# Stock Market as a 'Beauty Contest': Investor Beliefs and Price Bubbles sans Dividend Anchors 

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# Stock Market as a 'Beauty Contest': Investor Beliefs and Price Bubbles sans Dividend Anchors 

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#### Abstract

We experimentally explore if the absence of dividend anchors (from which investors can backward induct to arrive at the fundamental value) may help us understand the formation of security price bubbles. The fundamental value models assume that the investors (a) form rational expectations, (b) form higher-order beliefs, (c) the security matures in finite time, and (d) that these three conditions are common knowledge among the investors. We argue that when the deviation of security markets from these assumptions deprives the investors of any reasonable way of backward inducting the fundamental value of a security from its future dividends, its price is susceptible to floating freely. We create laboratory markets with exogenously and endogenously specified terminal values, and examine whether the absence of a dividend anchor generates price deviations from the fundamentals. We find that such deviations occur in sessions where it is difficult for investors to backward induct value from dividends. Bubble price levels appear to be indeterminate, and price predictions follow a first-order adaptive or trend process. These processes are consistent with the conjecture that the investors resort to forward induction when backward induction becomes difficult or impossible. Under these conditions, the allocative efficiency and the cross-sectional dispersion of wealth also become indeterminate, as compared to high efficiency and low dispersion in the absence of bubbles.


## JEL classification: G12; C91

Keywords: stock price bubbles, beauty contests, common knowledge, and market experiments

## Stock Market as a 'Beauty Contest': Investor Beliefs and Price Bubbles sans Dividend Anchors

## 1. Introduction

Cash dividends and capital gains from its sale constitute two parts of returns to equity. Given the relative importance of capital gains versus dividends over the typical holding periods for equity, buyers must consider the price at which they might be able to sell the security. ${ }^{1}$ The sale price depends on the value the other potential buyers may attribute to the security in the future; and that value, in turn, depends on such buyers' expectations of the sale price they would realize at the end of their own investment horizon. This dependence of security prices on the layers of higher order beliefs about future dividends opens the opportunity for prices to deviate from the fundamental value of securities. In this paper we identify the conditions for the convergence of security prices to their fundamental values, and present laboratory data on their divergence when these conditions are not met. We suggest that the violations of these conditions make it difficult, even impossible, for the investors to arrive at prices through backward induction from future dividends, giving rise to bubbles. We conjecture that when backward induction becomes too difficult to do, investors may abandon it for forward induction to arrive at prices unrelated to the fundamental value of the security.

Keynes (1937) compared the stock markets to newspaper beauty contests in which the prize goes to the reader whose choice most resembles the average pick. In security markets, investor beliefs about the others' beliefs (second-order beliefs) depend on the third-order beliefs which in turn depend on the fourth-order beliefs, and so on, if the dividends are paid beyond

[^0]their personal investment horizons. Investors have to form higher orders of beliefs, form them rationally, and backward induct their way through such beliefs to arrive at the valuation of the security and investment decision. Backward induction through these layers of beliefs requires additional common knowledge restrictions. When these assumptions are not satisfied, or if the demands of such a task on cognition extend beyond human capacity, the security price may lose its anchor to dividends and become indeterminate. There is no assurance that the security would be priced at the investors' own assessment of its fundamental value. A systematic investor decision-making mechanism may decouple market prices from the fundamental value of securities based on the investors' first-order beliefs. The gap created by this decoupling, positive or negative, may be defined as a "bubble."

Whether price bubbles are, or can be generated by the 'beauty-contest' mechanism is an empirical issue. Historical occurrence of price bubbles remains controversial. Economic historians tend to describe a run-up in prices followed by a sharp decline as a self-evident bubble (Galbraith 1988, Kindleberger 1978). Formal econometric studies, based on long-term recorded prices do not necessarily support the conclusion that bubbles ever existed (Hamilton and Whiteman 1985, Hamilton 1986, Campbell and Shiller 1987, Santoni 1987, Diba and Grossman 1988, Flood and Hodrick 1990, and Santoni and Dwyer 1990). Over the long run, stock prices seem to have covaried sufficiently with the actual dividends to raise questions about the persistence of bubbles. However, when other historical data such as pricing of closed-end mutual funds (DeLong and Shleifer 1991) and interest rates on broker loans (Rappoport and White 1993) are taken into account, the existence of bubbles becomes more plausible. Hamilton and Whiteman point out that the investors' information set about the prevailing fundamentals, and perhaps the information set of the contemporary observers, is larger than the data accessible
to econometricians. Still, it is difficult to identify any fundamental changes to support a 90 percent decline in US stock prices over a three-year period beginning in 1929.

Stock market bids, offers, and transactions are observable data. Investor beliefs of all orders, being inherently private, are not observable. Without the knowledge of investor beliefs, we cannot link the price rise to the gaps among various orders of beliefs or to the changes in these beliefs. The observation of a price rise followed by a steep fall is insufficient to infer the existence of a bubble because such price changes can be consistent with changes in the unobservable fundamental value of the firm. Data gathered from stock markets cannot help us distinguish between stock price changes due to a "beauty-contest" and a reassessment of the fundamental value.

Laboratory experiments can yield insights into such phenomena by letting the experimenter control the fundamental value through parameters assigned to the subjects. It is also possible to create conditions that make it difficult, if not impossible, for investors to backward induct. Such controlled treatments can help us examine the consequences of relaxing the conditions that are sufficient to achieve convergence between security prices and their fundamental values.

The main treatment in the experiment is differentiated by whether the terminal payoff is an exogenously specified dividend value, or an endogenous value given by subjects' own expectations of future prices. In addition to the main treatment, we use several other variations described in the following section to examine the robustness of any effects of the main treatment.

Briefly, we find that when the terminal payoff is endogenous, the security tends to lose its dividend anchor used by investors for backward induction to its fundamental value. Its price becomes susceptible to bubbles (defined as systematic deviations from the equilibrium prices based on the fundamental or first-order beliefs about the cash flows). When the terminal payoff
is exogenously specified, investors' beliefs about others' beliefs do not generate bubbles. In the presence of bubbles, prices are indeterminate and, consistent with forward induction, the price predictions can be better organized as first-order adaptive or trend processes. Finally, the allocative efficiency of markets is high and the dispersion of wealth is low in the absence of bubbles; both become unpredictable when bubbles arise. The dispersion of wealth increases with the magnitude of bubbles.

These laboratory observations suggest that bubbles are more likely to occur in markets for securities whose fundamental values are more difficult to backward induct from dividends. This may explain why high-growth and new technology stocks are believed to be more susceptible to bubbles. Our results support the possibility of free-floating prices, supported solely by the future expectations of investors. In such cases, actual and expected prices can support each other at any level. The higher dispersion of wealth confirms the widespread view of price bubbles as socially undesirable-to be suppressed through public policy.

The remainder of the paper is organized as follows: Section Two develops a model of asset price deviation from fundamentals when investment horizon is shorter than the date of maturity of the security; Section Three describes the experimental design and procedures; Section Four reports our laboratory results; and Section Five is a summary and discussion of our results and their implications.

## 2. The Valuations of Stocks using the Backward Induction

In standard theory, the security prices are, or tend towards, the fundamental values-the sum of discounted present value of expected future dividends-irrespective of investors' decision horizons. This proposition is derived through backward induction from the future into the present. We examine the critical dependence of the investors' ability to backward induct on several restrictive assumptions about their expectation-formation processes and their common
knowledge. When these assumptions are not met, the prices may lose their dividend anchors and form bubbles. The model helps guide the design of a critical experiment to examine the implications of the theory in laboratory.

## Short-term investor and the stock valuation

Following the standard treatment in finance, consider an investor $i$ who buys a security at time $t$, holds it for one period, and sells it at time $t+1$ immediately after receiving a cash dividend $D_{t+l}$. The value $V_{t}$ of the share to the investor at time $t$ (and its price $P_{t}$ in a market populated by such shareholders) is

$$
\begin{equation*}
P_{t}=V_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(P_{t+1}\right)}{(1+r)} \tag{1}
\end{equation*}
$$

where $P_{t+1}$ is the stock price at time $t+1, E_{t}(\cdot)$ is investor $i$ 's expectation at time $t$, and $r$ is discount rate. If the security matures at time $t+m$, its fundamental value $F_{t}$ at time $t$ is defined as the sum of discounted present values of future expected dividends:

$$
\begin{equation*}
F_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(D_{t+2}\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(D_{t+m}\right)}{(1+r)^{m}} . \tag{2}
\end{equation*}
$$

Price $P_{t}$ in (Equation 1) can take any value depending on $E_{t}\left(P_{t+1}\right)$. The standard textbook treatment equates $P_{t}$ to $F_{t}$. It can be supported by a strong set of assumptions about the current investors' expectations about the behavior and expectations of the future investors, and backward inducting from these expectations. We examine these assumptions, and the determination of prices when these assumptions do not hold in representative and heterogeneous investor models.

## Representative Investor Model

A representative investor model simplifies analysis by assuming that all investors within and across generations are homogenous in information sets, expectations, and behavior.

Determining $P_{t}$ in (1) requires further specification of how the investor forms expectation $E_{t}\left(P_{t+1}\right)$.

Assumption R1: Investors form rational expectations, i.e., they use all available information to form their expectations, and know that the price equation (1) holds every period.

