.edu/~rnau/411log.htm>

Characterizing Changes in a Time Series

- Time series data is a sequence of data points that is:
 - Time-ordered (and indexed), and
 - Measured in equal time intervals.
- A common data analysis problem is to characterize how the series has changed over time. These may include:
 - Changes in measured levels
 - Averages
 - Cycles
 - Trends
 - Rates of change
 - Averages
 - Acceleration or deceleration

Annual Percentage Change: Simple Year-Over Percentage Change

$$\left[\frac{x_{t=2}}{x_{t=1}} \right] - 1$$

where t is time (e.g., year)

Annual Percentage Change: Simple Year-Over Percentage Change

$$\left[\frac{x_{t=2} - x_{t=1}}{x_{t=1}} \right]$$

where t is time (e.g., year)

Characterizing Changes in a Time Series

- There are a variety of ways to evaluate and describe changes in a data series over time. We review five.
 - Each method has:
 - Advantages and disadvantages
 - Strengths and weaknesses
 - Optimal / appropriate applications
 - Potential to be misused or misapplied
 - We focus on annual data.
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Selected Methods

1. Average the annual percentage changes.
 2. Fit a line to the year-over annual growth rate of the series.
 3. Compute the two-point average annual rate of change.
 4. Use regression to fit a linear function to the original series.
 5. Compute the slope of a log linear regression of the original series.
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Method 1:

Average the Year-Over Percentage Changes

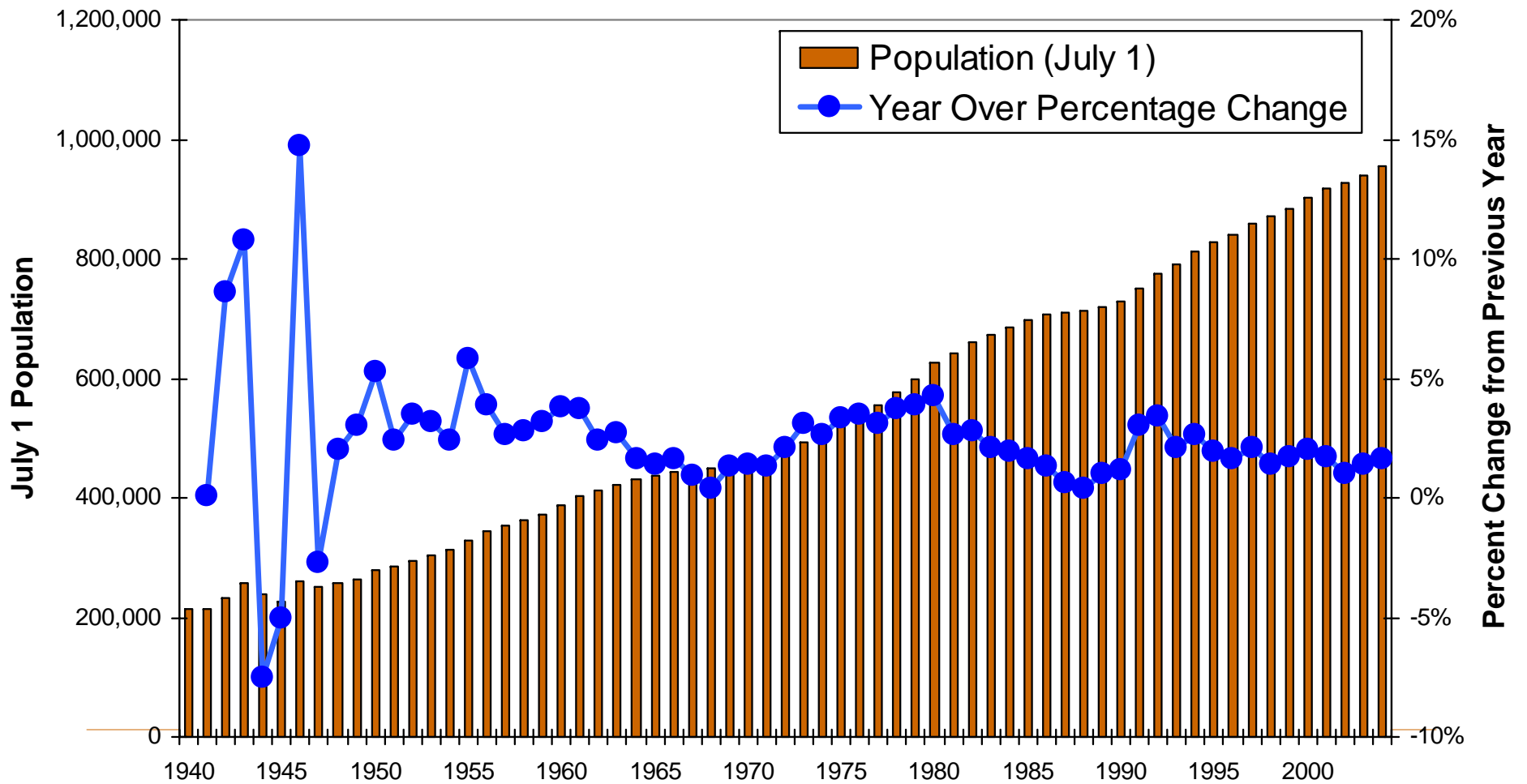
□ Procedure:

- Calculate annual percent changes for each pair of sequential data points in the series.
- Take the average of these computed year-over rates of change.
- This generates one number.