From R1, investor $i$ knows at time $t$ that equation (1) holds at period $t+1$,

$$
\begin{equation*}
P_{t+1}=\frac{E_{t+1}\left(D_{t+2}\right)}{(1+r)}+\frac{E_{t+1}\left(P_{t+2}\right)}{(1+r)} . \tag{3}
\end{equation*}
$$

Investor $i$ uses equation (3) to form his expectation of $P_{t+1}$ :

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)=\frac{E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right)}{(1+r)}, \tag{4}
\end{equation*}
$$

where $E_{t}\left(E_{t+1}(\cdot)\right)$ is investor $i$ 's expectation at period $t$ of the expectation of the investor $(i+1)$ about $(\cdot)$ at time $(t+1)$. Since the representative investor model does not distinguish between investors $i$ and (i+1), $i$ 's expectation at time $t$ of $(i+1)$ 's expectation at time $(t+1)$ is equal to $i$ 's expectation at time $t$ of his own expectation at time $(t+1)$. From the law of iterated expectations, today's expectation of next period's expectation about the future is the same as today's expectation about the future:

$$
\begin{align*}
E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right) & =E_{t}\left(D_{t+2}\right),  \tag{5}\\
E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right) & =E_{t}\left(P_{t+2}\right), \tag{6}
\end{align*}
$$

Using (5) and (6), we rewrite (4) as

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)=\frac{E_{t}\left(D_{t+2}\right)}{(1+r)}+\frac{E_{t}\left(P_{t+2}\right)}{(1+r)} . \tag{7}
\end{equation*}
$$

Equation (7) shows that investor $i$ needs to form $E_{t}\left(P_{t+2}\right)$ in turn to get $E_{t}\left(P_{t+1}\right)$. Substituting (7) into (1):

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(D_{t+2}\right)}{(1+r)^{2}}+\frac{E_{t}\left(P_{t+2}\right)}{(1+r)^{2}} . \tag{8}
\end{equation*}
$$

We also need $\mathrm{Et}(\mathrm{Pt}+2)$ in order to get the current period price, Pt .
From Assumption R1, investor $i$ at period $t$ could repeat the process of arriving at $E_{t}\left(P_{t+2}\right)$ and arrive $E_{t}\left(P_{t+1}\right)$; he updates (3) and (4) by one period, uses the law of iterated expectations to replace $E_{t}\left(P_{t+2}\right)$ in terms of $E_{t}\left(D_{t+3}\right)$ and $E_{t}\left(P_{t+3}\right)$. Repeated substitution yields the price equation:

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(D_{t+2}\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(D_{t+k}\right)}{(1+r)^{k}}+\frac{E_{t}\left(P_{t+k}\right)}{(1+r)^{k}} \tag{9}
\end{equation*}
$$

where $k \leq m-1$. Note that $P_{t}$ in (9) depends not only on the expectations of dividends for periods $(t+1)$ to $(t+k)$, but also on the expectation $E_{t}\left(P_{t+k}\right)$ of price at time $(t+k)$. No amount of repetition of the process would eliminate this term on the expectation of price unless we assume that the security matures in finite time.

Assumption R2: The security matures in finite time ( $t+m$ ); after paying dividend $D_{t+m}$, the security is worthless.

From Assumption R2, investor $i$ at time $t$ forms the expectation of the time $(t+m-l)$ price, i.e., the price one period before the date of maturity:

$$
\begin{equation*}
E_{t}\left(P_{t+m-1}\right)=\frac{E_{t}\left(D_{t+m}\right)}{(1+r)} \tag{10}
\end{equation*}
$$

Substituting (10) into (9),

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(D_{t+2}\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(D_{t+m}\right)}{(1+r)^{m}}=F_{t} \tag{11}
\end{equation*}
$$

completes the derivation of the fundamental value of the security as its price. When Assumptions 1 and 2 hold, the stock price is determined through the investor recursively forming a series of ( $m-1$ ) rational expectations and the corresponding backward inductions. This
valuation process yields the standard textbook case when the price $P_{t}$ is equal to the fundamental value of the security, $F_{t}$.

Suppose we relax the rational expectations Assumption R1 and the investors cannot form rational expectations beyond some period $t+k(1 \leq k \leq m-1)$. The investor will determine $P_{t}$ by equation (9) in terms of the dividend stream $E_{t}\left(D_{t+1}\right), \ldots E_{t}\left(D_{t+k}\right)$, and the expectation $E_{t}\left(P_{t+k}\right)$ of price at time $(t+k)$. Without the rational expectations assumption, we cannot substitute the expectations of dividends after period $(t+k)$ for $P_{t+k}$. In this case, the stock price determination process stops at equation (9), and therefore price $P_{t}$ would not be equal to the fundamental value $F_{t}$. If we cannot say anything more about $E_{t}\left(P_{t+k}\right), P_{t}$ is no longer anchored to the fundamental value $F_{t}$ by the future dividend stream through backward induction.

Next, suppose Assumption R2 does not hold, and the maturity of the security extends indefinitely $(m \rightarrow \infty)$. In this case, investor $i$ cannot use the terminal condition (10) even if he can form rational expectations far into the future. Without the use of the terminal condition, the investor cannot backward induct to arrive at the current value of the stock. Again, equation (9) describes the price, which is contingent on an undetermined terminal value. This price indeterminacy, arising from indefinite maturity, is called "rational bubbles" (see Blanchard and Watson 1982, and Tirole 1982, 1985).

While the security price level of rational bubbles is indeterminate, the deviation of price from the fundamentals (defined as a price bubble) must satisfy some conditions over time. Subtracting (2) from (9),

$$
\begin{align*}
& P_{t}-F_{t}=\frac{E_{t}\left(P_{t+k}\right)}{(1+r)^{k}}-\left[\frac{E_{t}\left(D_{t+k+1}\right)}{(1+r)^{k+1}}+\frac{E_{t}\left(D_{t+k+2}\right)}{(1+r)^{k+2}}+\ldots\right] \\
& =\frac{E_{t}\left(P_{t+k}\right)}{(1+r)^{k}}-\frac{E_{t}\left[\frac{\left(E_{t+k}\left(D_{t+k+1}\right)\right)}{(1+r)}+\frac{\left(E_{t+k}\left(D_{t+k+2}\right)\right)}{(1+r)^{2}}+\ldots\right]}{(1+r)^{k}}=\frac{E_{t}\left(P_{t+k}-F_{t+k}\right)}{(1+r)^{k}} \tag{12}
\end{align*}
$$

As this equation holds for any $k(\geq 1)$,

$$
\begin{align*}
E_{t}\left(P_{t+1}-F_{t+1}\right)= & (1+r)\left(P_{t}-F_{t}\right)  \tag{13}\\
& \ldots  \tag{14}\\
E_{t}\left(P_{t+k}-F_{t+k}\right)= & (1+r)^{k}\left(P_{t}-F_{t}\right)
\end{align*}
$$

That is, in a "rational bubble," the deviation of price from the fundamental value is expected to grow at the discount rate (see Flood and Hodrick (1990) survey).

To summarize, the representative investor model requires the rational expectations and the finite maturity assumptions to bring prices to equality with the fundamental value of a security. Without both these assumptions, backward induction from future dividends to fundamental value cannot be completed. Figure 1 shows that price $P_{t}$ is equal to the fundamental value $F_{t}$ only in a special case; otherwise the investors cannot backward induct from dividend expectations, and release prices from their dividend anchors. Heterogeneity of investors' information sets, expectations, and behavior makes the prices even more indeterminate.

## Heterogeneous Investor Model

In a market with heterogeneous investors, additional assumptions are necessary for the price to arrive at the fundamental value of the security through backward induction. Retaining the rational expectation assumption, the representative model story remains valid through equation (4). Heterogeneous investors ${ }^{2}$ may have differential information; the expectation of one generation of investors about the dividend expectation of another generation is not necessarily equal to the dividend expectation of the former. In other words, investor $i$ 's expectation at time $t$ of investor $(i+1)$ 's expectation at time $(t+1)$ of $D_{t+2}$ or $P_{t+2}$ may not be the same as his own

[^1]expectation of $D_{t+2}$ or $P_{t+2}$ and equations (5) and (6), and therefore equation (7) do not hold. $P_{t}$ is obtained by substituting (4) into (1).
\[

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)}{(1+r)^{2}}+\frac{E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right)}{(1+r)^{2}} . \tag{15}
\end{equation*}
$$

\]

Instead of $E_{t}\left(D_{t+2}\right)$ and $E_{t}\left(P_{t+2}\right)$ in equation (8), we have $E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)$ and $E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right)$ in equation (15). This means that the price of the security at time $t$ depends on the investor's expectations of other's expectations (second-order expectations). Therefore, in addition to the rational expectations assumption, we need an assumption about the investor's ability to form higher order expectations.

Assumption H1: Investors can form higher order expectations such as expectations of others' expectations (second-order), expectations of others' expectations of others' expectations (thirdorder), expectations of others' expectations of others' expectations of others' expectations (fourth-order), etc.

Equation (15) incorporates investor $i$ 's second-order expectations of dividend and price in period $(t+2)$. As we make further substitutions for expectations of future prices, higher order expectations of dividends and subsequent prices will appear in the price equation. To make such substitutions, we must specify how investors of generation $i$ expect the investors of $(i+1)$ and the subsequent generations to form their own expectations of dividends and prices. Expectation formation is private, and the investors of one generation do not necessarily know how investors of another generation form their expectations. A common knowledge assumption is needed to cope with this problem.

Assumption H2: Assumption R1 (that investors can form rational expectations) is common knowledge.

Assumption H3: Assumption H1(that investors can form higher order expectations) is common knowledge.

Under these two assumptions, investor $i$ at time $t$ knows that investor $(i+1)$ at time $(t+1)$ also forms rational expectations and second-order expectations as

$$
\begin{equation*}
E_{t+1}\left(P_{t+2}\right)=\frac{E_{t+1}\left(E_{t+2}\left(D_{t+3}\right)\right)}{(1+r)}+\frac{E_{t+1}\left(E_{t+2}\left(P_{t+3}\right)\right)}{(1+r)} \tag{16}
\end{equation*}
$$

Then, investor $i$ 's second-order expectation of $P_{t+2}$ is

$$
\begin{equation*}
E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right)=\frac{E_{t}\left(E_{t+1}\left(E_{t+2}\left(D_{t+3}\right)\right)\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(E_{t+2}\left(P_{t+3}\right)\right)\right)}{(1+r)} \tag{17}
\end{equation*}
$$

where $E_{t}\left(E_{t+1}\left(E_{t+2}(\cdot)\right)\right)$ is investor $i$ 's third-order expectation at time $t$. Substituting it into (15),

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)}{(1+r)^{2}}+\frac{E_{t}\left(E_{t+1}\left(E_{t+2}\left(D_{t+3}\right)\right)\right)}{(1+r)^{3}}+\frac{E_{t}\left(E_{t+1}\left(E_{t+2}\left(P_{t+3}\right)\right)\right)}{(1+r)^{3}} \tag{18}
\end{equation*}
$$

If investor $i$ can repeat this expectation formation process until he gets $k^{\text {th }}$ order expectation, we have

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(E_{t+1}\left(\ldots\left(E_{t+k-1}\left(D_{t+k}\right)\right) \ldots\right)\right)}{(1+r)^{k}}+\frac{E_{t}\left(E_{t+1}\left(\ldots\left(E_{t+k-1}\left(P_{t+k}\right)\right) \ldots\right)\right)}{(1+r)^{k}} \tag{19}
\end{equation*}
$$

$P_{t}$ still depends on $i$ 's $k^{\text {th }}$ order expectation of the price, $E_{t}\left(E_{t+l}\left(\ldots\left(E_{t+k-l}\left(P_{t+k}\right) \ldots\right)\right)\right.$. To end this recursive process, we need the finite maturity assumption $R 2$. However, $R 2$ is not sufficient in the heterogeneous investor model. We also need an assumption that the finite date of maturity is common knowledge.

Assumption H4: Assumption R2 (that the security matures at time ( $t+m$ )) is common knowledge among all investors.

Then, investor $i$ 's $(m-1)^{\text {th }}$-order expectation of $P_{t+m-l}$ is

$$
\begin{equation*}
\left.E_{t}\left(E_{t+1}\left(\ldots\left(E_{t+m-2}\left(P_{t+m-1}\right)\right) \ldots\right)\right)\right)=\frac{E_{t}\left(E_{t+1}\left(\ldots\left(E_{t+m-2}\left(E_{t+m-1}\left(D_{t+m}\right)\right)\right) \ldots\right)\right)}{(1+r)} \tag{20}
\end{equation*}
$$

Substituting (19) into (20), we arrive at investor $i$ 's value of the security through a series of ( $m-1$ ) backward inductions:

$$
\begin{equation*}
P_{t}=\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(E_{t+1}\left(D_{t+2}\right)\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(E_{t+1}\left(\ldots\left(E_{t+m-1}\left(D_{t+m}\right)\right) \ldots\right)\right)}{(1+r)^{m}} \tag{21}
\end{equation*}
$$

Note that the price given by equation (21) is different from the fundamental value $F_{t}$ in equation (2) because the former is based on the higher order expectations of investor $i$, and the latter is based only on his first-order expectations. Backward induction is possible without equality between the two sets of expectations, but it is not sufficient to make the price of the security equal to its fundamental value.

Figure 2 illustrates the price determination in a market consisting of investors who are heterogeneous across generations. As in Figure 1 for representative investors, the upper straight line denotes the determination of the security price through a sequence of the backward induction steps (BI (1), BI (2), BI (3), ..., Finite), when the security matures in a finite number of periods, and the assumptions R1-R2, and $\mathrm{H} 1-\mathrm{H} 3$ hold. However, this is only a special case among the possible investor expectation processes. The recursive backward induction process, (BI (1), BI (2), BI (3), ...) in Figure 1 demands only the formation of rational expectations; the process in Figure 2 makes far stronger demands of higher order expectations, including common knowledge of rational expectations, higher order expectations and the date of maturity. For example, in BI (2), investor $i$ needs not only to form his own rational expectation, $E_{t}\left(E_{t+1}\left(P_{t+2}\right)\right)$, but also to expect that the investors of the next generation $(i+1)$ also form a rational expectation $E_{t+1}\left(P_{t+2}\right)$. In BI (3), in addition to forming $E_{t}\left(E_{t+1}\left(E_{t+2}\left(P_{t+3}\right)\right)\right)$, investor $i$ must also expect that investor $(i+1)$ forms a rational expectation $E_{t+1}\left(E_{t+2}\left(P_{t+3}\right)\right)$ and that investor $(i+2)$ forms a rational expectation $E_{t+2}\left(P_{t+3}\right)$. In the latter stages of this process, the investor needs to form
higher order expectations rationally, and to have faith in the ability of the succeeding generations of investors, who he may not know and who may not even exist yet, to do the same.

Whether investors can and do form rational expectations, assuming others form similar expectations and conduct similar backward inductions is an open issue. Arrow (1986) argues that assuming not only each agent's rationality but also his knowledge of other agents' rationality is incompatible with the cognitive limits of the human being. We are unsure about the extent of the investors' ability to form higher order expectations. If a security matures in twenty periods ( $\mathrm{m}=$ 20), investors would have to form $1^{\text {st }}$ to $20^{\text {th }}$ order expectations of future dividends, to believe that investor $(i+1)$ forms $1^{\text {st }}$ to $19^{\text {th }}$ order expectations, to believe that investor $(i+1)$ believes that investor ( $i+2$ ) forms investors form $1^{\text {st }}$ to $18^{\text {th }}$ order expectations, and so on. This kind of expectation formation might be possible for securities whose future cash proceeds are almost certain or easily predictable (e.g., treasury notes); for equity shares of publicly traded firms, such conditions are not likely to be fulfilled. In the stock markets it is quite possible that the cognitive demands of such a valuation process exceed the individual human capacity, so as to preclude the security prices from approximating their fundamental values. If security prices are no longer anchored by expected future dividends, they may become indeterminate and form bubbles.

## Uncertainty, maturity, and duration of stocks and price bubbles

Both the representative and heterogeneous investor models suggest that the standard textbook treatment of securities valuation to its fundamentals requires strong restrictive assumptions about maturity, investors' expectation formation, and common knowledge. The difficulty of fulfilling these assumptions raises the possibility of prices deviating from fundamental values (bubbles). These models can also yield insights into the kinds of securities that are susceptible to bubbles.

First, securities with longer maturities should be more susceptible to bubbles because their valuation requires higher orders of investor rational expectations, common knowledge about such expectations, as well as the backward induction process-all less likely as the maturity of the security stretches into the future. As much as investors may themselves form rational expectations, their willingness to attribute such expectations to other current or future investors cannot be taken for granted. Further, given their limited cognitive ability, forming expectations far into the future, rational or otherwise, cannot be taken for granted; common knowledge of rational expectations formation; and backward induction processes far into the future is even less plausible. The longer the maturity of the security, the more likely it is that the investors simply abandon their attempt to conduct backward induction through multiple layers of expectations.

Second, long duration securities are more susceptible to bubbles ${ }^{3}$. Consider two components of price $P_{t}$ in equation (9), the discounted dividends part $\left(\frac{E_{t}\left(D_{t+1}\right)}{(1+r)}+\frac{E_{t}\left(D_{t+2}\right)}{(1+r)^{2}}+\ldots+\frac{E_{t}\left(D_{t+k}\right)}{(1+r)^{k}}\right)$ and the discounted expected future price part $\left(\frac{E_{t}\left(P_{t+k}\right)}{(1+r)^{k}}\right)$. As the duration of the security increases, the second part becomes a larger fraction of the price. The susceptibility of this part of the price to instability due to the failure of backward induction suggests that the prices of securities with longer durations are more likely to deviate from the fundamentals.

Third, bubbles are more likely to occur when the future dividend has higher uncertainty. In such cases, it is difficult for the investors to conjecture what others think (and what others

[^2]think about what others think) about the future prospects of dividends. It becomes more likely that they fail to backward induct and the prices lose their dividend anchors.

These implications are consistent with casual observations of stock market behavior. Historically, price bubbles have often been attributed to securities with longer maturity or duration, and greater uncertainty about the fundamentals (see Blanchard and Watson 1982 and Ackley 1983). During the 1920s, RCA, a new technology firm that had never paid dividends, saw a much greater rise and fall in its share price than the railroad industry with its established technology and dividend record. Similar anecdotes have recently been told the "dotcom bubble." Our models explain why high-growth and new technology equity securities should be more susceptible to bubbles: it should be more difficult for investors to backward induct the value of such securities from dividend expectations; absent backward induction, prices lose their dividend anchors and become indeterminate.

The ability of human traders to conduct backward induction under various conditions is both a logical as well as an empirical issue. We designed and conducted the following laboratory experiment to explore the empirical relevance of our theoretical model and its predictions.

## 3. The Experimental Design

We created double auction markets for multiple units of a security on a computer network in a laboratory. The security paid a single liquidating dividend to its holders at the end of its life, which was divided into many trading periods of three minutes each. Participating subjects were randomly assigned to one of two roles-investors and predictors. Each investor was endowed with 10 securities and 10,000 points of "cash" at the beginning of period 1 , and could trade freely through the multiple periods without going short on securities or "cash." At the end of the session, the securities held by investors were liquidated by paying them either a dividend or a
predicted price (as described later under the Main Treatment). The investors could make money through trading and terminal liquidation of their securities.

The predictors studied all the instructions given to the investors; they did not get endowments of cash or securities, could not trade, and only knew the range of the traders' terminal dividends. At the end of each period they were asked to predict the average price of the security transactions for the following period. Their earnings decreased with the absolute difference between their prediction and the actual average transaction price for the period ( N minus absolute prediction error; see Table 2 for value of N ). In addition to these earnings, all subjects earned $\$ 3$ (in Sessions 1-6) or $\$ 5$ (in Sessions 7-11) if they arrived in the laboratory punctually.

## Main Treatment: Exogenous or Endogenous Terminal Payoff

In five sessions (numbered chronologically $3,4,5,6$, and 7 in Table 1), the investors were informed that there would be 15 periods in the session, and that their securities would pay a terminal dividend (pre-written on their respective cards). This environment is designed to correspond to the topmost line in Figures 1 and 2, and we expected that it would make it relatively easy for the subjects to backward induct the fundamental value of the security. We call these exogenous terminal payoff sessions.