□ Does not capture:

- Trends
- Cycles
- Acceleration or deceleration of growth rates

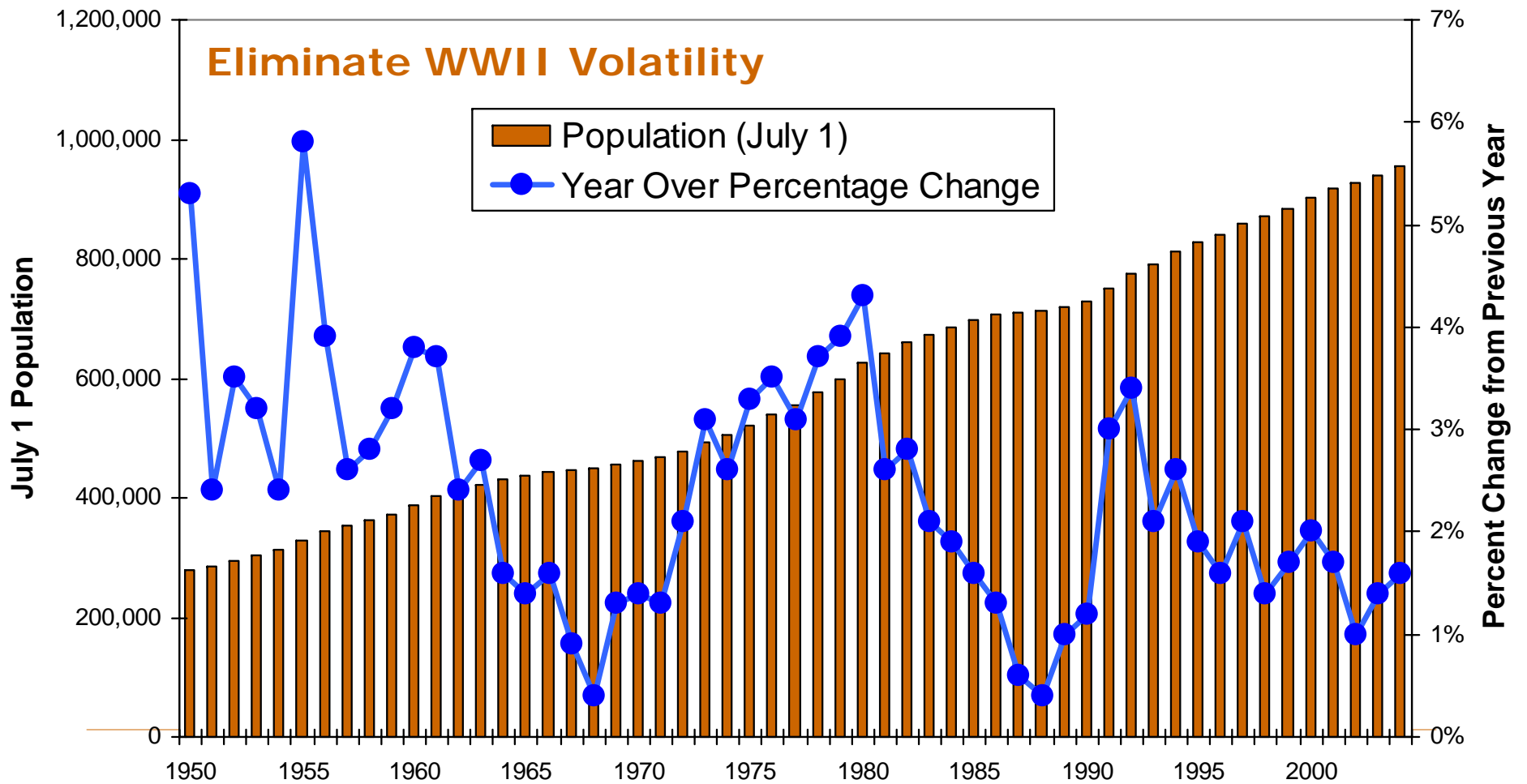
Salt Lake County Population: 1940-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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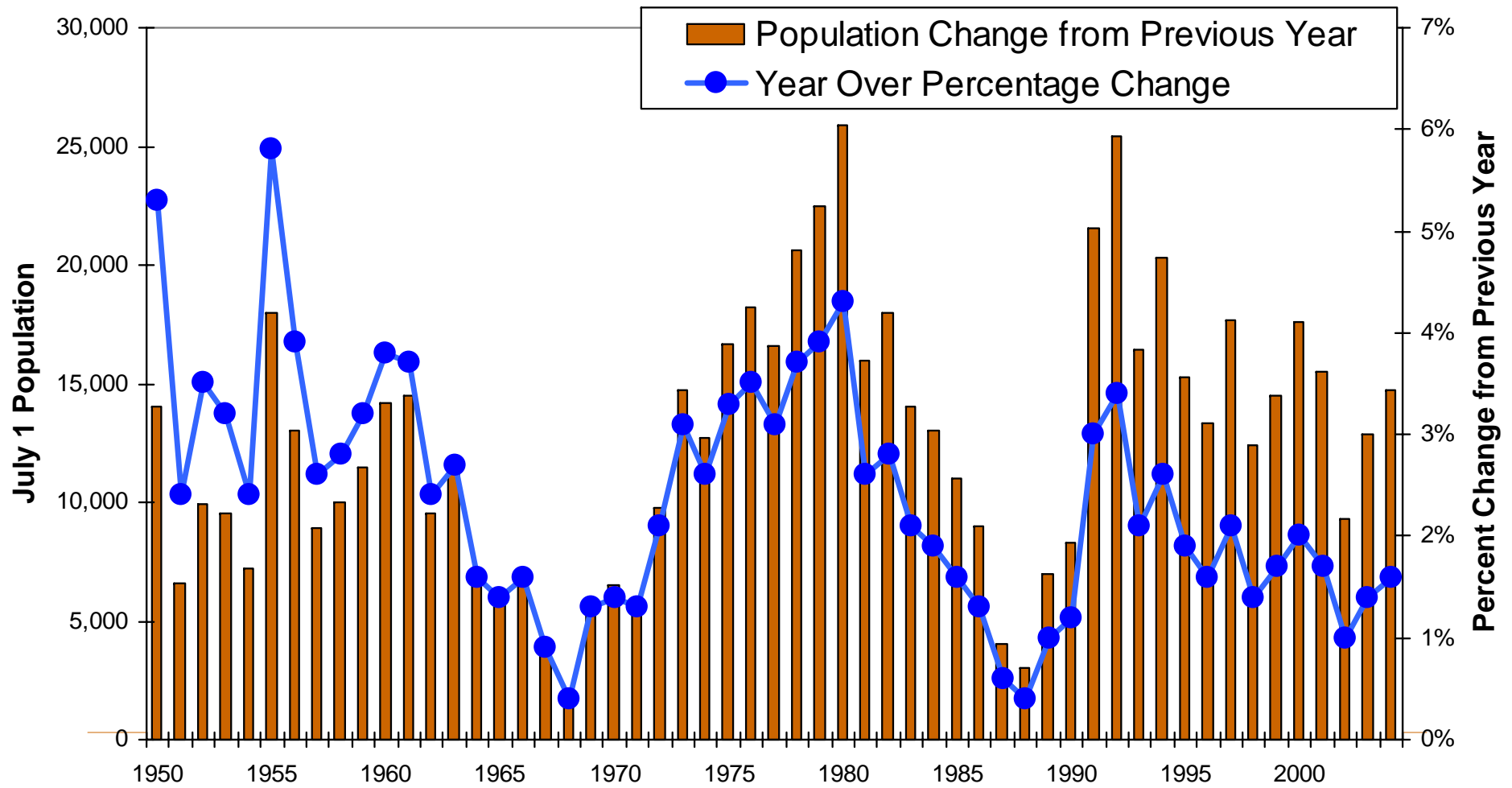
Salt Lake County Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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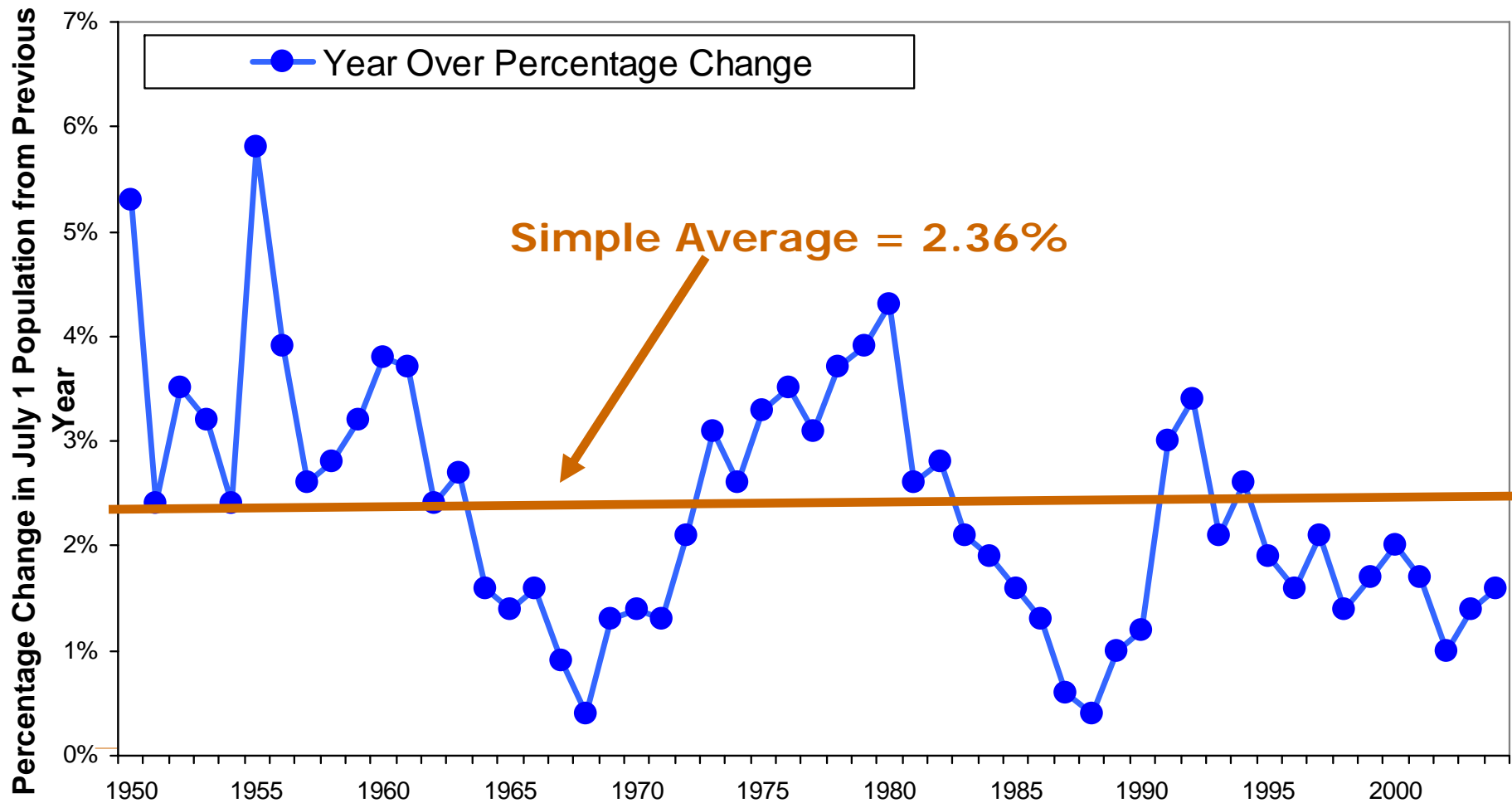
Salt Lake County: Annual Changes in July 1 Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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Salt Lake County: Year-Over Percentage Changes in July 1 Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