In six sessions (numbered $1,2,8,9,10$ and 11 in Table 1) the security paid a terminal payoff, which was determined endogenously as the average of the predictors' forecasts. The subjects were informed that the session would last for no more than 30 periods of trading of three minutes each. They were told that if the session were to end in period 30, their security holding would be liquidated by paying the dividend pre-written on their respective cards. However, they were also informed that "in the likely case" their session ended earlier (at the experimenter's discretion) their security holdings would be liquidated at the average transaction price predicted
for the period immediately following the termination. In other words, the average predicted price would replace the pre-written dividend as the liquidation value of their securities. Although they were not informed about the actual number of periods in the session until it was terminated, they would have estimated that the session for which they had been recruited into the laboratory session would end well before Period 30. Under this treatment, the security pays an endogenously determined terminal payoff instead of an exogenously specified dividend. Forming higher order beliefs for all these periods, and backward inducting their way through them to estimate the terminal value of the security at an uncertain terminal date was a challenging task. This treatment was intended to create conditions approximating the lower branches in Figures 1 and 2.

## Robustness Variations

As shown in the five sections of Table 1, the main treatment of endogenous-exogenous terminal payoff was supplemented by five variations to examine the robustness of the main treatment to other plausible experimental conditions.

## 1. Potential inequality between the first and higher-order beliefs;

In Sessions 1 and 8 (for endogenous treatment) and Sessions 3, 5, 6, and 7 (for exogenous treatment) there existed a potential for a gap between the first-order (fundamental) and the second (or higher) order beliefs of investors. For example, as shown in Table 2, dividend cards distributed in Sessions 1 and 8 had a terminal dividend of 40 points for two investors, and 75 points for the other two investors. It was publicly announced to all traders and predictors that none of the dividend numbers on the cards given to the investors lay outside the 10-300 point range. The information about this range had some chance of creating a non-zero subjective probability in the minds of investors that the other investors may have dividends as high as 300 points, even though they knew with certainty that their own dividend, either 40 or 75 points, was
well below this limit. Similar subjective distribution may have been induced among the predictors. Unlike Sessions 1, 3, 5, 6, 7 and 8, there was no gap between the first and second (or higher) order beliefs of the investors in Sessions 2, 4, 9, 10 and 11. For example, in sessions 2 and 9 , two or three investors were given cards informing them that their own dividend was 70 points while another three had 130 points as their dividend. It was publicly announced to subjects that none of the investor dividends lay outside the 70-130 point range.

## 2. Hetrogeneity of pre-determined dividends:

In Sessions 1-5 and 7-9, the terminal dividends written on the cards given to the traders were not identical across traders (e. g., 40 for two traders and 75 for two traders in Session 1). In contrast, the dividends were identical for all the traders in Sessions 6, 10 and 11.

## 3. Common knowledge of predetermined dividends;

In Sessions 10 and 11, the predetermined dividends written on the trader cards were made common knowledge through a public announcement. In all the other sessions, the predetermined dividends on the cards given to the traders were private knowledge.

## 4. Verification of understanding of instructions:

In Sessions 4-11, the instructions were followed by a written questionnaire and answer sheet to the questionnaire to help the subjects and the experimenter verify the former's understanding of the instructions and procedures. In addition, Sessions 6-11 included a review of each subject's answers by the experimenter, and an explanation of the relevant part of the procedures in case of any errors in the subject's answers.

## 5. Subjects paid by absolute or relative performance:

In Sessions 1-7, points earned by the subjects were converted into US dollars at a rate announced during the course of the instructions. In Sessions 8-11, the total dollar amount to be
paid to the n traders (and to the m predictors) was announced at the outset. This amount was allocated to individuals in proportion to the number of points earned in the session.

These five robustness variations, as well as the main treatment, are summarized in Table 1.

## Experimental Procedures

The experimental procedures common to all markets were as follows. We summarize information about the eleven sessions in Table 2. Each session consisted of some 12 to 17 periods, and each period consisted of three minutes of trading, succeeded by a one or two minute break for paperwork. At the start of the session, each investor received 10 shares and "cash" of 10,000 points. The investors could buy/sell securities if they had cash to pay for them or shares to sell. Short sales were prohibited. Securities and cash were carried over from one trading period to the next. The endowment of securities or cash was not replenished.

Before a session started, each investor drew a Dividend Card, which showed his/her terminal dividend per share. The investor would receive this amount at the end of period 30 in an endogenous terminal payoff treatment (see Table 1) only if the session were to last for 30 periods. In the exogenous terminal payoff treatment (see Table 1), this would be the actual terminal dividend at the end of the last period announced at the outset of the session. This personal dividend per share was each investor's private information (except that it was common knowledge in Sessions 10 and 11). They were told that the dividend might not be the same for all investors, and that the personal dividends of investors lay within the publicly announced range.

The final earnings of each investor were equal to the cash balance at the end of the last period's trading, plus the end-of-session payoff, minus the initial cash provided at the beginning of the session. In the endogenous terminal payoff treatment, the end of session payoff was
[average predicted price $\times$ the number of shares he or she hold at the end of the session] if the session ended before period 30 (this always was the case); and it was [his or her dividend per share on the Dividend Card $\times$ the number of shares he or she held at the end of the session] if the session lasted for 30 periods (this was never the case). In the exogenous terminal payoff treatment, the end-of-session payoff was [his or her dividend per share on the Dividend Card $\times$ the number of shares he or she held at the end of the session]. With the exception of Sessions 811 which used relative performance evaluation, the investor's final earnings in all other sessions were converted from points into US dollars at a predetermined rate, and paid in cash at the end of the experiment (along with a $\$ 3$ or $\$ 5$ punctuality bonus).

Trading was by continuous double auction, implemented with the Caplab ${ }^{\text {TM }}$ software. Each investor was free to make bids (proposals to buy shares) and asks (proposals to sell shares) by entering the price and quantity through their mouse and keyboard during trading periods. The computer reflected the number of shares he or she had, cash balance, market bid and ask price, and the price of the most recent transaction (see Instruction Set 2 for Trading Screen Operation in the Appendix).

All the sessions had predictors as well as investors. At the beginning of the session, each subject's role (investor or predictor) was determined by lots. The predictors guessed the prices at which the investors traded securities during each period. At the end of each period, they were asked to predict the average stock price of the following period by writing it down on their Price Prediction Sheet. The experimenter gathered this information before starting trading for the period. At the end of each period, the experimenter wrote the predicted price (averaged across all the predictors) on the board for all to see. The predictors' profits depended on the accuracy of his or her prediction. For each period, they earned [Constant N - the absolute difference between the
prediction and the actual trading price] points. If this value was negative, they earned zero points for the period. Constant N was the same for all the predictors in one session, but differed across sessions. Their total earnings for all periods were converted from points into US dollars at a predetermined rate known to them (except in Sessions 8-11 that used relative performance evaluation).

The sequence of activities in a session was as follows: (1) Instruction Sets (general instructions, investor instructions, predictor instructions, and trading screen instructions) were distributed and read out aloud. The subjects could ask questions at any time. (2) All the subjects participated in the trial session (2-3 rounds) until they got used to the trading screen operation using Caplab ${ }^{\mathrm{TM}}$. (3) Each subject drew a slip of paper from a bag that determined his or her role.
(4) Each investor randomly picked a Dividend Card on which his or her dividend was written.
(5) Trading period 1 of the session began and was followed by other periods.

Additionally, in Sessions 5-11, Step (2) above was followed by (2A), consisting of a questionnaire to the subjects testing their understanding of the rules, followed by the distribution of a written answer sheet to the questions asked. In Sessions 6-11, Step (2A) was followed by Step (2B) in which the experimenter read each subject's written answers, and explained the relevant rules to the subject whenever the answers were wrong.

This papers reports on all eleven experimental sessions shown in Tables 1 and 2. The sessions were held at Yale University in the fall of 2001 through the summer of 2002. All the subjects were undergraduate students, who had not participated in any previous research experiments with stock markets. Each session lasted, on the average for two hours and 30 minutes.

## 4. Experimental Results

Figures 3-13 show the price and allocation data from the eleven laboratory sessions. Each figure shows the time series of transaction prices (black diamond markers) with the average price for each period written at the top of the chart. The dotted line shows the average predicted price for the period and the thick solid line indicates the market equilibrium price based on the fundamental value of the shares. The market equilibrium price is the higher of the two dividend values in the heterogeneous dividends sessions (e.g. 150 in Session 3) and the unique dividend value in the homogenous dividends sessions (e.g. 75 in Session 11). The thin solid line shows the upper limit of the publicly announced range of dividends (300 in session 3), which is also, presumably, the upper limit of the investors' and the predictors' second (or higher) order beliefs about dividends.

The small dots plotted against the $y$-axis on the right hand scales track the allocation of securities relative to the initial endowment ( 0 percent) after each transaction. If all the securities were to be transferred to the investors who had the higher dividend (fundamental value) on their cards, the allocative efficiency would be 100 percent; if all the securities were to be transferred to the investors who had the lower dividend on their cards, the allocative efficiency would be 100 percent.

For example, in Session 1 (endogenous terminal payoff treatment, Figure 8) the transaction prices (black diamonds) remained in the $80-85$ range throughout the session and stabilized at around 83 , about 10 percent above the fundamental value of 75 (thick solid line), but well below the upper limit of the potential second-order beliefs of investors (thin solid line at 300). The average predicted price remained close to 83 throughout (dotted line). The allocation of securities between the high and the low dividend investors hovered near the initial allocation
for the first six periods, and then the higher dividend investors steadily bought all but 1 of the 40 securities by the end of the $12^{\text {th }}$ period.

We organize the data around five results.
Result 1: When it is easy for the investors to backward induct the value of the security, the security prices converge to the equilibrium level derived from the fundamental values of individual investors.

In the exogenous terminal payoff sessions (Sessions 3, 4, 5, 6, and 7) all the traders knew that there would be 15 periods in the session and a terminal dividend would be paid at the end of the session. Therefore, they could backward induct the value of the security easily. In this zerodiscount rate and ample liquidity environment, the fundamental (equilibrium) price for all fifteen periods of these sessions is equal to the highest dividend across traders. Figures $4-7$ show that the prices in these markets are determined by this highest dividend, and they converge to the fundamental level. A bubble was observed in Session 3 (Figure 3); it did not reappear in the subsequent sessions (Sessions 4-7) after the experimental instructions were modified to include a questionnaire to test the subject comprehension of the experimental instructions, and additional instruction as necessary. In these four sessions, the prices exhibit a strong tendency to converge to the fundamental value. ${ }^{4}$

Biais and Bossaerts' (1998) model predicts that price bubbles should be observed when there is a possibility that investors' higher order beliefs differ from their first order beliefs. We

[^3]can think of two reasons why we did not observe bubbles in Sessions 5, 6, and 7 in which the first and higher-order beliefs might have diverged. First, it is possible that investors traded conservatively. They knew that the only dividend they were to receive was the liquidating dividend at the end of the session. They did not have to speculate and develop buy-and-resell strategies based on others' beliefs about dividends. In Allen, Morris, and Shin's (2002) model risk averse investors engage in such behavior. Second, it is possible that we did not succeed in our attempt to induce our laboratory subjects to develop divergent first and higher order beliefs about dividends. We asked the subjects "not to assume that other subjects have the same dividends as their own" and announced a range containing all investors' dividends, which was wider than the actual distribution of dividends. Perhaps this procedure does not necessarily induce divergent first and higher order beliefs.

The bubble did not occur after the experimental procedures were adjusted to test and correct any misunderstandings subjects may have had about the rules of the market in the exogenous terminal payoff sessions. Lei et al. (2001) also attribute the bubbles they observed in the laboratory to the subjects' lack of understanding of the structure of the asset, the nature of the task, or the opportunities available to them. A bubble was observed in Session 3 before we started testing the subjects' understanding of the rules; no bubbles were observed in the later four sessions where we used questionnaires (Session 4 and 5) and verified and corrected any misunderstanding of the rules (Session 6 and 7). Lack of understanding of the instructions is a laboratory artifact that must be carefully guarded against in bubble experiments.

The data for Sessions 4, 5, 6 and 7 (but not from Session 3) provide a strong support for Result 1.

Result 2: When it is difficult for investors to backward induct the value of the security,
a) Prices may deviate from the fundamental values to form bubbles, and

## b) Prices are indeterminate and free-floating.

Figures 8-13 show that, in contrast to the exogenous terminal payoff treatment of Sessions 3-7, Sessions 1, 2, 8-11 exhibit a strong tendency to generate bubbles. In these sessions, nominally 30 periods long, the subjects were informed that the sessions were likely to end earlier at the experimenter's discretion. The dividends were to be used for the liquidation of securities only if the session ended in period 30 ; otherwise the mean predicted price for the period immediately following the last period was to replace the liquidating dividend. Given the time constraint in all these sessions, it was quite clear to the subjects that the sessions would end well before period 30 , and they did. The dividend numbers seem to have played a role in the determination of prices in the early periods of some of these sessions (e.g., Sessions 1, 8 and 10). However, as these sessions progressed (see Figures 8-13) the prices often ceased to have any meaningful relationship with the dividends (they were about 10 to 900 percent higher or lower than the equilibrium price based on liquidating dividend values). Given the high probability of these sessions ending before period 30, and the difficulty of backward inducting the value of the security from the dividends as far as 30 trading periods away, the dividends ceased to play a role in the determination of prices in the security markets. Also, we should note that the possibility of a gap between the first and the second-order beliefs in Sessions 1 and 8 does not seem to affect the generation and magnitude of bubbles. We observed a huge bubble in Sessions 2 and 10, in which we made no attempt to induce a divergence between the first and the second (higher) order beliefs.

Both the levels and the paths of prices are indeterminate in all six sessions with the endogenous terminal payoff treatment. In Session 1 (Figure 8), both the transaction as well as the average predicted prices settled down to about 83 early on, and stayed there throughout at a level about 10 percent above the fundamental value. Similarly, in Session 11 (Figure 13), prices and
predictions settled down in the neighborhood of 50 , about 33 percent below the fundamental value, and stayed there. Prices in both these sessions conform to the definition of bubbles, one positive and the other negative. Both were relatively stable, with the transaction prices and the price predictions reinforcing each other. In Session 1, there was no reason for any investor to pay a price of 83 for these securities except on the basis of the expectation that at the end of the session he/she will get the average predicted price, not the dividend written on the card. A similar argument applies to Session 11.

In Session 2 (Figure 9), prices started out low, and increased slowly until an explosive spurt from the middle of period 3 to the middle of period 4. It settled noisily around 900 , almost seven times the fundamental price, until the end of the session. The fundamental value (130) hardly played any role in determining the transaction prices. Expectation of the future prices in the market seems to take over as the primary determinant for prices. The investors' expectations of high prices in the future seemed to sustain the realization of high prices. We have no explanation as to why the price rose to the level of 900-1000 in the first place, and then stopped rising. Once prices reached that level, and the predictors started predicting that price, there was little reason for the prices to return to their fundamental levels. Period 30, where dividends would have become the determining factor, was still too remote a possibility when the session ended in period 15.

In Session 10 (Figure 12), we also observed a huge bubble. In this session, the liquidating dividend (150) was common knowledge among all investors. However, the dividend seemed to play an anchoring role only during the early periods. In period 3-5, the price rose around 200 and continued to rise, even accelerating in periods 9-10, reaching above 1500 in periods $13-14$, about ten times as the fundamental value, before falling back about 100 points in period 15 .

It is interesting to note the dynamics of transaction and predicted prices. In period 10 the average price prediction was 342 and the actual transactions averages at 390 . In period 11 the predictions rose to 465 , and the transactions rose to 520 . In period 12 the predictions and the transactions rose to 815 and 995 respectively. Finally the price prediction rose to 1,500 in period 13 when the actual average price was 1526. Increases in the transaction and the predicted prices seem to reinforce each other. Shiller (2000) argues that this "amplification" mechanism is created by investors' psychological factors and emotions ${ }^{5}$. Our data suggest an alternative driver for the amplification mechanism-the investors difficulty in backward inducting the value of securities.

Sessions 8 and 9 (Figures 10 and 11), on the other hand, exhibit a slow and steady growth of prices through most of the session. The growing prices we observe in this session are not consistent with rational bubbles prediction of stable prices in a zero discount rate environment that obtained in the laboratory. In these two sessions, we observe the amplification mechanisms as well. It seems that the actual price increases are followed by the increases in the price predictions, which in turn help raise the transaction prices ${ }^{6}$.

In the endogenous terminal payoff sessions investors find it difficult to backward induct the value of the security, and neither the first nor the higher order beliefs about dividends can help anchor the prices; the prices become indeterminate and float freely. Investor anticipation of the future prices becomes the sole determinant of current prices. In the absence of any other

[^4]anchors, investor anticipations themselves seem to depend on the past prices. In other words, investors and predictors may form their expectations of future prices through forward, not backward, induction.

Result 3: When it is difficult for investors to backward induct the value of the security, price expectations are consistent with forward induction; when backward induction is easy to do, price expectations are consistent with backward induction.

We explore how the price predictions are formed by comparing a backward induction (or the fundamental) model and two forward induction models. First consider the investors who backward induct price from the future dividends through a statistical adjustment process:

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)=P_{t}+\alpha\left(D-P_{t}\right) \quad 0<\alpha \leq 1 . \tag{22}
\end{equation*}
$$

D is the terminal dividend value (the largest dividend in heterogeneous dividends sessions) and $\alpha$ is the adjustment coefficient. We can contrast the backward induction forecast process (22) with two candidates for forward induction, an adaptive expectations model (23), and a trend expectations model (24):

$$
\begin{align*}
& E_{t}\left(P_{t+1}\right)=E_{t-1}\left(P_{t}\right)+\beta\left(P_{t}-E_{t-1}\left(P_{t}\right)\right) \quad 0<\beta<1,  \tag{23}\\
& E_{t}\left(P_{t+1}\right)=P_{t}+\gamma\left(P_{t}-P_{t-1}\right) \quad \gamma \neq 0 . \tag{24}
\end{align*}
$$

We can compare each of the two forward induction models against the backward induction model, one at a time. To compare the fundamental and the adaptive models, we can write them as:

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)-E_{t-1}\left(P_{t}\right)=a_{1}\left(P_{t}-E_{t-1}\left(P_{t}\right)\right)+a_{2}\left(D-P_{t}\right)+u_{t} \tag{25}
\end{equation*}
$$

where $a_{1}=1$ and $0<a_{2}<1$ would support the fundamental model and $0<a_{1}<1$ and $a_{2}=0$ would support the adaptive model. Similarly, to compare the fundamental and the trend models, we can write them as:

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)-P_{t}=b_{1}\left(P_{t}-P_{t-1}\right)+b_{2}\left(D-P_{t}\right)+v_{t} \tag{26}
\end{equation*}
$$

where $b_{1}=0$ and $0<b_{2}<1$ would support the fundamental model, and $b_{1} \neq 0$ and $b_{2}=0$ would support the trend model. We estimate these models with the prediction data from the exogenous and endogenous terminal payoff sessions.

For the exogenous terminal payoff sessions ( $3,4,5,6$ and 7 ), the estimated coefficients support the backward induction model over both the forward prediction models:

Fundamental vs. Adaptive model (standard errors shown in parentheses)

Since $a_{1}=1$ and $0<a_{2}<1$, the data support the fundamental model and the adaptive model is not supported.

Fundamental vs. Trend model (standard errors shown in parentheses)

Since $b_{1}=0$ and $0<b_{2}<1$, the data support the fundamental model and the trend model is not supported.