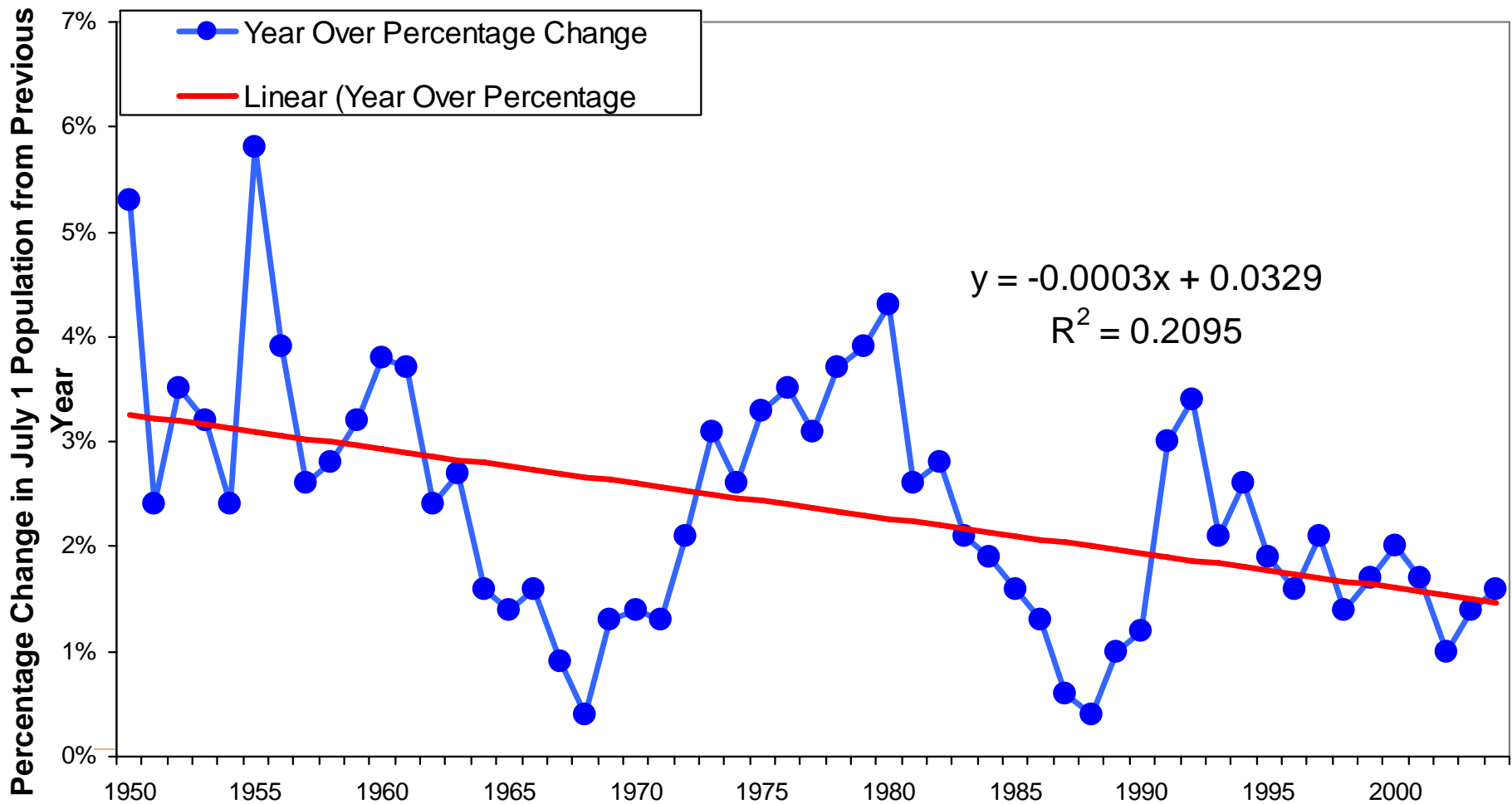
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Method 2:

Fit a Line to the Annual Rates of Change Series

- Procedure:
 - Calculate annual percent changes for each year of a series.
 - Plot the series
 - Fit a linear regression trend line.
- The slope of this trend line shows how the growth rate has changed, on average, over time.
 - Positive slope → Acceleration
 - Negative Slope → Deceleration
 - Flat Line → Constant Rate of Growth
- Does not capture:
 - Cycles
 - Long waves

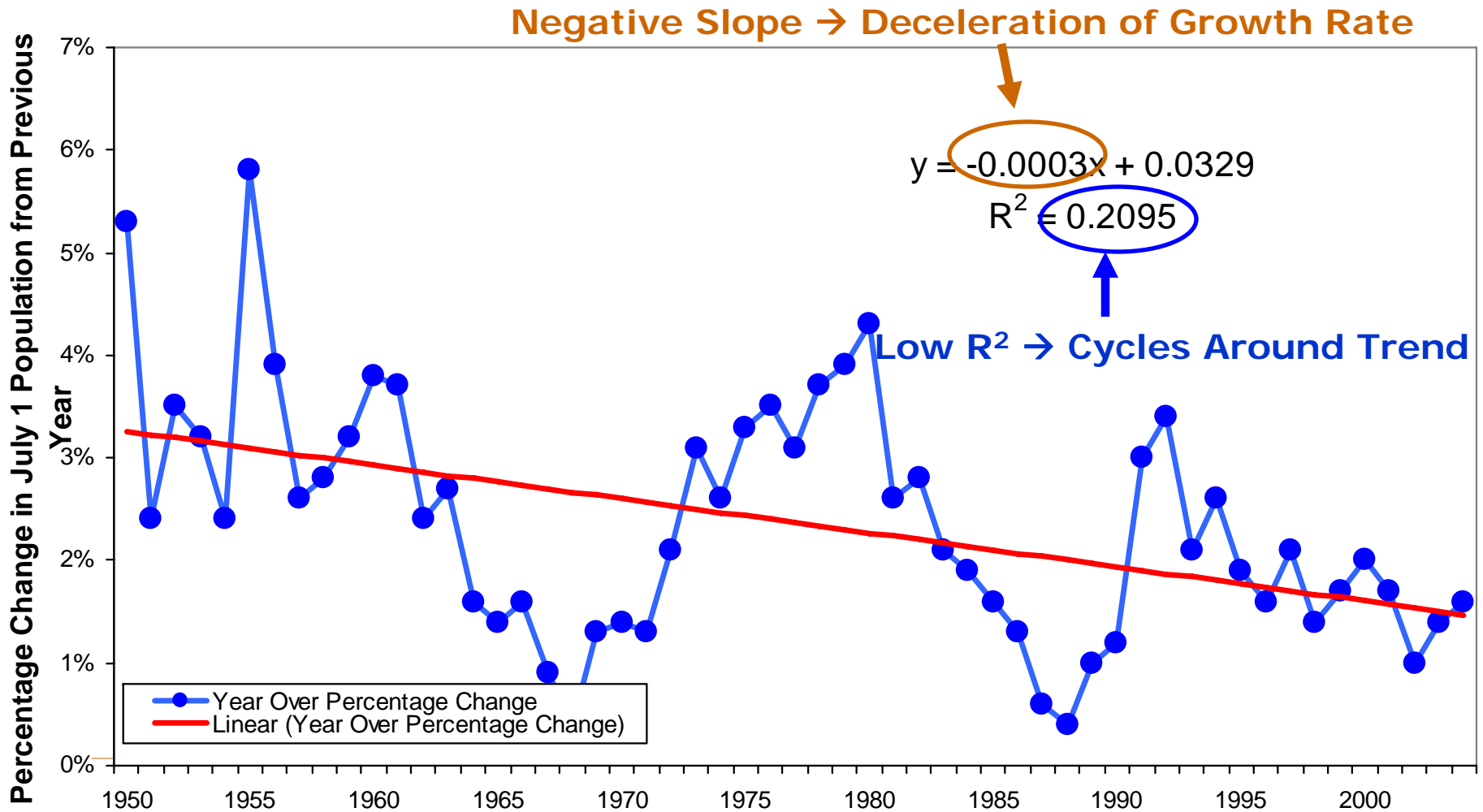
Salt Lake County: Year-Over Percentage Changes in July 1 Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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Salt Lake County: Year-Over Percentage Changes in July 1 Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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Method 3:

Average Annual Rate of Change AARC

- Procedure:
 - Select two data points – a beginning point and ending point.
 - Calculate AARC
 - AARC formula is derived from the present value calculation.
 - The result is completely determined by the two points chosen for the calculation.
 - The entire series is characterized by two points. Does not capture:
 - Cycles
 - Long wages
-

Average Annual Percentage Change

$$\sqrt[\Delta t]{\frac{x_{t=2}}{x_{t=1}}} - 1 = r$$

where t is time (years)

Example: AARC of Salt Lake County Population 1950 to 2004

July 1, 2004 Population

$$(2004-1950) \sqrt{\frac{955,166}{279,000}} - 1 = 2.31\%$$

July 1, 1950 Population

Example: AARC of Salt Lake County Population 1950 to 2004

$$\sqrt[54]{3.42453} - 1 = 2.31\%$$

Note: Roots are entered in Excel formulas with ^

The 54th root formula: =3.42353^(1/54)

AARC is Derived from Present Value Formula

$$FV = PV * (1 + r)^{\Delta t}$$

Where

FV: Future Value

PV: Present Value

t: time

Δt : change in time

AARC Formula Derivation

$$FV = PV * (1 + r)^{\Delta t}$$

$$\frac{FV}{PV} = (1 + r)^{\Delta t}$$

$$\sqrt[\Delta t]{\frac{FV}{PV}} = (1 + r)$$

$$\sqrt[\Delta t]{\frac{FV}{PV}} - 1 = r$$

← Divide both sides by PV

← Take the Δt root of both sides

← Subtract one from both sides

Average Annual Percentage Change

□ Problems:

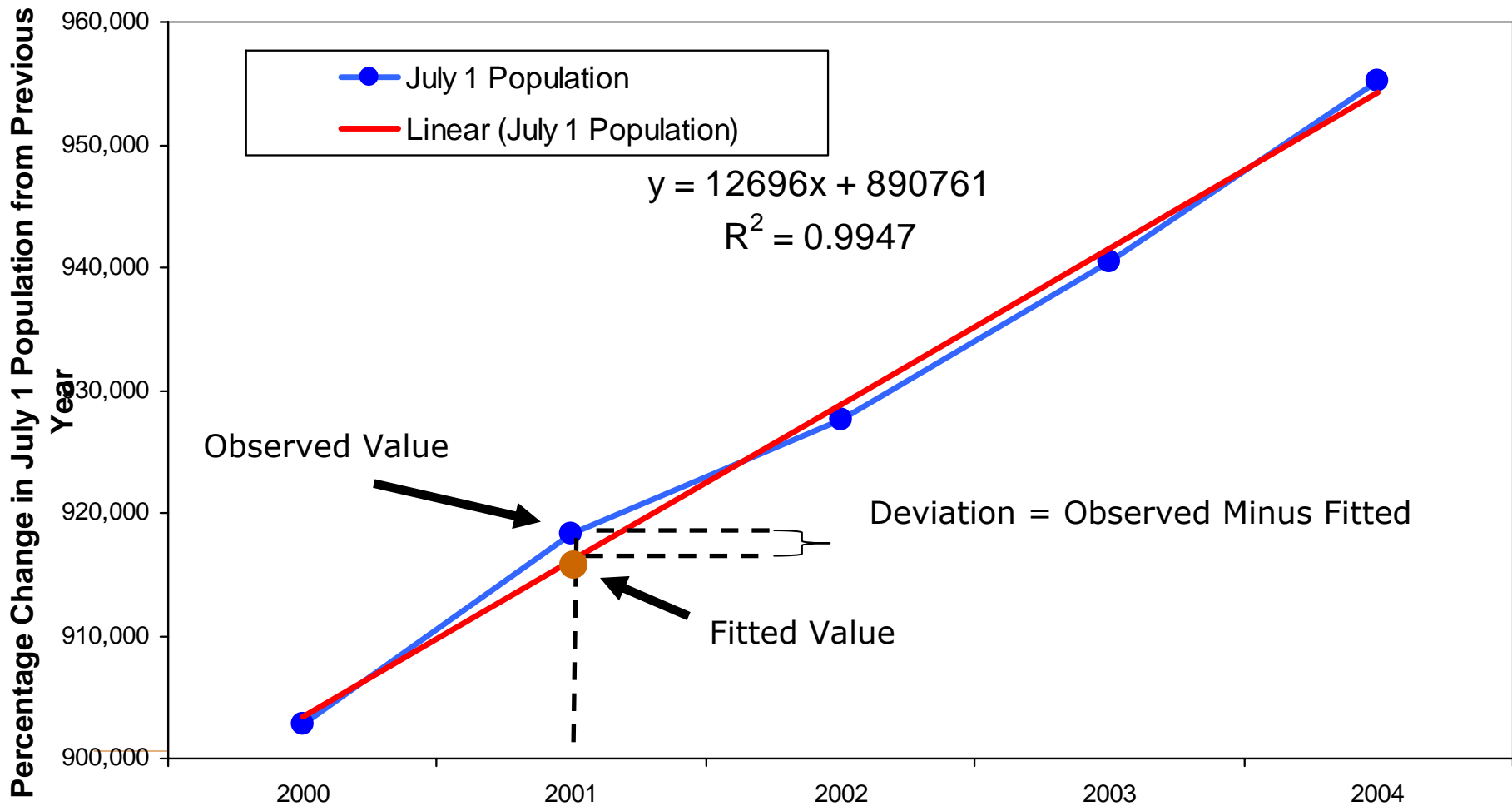
- Only considers two points
- Results can vary depending upon which two points are selected
- Can lead to misrepresentation of the data
- Does not capture fluctuations in the data.

Method 4:

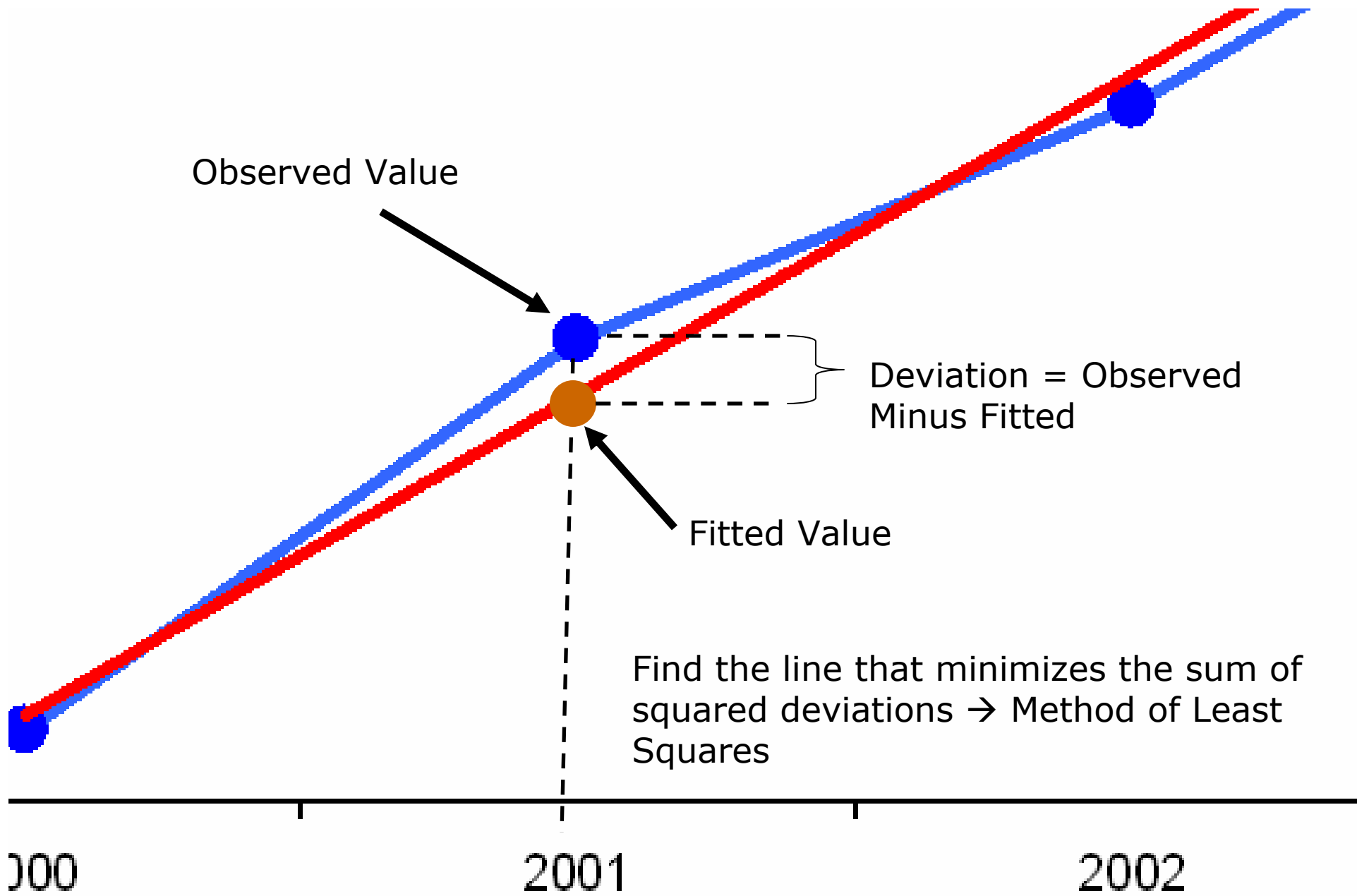
Use Regression to Fit a Line to the Original Series

- Procedure in Excel:
 - Graph the original data set
 - Add a trend line – linear
 - Method of least squares
 - Finds line that minimizes the sum of squared deviation.
 - The slope of the line indicates the absolute increments of change.
 - A line (with positive slope) has declining rates of growth.
 - Alternative functions can be fitted
 - Used also for extrapolation and interpolation
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Salt Lake County: July 1 Population - 2000-04