On the other hand, for the endogenous terminal payoff sessions (1, 2, 8, 9, 10 and 11), the estimated coefficients support the trend model over the fundamental model; the results of the comparison of the adaptive and the fundamental model are ambiguous:

Fundamental vs. Adaptive model (standard errors shown in parentheses)

$$
E_{t}\left(P_{t+1}\right)-E_{t-1}\left(P_{t}\right)=\underset{(0.154)}{1.212}\left(P_{t}-E_{t-1}\left(P_{t}\right)\right)-\underset{(0.038)}{0.087}\left(D-P_{t}\right) \quad \text { Adjusted } \mathrm{R}^{2}=0.66, \mathrm{~N}=166 .
$$

Since $a_{1}=1$ and $a_{2}<0$, neither the fundamental nor the adaptive model fits the data well.
Fundamental vs. Trend model (standard errors shown in parentheses)

$$
E_{t}\left(P_{t+1}\right)-P_{t}=\underset{(0.5012)}{0.508}\left(P_{t}-P_{t-1}\right)-\underset{(0.028)}{0.024\left(D-P_{t}\right) \quad \quad \text { Adjusted } \mathrm{R}^{2}=0.43, \mathrm{~N}=166 . . . ~ . ~}
$$

Since $b_{1} \neq 0$ and $b_{2}=0$, the data support the trend model and the fundamental model is not supported.

The above results indicate that in the endogenous terminal payoff sessions price expectations are formed by forward, not backward, induction. To judge whether investors (predictors) use the adaptive, the trend, or some combination of the two models to form their expectations through forward induction, we also consider a hybrid model (27) in which the first two terms represent an adaptive process and the third term represents a trend process:

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)=\beta P_{t}+(1-\beta) E_{t-1}\left(P_{t}\right)+\gamma\left(P_{t}-P_{t-1}\right) \quad 0<\beta<1, \text { and } \gamma \neq 0 . \tag{27}
\end{equation*}
$$

To compare the validity of these three forward induction models, we estimate the following equation.

$$
\begin{equation*}
E_{t}\left(P_{t+1}\right)-E_{t-1}\left(P_{t}\right)=c_{1}\left(P_{t}-E_{t-1}\left(P_{t}\right)\right)+c_{2}\left(P_{t}-P_{t-1}\right)+w_{t} \tag{28}
\end{equation*}
$$

$0<c 1<1$ and $c 2=0$ support the adaptive model, $c 1=1$ and $c 2 \neq 0$ support the trend model, and $0<c l<1$ and $c 2 \neq 0$ support the hybrid model. The estimation of equation (28) with the data from endogenous Sessions 1-2 and 8-11 yields (standard errors shown in parentheses)

$$
E_{t}\left(P_{t+1}\right)-E_{t-1}\left(P_{t}\right)=\underset{(0.164)}{0.590\left(P_{t}-E_{t-1}\left(P_{t}\right)\right)+\underset{(0.149)}{0.770}\left(P_{t}-P_{t-1}\right) \quad \text { Adjusted } \mathrm{R}^{2}=0.81, \mathrm{~N}=166 . . . ~ . ~}
$$

Since $0<c_{1}<1$ and $c_{2} \neq 0$, the data support the hybrid model. The investors seem to use a combination of adaptive and trend models to do forward induction when they find it difficult to do backward induction.

Result 4: Allocative efficiency is high when backward induction is easy to do, and unpredictable when backward induction is difficult.

Allocative efficiency (the percent of securities transferred toward the fundamental value equilibrium allocation) for Sessions 1-5 and 7-9 is plotted in Figures 3-5 and 7-11 with small dots on the right hand scale ${ }^{7}$. During the three exogenous terminal payoff sessions (Sessions 4, 5 and 7) almost all the securities ended up in the hands of the high fundamental value traders, and the allocative efficiency approached 100 percent at the end of each session. In Session 3, the efficiency hovered around zero during the bubble phase (Periods 1-9). The bubble collapsed in the last five periods as the securities were transferred to the high-dividend investors ${ }^{8}$.

The bubble economies of endogenous terminal payoff Sessions 8 and 9 had negative efficiencies (the securities transferred to low-dividend investors), and Session 2 had an efficiency of only 20 percent. When the dividends are replaced by endogenously determined predicted prices as the terminal payoff, there is no reason to expect either high or low efficiencies in the market. Every trader would get the same terminal payoff from holding the security, and the market exerts no pressure to make the allocation of resources more efficient as defined by the the ultimate (period 30) dividends. However, in Session 1, the efficiency rose to almost 100 percent. Since the investors could be reasonably sure that they will get the average predicted price that hovered around 83 through virtually the entire session, there was no pressure for the securities to be transferred to the higher-dividend investors. In fact, 39 of the 40 securities were held by one of the two high-dividend investors at the end of Period 12. One low-dividend investor held one security while one high-dividend and one low-dividend investor held no securities. It is

[^5]therefore plausible that the transfer of securities to one high dividend investor could have been the outcome of idiosyncratic trading strategies of the investors, and the securities could just as easily have ended up in the hands of any of the other three investors.

Uncertain allocative efficiency of bubble-prone markets is a strong argument for public policies aimed at discouraging the formation of price bubbles. Bubbles also have significant distributive consequences in the form of increasing the dispersion of wealth among agents.

Result 5: The cross-sectional dispersion of investor wealth increases with the size of bubbles.
The adjusted the profits of individual traders (= individual trader's profit in points / the fundamental value of the initial endowment of 10 shares - the cross-sectional average of this ratio) for each of the eleven sessions are shown in Figure 14. The standard deviations of the adjusted profits are shown in parentheses under the session numbers.

Of the five exogenous terminal payoff sessions, only Session 3, which had a large price bubble, shows a significant cross-sectional dispersion of profits (the standard deviation of the adjusted profits is 3.88 ). Dispersion in all other sessions is close to zero. Of the six endogenous terminal payoff Sessions 1 and 11 had small but stable bubbles; and the dispersion of adjusted profits is these sessions is close to zero. In the other four sessions we observed large price bubbles, and all of them show large dispersion of individual profits. The data suggest that the dispersion of wealth increases with the magnitude of the price bubble.

## 5. Discussion and Concluding Remarks

Investors' difficulty in using backward induction to arrive at the fundamental value of securities gives rise to price bubbles. When the realization dividend cash flows that form the basis for fundamental valuation is distant, vague, and well beyond their investment horizon,
investors find it difficult, even impossible, to use dividends as anchors for backward inducting the fundamental value of securities. Such backward induction involves working through the higher order beliefs of the future generations of investors who might hold the securities between their sale by the current investors and the ultimate realizations of cash through dividends or liquidation. Cognitive demands of forming higher order beliefs, and backward induction through multiple stages seem to be beyond the conscious capabilities of most people. When decoupled from the future realization of cash, prices can float free of the fundamental value to form bubbles.

When backward induction becomes difficult or impossible, investors may resort to forward induction, and adjust their expectations on the basis of observed prices. Such forward induction adds volatility to prices when the markets are open, as suggested by French and Roll (1986).

Keynes suggested a newspaper beauty contest metaphor for security price bubbles. It is difficult to verify the empirical validity of such a story with data from the field due to the unobservability of investor beliefs. We created laboratory markets where we could control the fundamental valuation. We populated these markets with investors and predictors, and controlled the terminal payoff of securities so we could examine the consequences of varying the investor difficulty in backward induction of the fundamental value from the future dividends.

In the eleven laboratory sessions of this experiment, we observe that when it becomes difficult for the investors to backward induct the fundamental value of the security from the future dividends, the security prices become susceptible to bubbles. In contrast, when backward induction is possible and easy to do, security prices converge to the fundamental value equilibrium. Second, bubble prices are indeterminate, exhibiting free-floating character. Without a future anchor of dividends from which they could backward induct, investors tend to forward
induct in bubble markets, using an approximation of first-order adaptive or trend process. Third, the allocative efficiency of markets is high in the presence of dividend anchors and becomes indeterminate in absence of anchors. Since the efficient allocation of capital, not gambling, is supposed to be the social function of security markets, it is understandable that policy makers see bubbles as a public enemy. Evidence on the tendency of bubbles to increase the cross-sectional dispersion of wealth serves as another reason for policy intervention in markets.

Lei et al. (2001) modified the Smith et al. (1988) experiment by eliminating the possibility of speculative trades, and found that the bubble phenomenon reported in the Smith et al. study persisted. They attribute the bubbles in their experiment to a lack of understanding on the part of subjects about the structure of the asset or the nature of the task and opportunities facing them. They point to the possibility of a lack of correspondence between the experimenter's definition and the subjects' beliefs about the experimental task and environment. In complex tasks where decisions depend critically on agent beliefs, experimenter must ensure that the subjects' beliefs correspond to those postulated in the experimental design. In our sessions 4-11, we used questionnaires to verify this correspondence and such lack of correspondence is not a likely source of bubbles in those markets. Instead, they arise from the virtual impossibility of forming and backward inducting through many orders of beliefs in appropriate circumstances.

The Implications of our model and experimental results include the influence of investment horizon, maturity and duration on the likelihood and severity of bubbles when they do occur. If traders with short investment horizons-day traders-dominate the market, bubbles are morel likely. Securities with longer time horizons should be more susceptible to bubbles because, other things being equal, any investor is more likely to liquidate his holdings at market price for longer horizon securities, increasing the significance of higher-order beliefs relative to
first-order beliefs in valuation of the security. A similar argument is applicable to the duration of securities.

Shiller (1981) concludes that security prices are too volatile, given the fundamentals. Blanchard and Watson (1982) argue that bubbles are more likely to occur when there is greater uncertainty about the fundamentals. The securities of firms with new technologies and high growth prospects are believed to be more bubble-prone than the firms with established technologies and dividend records. French and Roll (1986) observed that the volatility is greater when the markets are open, and market trading itself seems to create volatility. Our model and experimental results put this work and observations in a consistent framework.