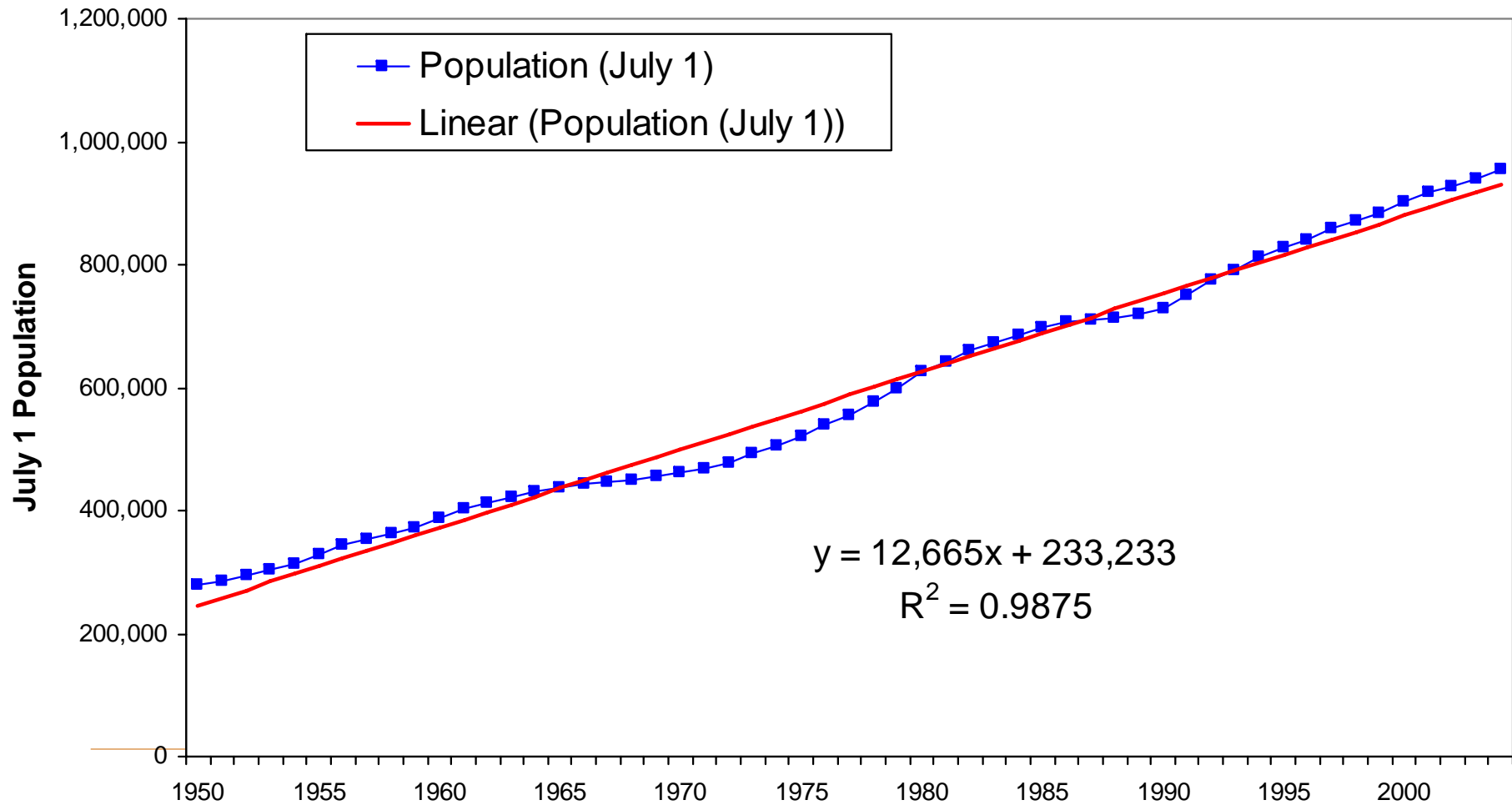


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ulation Estimates Committee, www.governor.stat

Salt Lake County Population: 1950-2004



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Method 5: Log Linear Regression

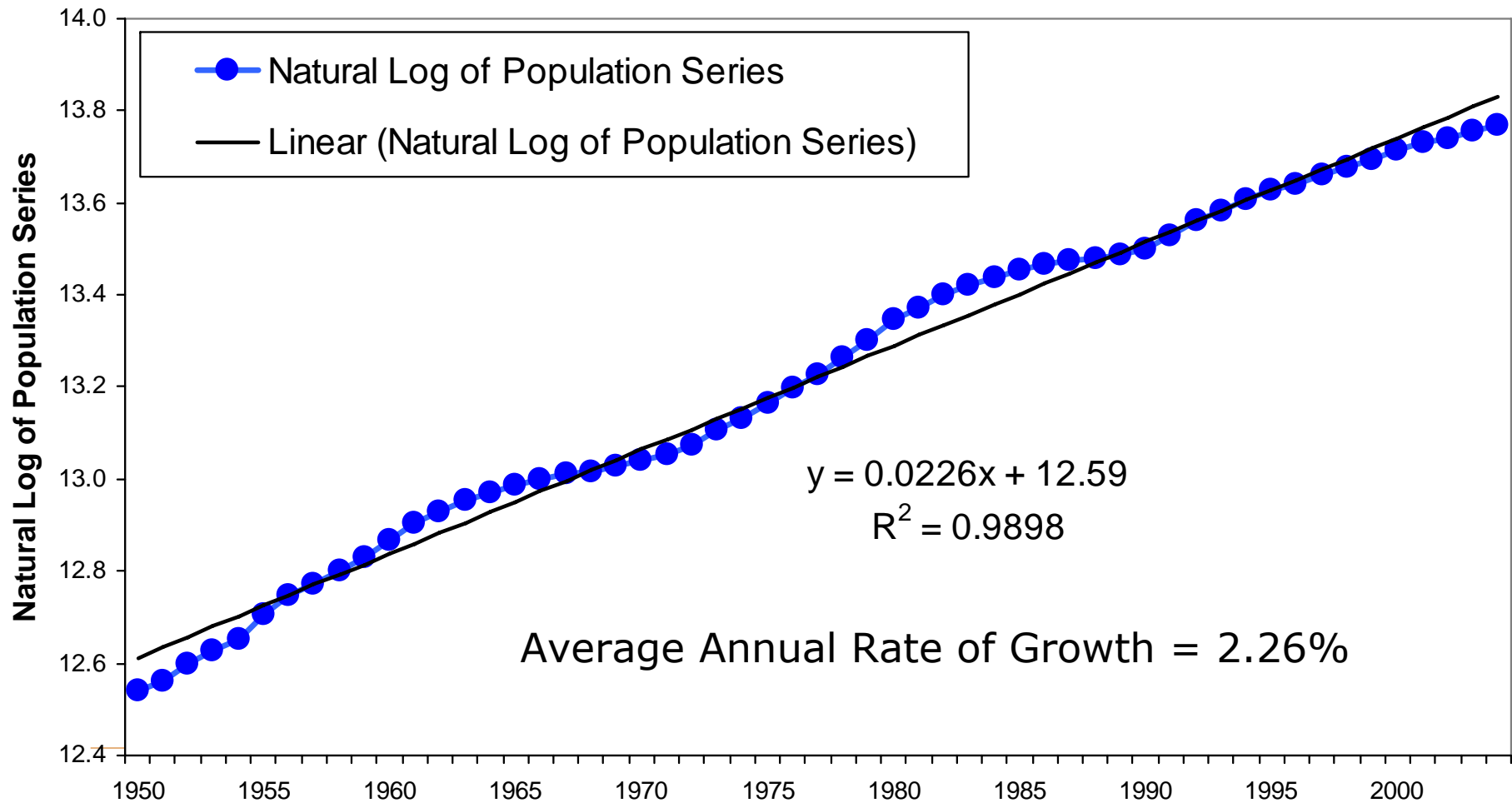
- Procedure in Excel:
 - Compute the natural log of the series
 - Graph this logged series
 - Add a linear trend line to the logged series
 - The slope of the line is a measure of average annual change
 - <http://www.duke.edu/~rnau/411log.htm#difference>
- Advantage
 - Includes all points in the series in the computation