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Table 1: Experimental Design

|  |  | Main Treatment: Terminal Payoff |  |
| :---: | :---: | :---: | :---: |
| Robustness Variations |  | Endogenous | Exogenous |
| Subsidiary Treatment 1: <br> Potential inequality of the first and higher order beliefs about dividends | Potential for a gap between first and higher order beliefs | Sessions 1, 8 | Sessions 3, 5, 6, and 7 |
|  | Equality between first and higher order beliefs | Sessions 2, 9, 10, and 11 | Session 4 |
| Subsidiary Treatment 2: Heterogeneity of pre-determined dividends | Non-identical pre-written dividends | Sessions 1, 2, 8, and 9 | Sessions 3, 4, 5, and 7 |
|  | Identical pre-written dividends | Sessions 10 and 11 | Session 6 |
| Subsidiary Treatment 3: Common Knowledge of pre-determined dividends | Dividends common knowledge | Sessions 10 and 11 |  |
|  | Dividends not common knowledge | Sessions 1, 2, 8, and 9 | Sessions 3, 4, 5, 6, and 7 |
| Subsidiary Treatment 4: Verification of proper understanding of the instructions | No questionnaire, answer, verification and correction | Sessions 1, and 2 | Session 3 |
|  | Questionnaire, answer, No verification and correction |  | Sessions 4 and 5 |
|  | Questionnaire, answer, verification and correction | Sessions 8, 9, 10, 11 | Sessions 6 and 7 |
| Subsidiary Treatment 5: Subjects paid by absolute or relative performance | Payoff based on absolute performance | Sessions 1 and 2 | Sessions 3, 4, 5, 6 and 7 |
|  | Payoff based on relative performance | Sessions 8, 9, 10, 11 |  |

Table 2: Experimental Design

| $\stackrel{\lambda}{i}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \cong \\ & \stackrel{0}{\mathrm{Z}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 09/21/01 | 1 | Predicted Value | 40 for 2 <br> 75 for 2 | [10, 300] | 10 | 10,000 | 4 | 2 | 150 | 0.025 | 12 | $\leq 30$ | No | No |  |
| 09/29/01 | 2 | Predicted Value | $\begin{gathered} 70 \text { for } 3 \\ 130 \text { for } 3 \end{gathered}$ | [70, 130] | 10 | 10,000 | 6 | 2 | 100 | 0.015 | 15 | $\leq 30$ | No | No |  |
| 09/30/01 | 3 | Dividend | $\begin{gathered} 80 \text { for } 3 \\ 150 \text { for } 3 \end{gathered}$ | [40, 300] | 10 | 10,000 | 6 | 1 | 100 | 0.015 | 15 | 15 | No | No |  |
| 11/16/01 | 4 | Dividend | $\begin{aligned} & 120 \text { for } 2 \\ & 205 \text { for } 3 \end{aligned}$ | [120, 205] | 10 | 10,000 | 5 | 2 | 150 | 0.01 | 15 | 15 | Yes | No |  |
| 12/01/01 | 5 | Dividend | $\begin{aligned} & 105 \text { for } 2 \\ & 175 \text { for } 3 \end{aligned}$ | [55, 350] | 10 | 10,000 | 5 | 2 | 130 | 0.012 | 15 | 15 | Yes | No | $\begin{gathered} \text { One } \\ \text { predictor } \\ \text { violated } \\ \text { instructions } \end{gathered}$ |
| 02/15/02 | 6 | Dividend | 150 for 5 | [40, 300] | 10 | 10,000 | 5 | 1 | 100 | 0.015 | 15 | 15 | Yes | Yes |  |
| 02/22/02 | 7 | Dividend | $\begin{aligned} & 105 \text { for } 3 \\ & 175 \text { for } 3 \end{aligned}$ | [55, 350] | 10 | 10,000 | 6 | 2 | 130 | 0.012 | 15 | 15 | Yes | Yes |  |
| 03/01/02 | 8 | Predicted Value | 40 for 2 <br> 75 for 2 | [10, 300] | 10 | 10,000 | 4 | 2 | 150 | [Points/ <br> Average] $\times \$ 25$ | 17 | $\leq 30$ | Yes | Yes |  |
| 04/26/02 | 9 | Predicted Value | $\begin{gathered} 70 \text { for } 2 \\ 130 \text { for } 3 \end{gathered}$ | [70, 130] | 10 | 10,000 | 5 | 2 | 150 | [Points/ Average] $\times \$ 25$ | 15 | $\leq 30$ | Yes | Yes |  |
| 07/11/02 | 10 | Predicted Value | 150 for 8 | 150 | 10 | 10,000 | 8 | 2 | 100 | [Points/ <br> Average] $\times \$ 25$ | 15 | $\leq 30$ | Yes | Yes |  |
| 07/12/02 | 11 | Predicted Value | 75 for 6 | 75 | 10 | 10,000 | 6 | 2 | 200 | [Points/ <br> Average] $\times \$ 25$ | 15 | $\leq 30$ | Yes | Yes |  |

*Predictor compensation $=\max (0,(\mathrm{~N}-$ absolute price prediction error $))$

Keynes' Beauty Contest and Stock Price Bubbles, 4/20/04

Fig 1: Backward Induction (Representative Investor Case)

Backward induction is possible.


BI (1): Investor $i$ forms a rational expectation of $E_{t}\left(P_{t+1}\right)$.
BI (2): Investor $i$ forms a rational expectation of $E_{t}\left(P_{t+2}\right)$.
Finite: Stock has a finite maturity.
Infinite: Stock has an indefinite maturity.

Fig 2: Backward Induction (Heterogeneous Investor Case)

Backward induction is possible.


BI (1): Investor i forms a rational expectation of $\mathrm{Et}(\mathrm{Pt}+1)$.
BI (2): Investor i forms a rational expectation of $\mathrm{Et}(\mathrm{Et}+1(\mathrm{Pt}+2))$. Investor i expects that investor $(\mathrm{i}+1)$ forms a rational expectation of $\mathrm{Et}+1(\mathrm{Pt}+2)$.

Finite: Stock has a finite maturity. Infinite: Stock has indefinite maturity.
$E_{t}{ }^{\text {(s) }}(\cdot)$ : Investor i's s ${ }^{\text {th }}$ order expectation of ( $\cdot$ )

Figure 3: Stock Prices and Efficiency of Allocations for Session 3
(Exogenous Terminal Payoff Session)


Figure 4: Stock Prices and Efficiency of Allocations for Session 4
(Exogenous Terminal Payoff Session)


* Two transaction in period 2 occurred at 2,155 because the bidder said he inadvertly added 5 s to the intended bids of 215 .

Figure 5: Stock Prices and Efficiency of Allocations for Session 5
(Exogenous Terminal Payoff Session)


- Transaction Price ___Fundamental Price ----- Predicted Price __ Maximum Possible Dividend . Efficiency of Allocations

Figure 6: Stock Prices for Session 6
(Exogenous Terminal Payoff Session)


* One transaction in period 2 occurred at 5,050 because the bidder said he inadvertently added 50 to the intended bids of 50 . Allocative efficiency of this market is undefined because all investors had identical dividends.

Figure 7: Stock Prices and Efficiency of Allocations for Session 7

## (Exogenous Terminal Payoff Session)



$$
\begin{array}{ccc}
\text { - } & \text { Transaction Price } & \text { Fundamental Price } \\
\text { Maximum Possible Dividend } & \text { Efficiency of Allocations } \\
\hline
\end{array}
$$

* One transaction in period 9 occurred at 1 because of mis-ask

Figure 8: Stock Prices and Efficiency of Allocations for Session 1
(Endogenous Terminal Payoff Session)


Figure 9: Stock Prices and Efficiency of Allocations for Session 2
(Endogenous Terminal Payoff Session)


- Transaction Price ——_Fundamental Price ---- Predicted Price . Efficiency of Allocation
* One transaction in period 10 occurred at 8.970 , because the bidder said she inadvertently added a zero to the intended bid of 897

Figure 10: Stock Prices and Efficiency of Allocations for Session 8
(Endogenous Terminal Payoff Session)


Figure 11: Stock Prices and Efficiency of Allocations for Session 9
(Endogenous Terminal Payoff Session)


- Transaction Price ——Fundamental Price ---- Predicted Price . Efficiency of Allocations

Figure 12: Stock Prices for Session 10

## (Endogenous Terminal Payoff Session)



* One transaction in period 9 occurred at 2,260 because the bidder said he inadvertently added 2 to the intended bids of 260 Allocative efficiency of this market is undefined because all investors had identical dividends.

Figure 13: Stock Prices for Session 11
(Endogenous Terminal Payoff Session)


* One transaction in period 6 occurred at 200 because the bidder said he inadvertently added 0 to the intended bids of 20. Allocative efficiency of this market is undefined because all investors had identical dividends.

Figure 14: Dispersion of Investor Profits


## Appendix:

## Instruction Sheets for the Subjects for Session 2

## General Instructions

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

In this session, we conduct a market in which you can trade an object we shall call "shares". You will be assigned the role of either an investor or a predictor. At the beginning of session, your role (investor or predictor) is determined by lots: "I" means investor and " P " means predictor. Your role does not change during the session.

If you are assigned to the role of an investor, you may buy/sell shares in the market. At the end of the session, your total points gained from the market will be converted into U.S. dollars at $\$ 0.015$ per point and paid to you in cash. The more points you earn, the more dollars you will take home with you.

If you are assigned to the role of a predictor, you will predict the average market price at which the investors trade shares each period. What you earn each period depends on the accuracy of your price prediction. The more accurately you predict the share prices, the more dollars you will take home with you.

During this session, neither investors nor predictors are allowed to talk to any other participant. Also, you are to follow the various instructions given by experimenter. Violation of instructions risks forfeiting your earnings.

## Investors Instructions

If you are assigned to role of an investor, you may trade shares. At the start of the session, you are given 10 shares. You may sell these shares or keep them until the end of the session. You are also provided with an initial "cash" of 10,000 points. You may use the points to purchase shares or keep them until the end of the session. Trading in shares should follow the rules to be explained later.

The session consists of many market periods of three minutes each. That is, period 1 takes place during the first three minutes of trading, period 2 in the second three minutes, period 3 in the third three minutes, and so on. The number of periods will not exceed 30.

To buy shares you should have cash to pay for them. Buying a share reduces your cash balance by the purchase price. You may sell any shares you have. Selling increases your cash balance by the sale price.

If you keep a share, then you receive a dividend on it at the end of period 30. It is written on the Dividend Card given to you before period 1. Dividend is your private information, and it may be different for different investors. You are not informed of other's dividends. The range of dividends will be announced at the beginning of period 1.

In the likely case that the session ends before period 30, each share you hold at the end of the last period (i.e., at the end of the session) will be converted into points at the average of the predictions of the next period's price made by the predictors.

Your profits in this experiment are the sum of profits from two sources: trading profits and end-of-session payoff. Your trading profits are equal to the change in your cash balances due to trading, which is calculated as (your cash balance at the end of last period - your initial cash provided at the beginning of the experiment). Your end-ofsession payoff will depend on the number of shares you hold at the end. If the session lasts for 30 periods, this payoff will be (the number of shares you hold at that time $\times$ your dividend per share on the Dividend Card). If the session ends earlier, this payoff will be (the number of shares you hold at that time $\times$ average predicted price). Your total profits will be converted from points into U.S. dollars at the rate of $\$ 0.015$ per point and paid to you in cash.

Investor's Record Sheet
Name $\qquad$ Number $\qquad$ Date $\qquad$

| Period | Shares | Cash | Profits |
| :--- | :--- | :--- | :--- |
| 0 | 10 | 10,000 |  |
| 1 |  |  |  |
| 2 |  |  |  |
| $\ldots$ |  |  |  |
| 29 |  |  |  |
| 30 | Last Cash Balance - Initial Cash of $10,000=$ |  |  |
| Trading Profits | Last share holding $\times$ Dividend per share or <br> Last share holding $\times$ Average Prediction $=$ |  |  |
| Final Payoff | In Points |  |  |
| Total Profits | In dollars (points $\times \$ 0.015)=$ |  |  |
| Total Profits |  |  |  |

## Predictors Instructions

Predictors are asked to predict the prices at which the investors trade shares in each three minute period. At the beginning of each period, you predict the average stock price of the period by writing it down in the column of My Predicted Price on your Price Prediction Sheet. We shall gather this information before starting trading for the period. At the end of each period, we shall write on the board the average predicted price for all to see.

During the period, you can see bids, asks and transaction prices on the big screen in the room.

Your profit for the period depends on the accuracy of your price prediction. Each period, you will earn 100 minus the absolute difference between your prediction and the actual trading price (averaged across all transactions in the market) shown on the big screen. For example, suppose, you predict a price of 960 and the actual average price is 980 , you have a prediction error of 20 points, and your prediction earnings will be 100 minus 20 which is 80 . On the other hand, if the actual average transaction price turns out
to be 900 , you have prediction of error of 60 points, and your prediction earnings will be $100-60=40$.

You should record the actual average price, your prediction error (Absolute Difference), and your earning on your Price Prediction Sheet at the end of each period. At the end of the experiment, you should add your prediction earnings for all periods. Your total earnings are converted from points into U.S. dollars at the rate of $\$ 0.015$ per point and paid to you in cash.

## Price Prediction Sheet

Name $\qquad$ Date $\qquad$
Predictor Number $\qquad$

| Period No. | My <br> Predicted <br> Price <br> (PP) | Actual <br> Average <br> Price <br> (AP) | Absolute <br> Difference <br> $\|\mathrm{PP}-\mathrm{AP}\|$ | My Earning <br> from <br> Prediction <br> $100-\|\mathrm{PP}-\mathrm{AP}\|$ | My <br> Cumulative <br> Earnings in <br> Points |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| $\ldots$ |  |  |  |  |  |
| 29 |  |  |  |  |  |
| 30 |  |  |  |  |  |

Your Dollar Earnings (points $\times \mathbf{\$ 0 . 0 1 5 )}$ ) $\qquad$

## Instruction Set 2 (Trading Screen Operation)

1. Figure A1 shows an active trading screen belonging to Investor 1.
2. Your identification number is shown in the title bar.
3. The number of shares you have are shown in the light blue section under the "Hold." label. This number increases when you buy and decreases when you sell shares.

Figure A1

4. The clock in your CapLab screen (Figure A2) starts ticking down from a preset time allowed for trading, and shows the amount of time remaining till the end of trading.
5. Your cash balance is shown in the CapLab screen in green font in Figure A2.
6. Submit your bids (proposals to buy shares) by entering price and quantity under the "My Bid" column in brown section and pressing the enter key. Do the same for submitting asks (proposals to sell shares) under the "My Ask" column in deep yellow section. Note that default quantity is 1 share.

FigureA 2

7. When you submit a bid (proposal to buy), the computer checks if your bid price is higher than the existing market bid and if you have enough cash to pay for the purchase. If both the answers are affirmative, the market bid is replaced by your bid on the screen in the light brown section.
8. When you submit an ask (a proposal to sell), the computer checks if your ask price is lower than the existing market ask and if you have the shares you propose to sell. If both the answers are affirmative, the market ask is replaced by your ask in the light yellow section of the screen.
9. CapLab warns you whenever your own bid (or ask) is the market bid (or ask) by displaying them in red font on your trading screens.
10. You can change your bid/ask price by one unit with each click on the plus or minus buttons.
11. Please remember that your bid/ask is not submitted until you press the enter key.
12. Caplab shows the identity of the investor who made the market bid/ask next to price and quantity. An S1 label under the Market Bid or Market Ask columns means investor number 1 made it.
13. You can buy shares in two different ways:

- You can submit a bid under My Bid (brown section) and wait for someone else to accept it
- If you see a market ask price in light yellow section at which you would like to buy, submit the same price and appropriate quantity under My Bid.

14. You can sell shares in two different ways:

- You can submit an ask under My Ask (deep yellow section) and wait until someone to accept it.
- If you see a market bid price (in light brown section) at which you would like to sell, submit the same price and appropriate quantity under My Ask (brown section).

15. If bid and ask cross (bid is above ask), transaction is executed at the price equal to the bid or ask that came first.
16. Whenever a transaction takes place, you will see the unit price of the latest transaction on the right hand side in the light blue section under label "L. Price."
17. This market has no book or queue. When a better one overtakes an unaccepted bid/ask, the latter is simply flushed from the system. It does not stay in the memory.
18. At the end of each period, the average trading price for the period will be shown under the heading "Avg. Price" at the right end of the trading screen. You may ignore the "BOP Hold." and "Value" columns.

## Questionnaire Given to Subjects at the End of Instructions/Trial Session

(Used in Sessions 4-9 only, answers were supplied after they filled their own answers)
Name $\qquad$ Date
Prediction Questions

1. A predictor predicts a price which is 5 points more than the actual average price of the period. What is the predictor's profit (in points)?
$150-5=145$
2. A predictor predicts a price which is 8 points less than the actual average price of the period. What is the predictor's profit (in points)?
$150-8=142$
Investor Questions
3. If you buy a share, what happens on your cash balance?

My cash balance decreases by the purchase price.
4. If you sell a share, what happens on your cash balance?

My cash balance increases by the sale price.
5. Can you buy the shares when you do not have enough cash to pay for the purchase?

No.
6. Can you sell the shares when you do not have the shares you propose to sell?

No.
7. What is the way of buying a share?

Submit a bid.
8. What is the way of selling a share?

Submit an ask.
9. How many points would you earn from each share you have?

It depends on whether I sell it to others or I keep it until the end of the last period of the session.
If I sell the share to another investor, I get the sale price.
If I keep the share until the end of the last period of the session, I receive a
dividend at the end of period 15.
10. How much is the dividend per share?

It is shown on the dividend card given to me before period 1 of the session.
11. Is the dividend same for all the investors?

Not necessarily. The dividend of each investor is written in his/her dividend card.
But it lies in the dividend range to be announced before period 1 of the session.
General Questions.
12. How many trading periods are there during the session?

15 periods.
13. For how many minutes does one trading period last?

3 minutes
14. What is the rate at which your points are converted into real dollars?
$\$ 0.01$ per point
15. Are you allowed to talk, use email, or surf the web during the session? No.


[^0]:    ${ }^{1}$ Even for investors who do hold a security for, say 12 months, dividend is only a part of the total return on the investment. Further, Fama and French (2001) found that the percent of firms that pay dividends declined from 53 percent in 1973 to 21 percent in 1999. Since many investors trade in and out of a security before any dividend is

[^1]:    ${ }^{2}$ We assume that investors are homogenous within each generation but heterogeneous across generations.

[^2]:    ${ }^{3}$ Duration is the first derivative of the present value of a security with respect to the discount rate, or the weighted average time of cash flows associated with the security.

[^3]:    ${ }^{4}$ We conjecture that in Session 3, a bubble formed because at least some of the subjects did not fully understand the instructions about the structure of the security. In period 4 the price rose to 320 , although the subjects knew that the maximum possible dividend in this market was only 300 . In period 15 , some transactions occurred at a price of 1 ; we suspect that the seller forgot that the securities earn a liquidating dividend at the end of the last period. In Session 5 (Figure 5), the price dropped below the fundamental value (175) in period 15 . The experimenter failed to disable the trading function on the predictors' computers. Although the instructions prohibited predictors from trading, one of the predictors traded anyway, and 14 of the 17 trades in period 15 were sales by this predictor. We confirmed that this predictor's trading had no significant effect on the convergence pattern to the fundamental value during the first 14 periods of the session.

[^4]:    5 "Investors, their confidence and expectations buoyed by past price increases, bid up stock prices further, thereby enticing more investors to do the same, so that the cycle repeats again and again, resulting in an amplified response to the original precipitating factors. The feedback mechanism is widely mentioned in popular discourse as merely a hypothesis, often regarded as unproven. In fact, there is some evidence in support of such a feedback mechanism, as we shall see." (Shiller (2000, pp.44)).
    ${ }^{6}$ Note that the predictors recorded their predictions at the beginning of each period and the average of these predictions were not announced to the investors till the end of the period. Since the information sets of the predictors and the investors essentially the same (except for any private dividends), it is reasonable use the predictors' predictions as proxies for the expectations held by the investors in the experiment. We had different subjects play

[^5]:    ${ }^{7}$ In sessions 6 (Figure 6), 10 (Figure 12) and 11 (Figure 13), all investor had identical dividends and the allocative efficiency was undefined
    ${ }^{8}$ The efficiency dropped just before the end of Session 3. After the session ended, a high-dividend trader who had accumulated a large number of securities told us that in period 15 he forgot that there was a terminal dividend, panicked, and sold off many securities at a price of 1 .