The Logarithm Transformation

“Trend in logged units = percentage growth: Because changes in the natural logarithm are (almost) equal to *percentage* changes in the original series, it follows that the slope of a trend line fitted to logged data is equal to the average *percentage* growth in the original series .” **Robert F. Nau**

<http://www.duke.edu/~rnau/411log.htm>

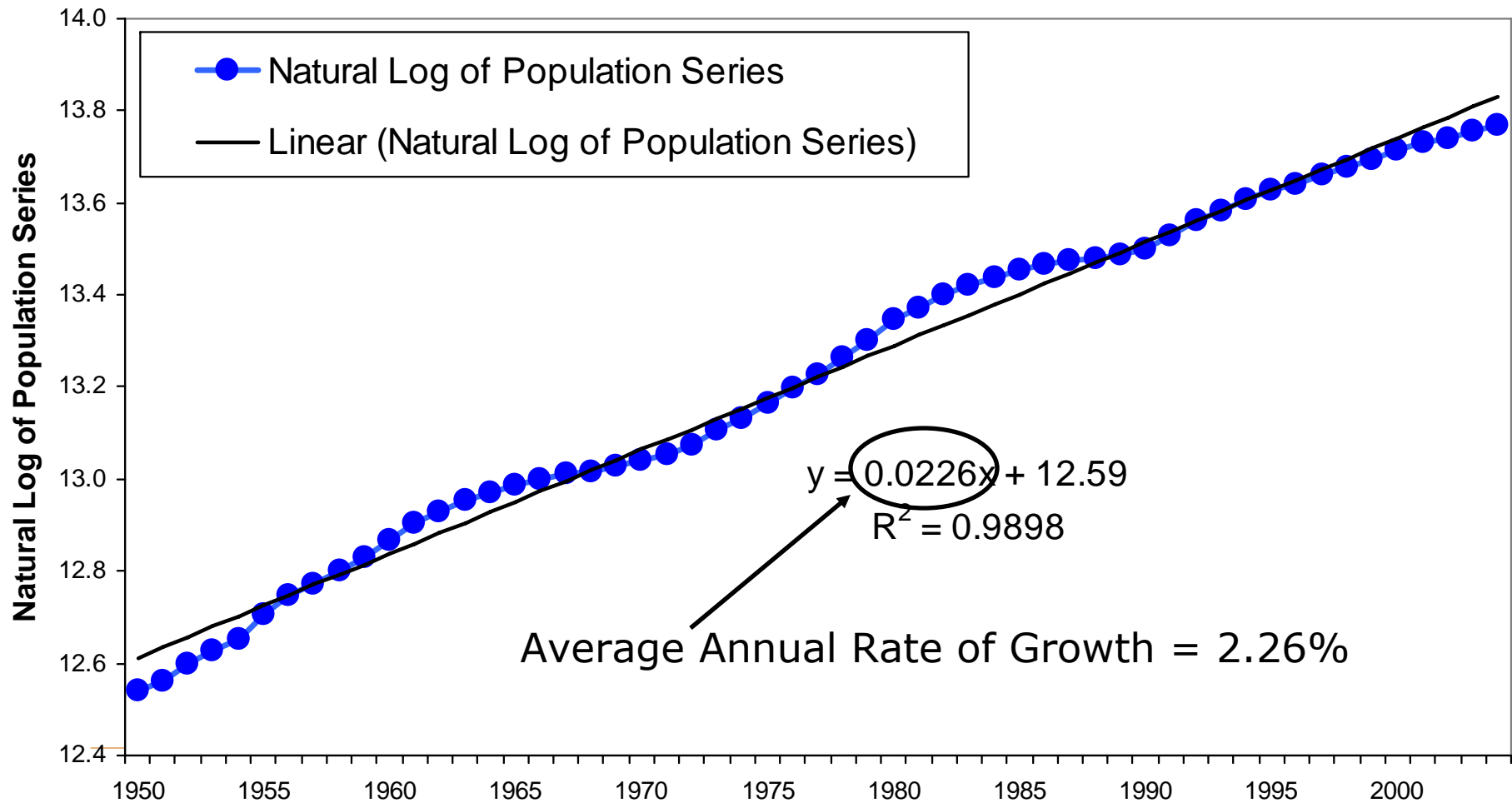
Salt Lake County: Log Linear Regression of Population: 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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Salt Lake County: Log Linear Regression of Population → 1950-2004



Source: Utah Population Estimates Committee, www.governor.state.ut.us/dea Downloaded 10/02/2005.

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Characterizing Changes in a Time Series

- Often we need to summarize what has happened to a variable over a period of time.
- Various methods exist. We review five:
 1. Average the annual percentage changes.
 2. Fit a line to the annual growth rate series.
 3. Compute the two-point average annual rate of change.
 4. Use regression to fit a function to the series.
 5. Take the slope of a log linear regression of the series.
- Each has advantages and disadvantages