A Challenger to the Limit Order Book: The NYSE Specialist*

Job Market Paper

Sabrina Buti†

Abstract

This paper gives a new answer to the challenging question raised by Glosten (1994): "Is the electronic order book inevitable?". While the order book enables traders to compete to supply anonymous liquidity, the specialist system enables one to reap the benefits from repeated interaction. We compare a competitive limit order book and a limit order book with a specialist, like the NYSE. Thanks to non-anonymous interaction, mediated by brokers, uninformed investors can obtain good liquidity from the specialist. This, however, creates an adverse selection problem on the limit order book. Market liquidity and social welfare are improved by the specialist if adverse selection is severe and if brokers have long horizon, so that reputation becomes a matter of concern for them. In contrast, if asymmetric information is limited, spreads are wider and utilitarian welfare is lower when the specialist competes with the limit order book than in a pure limit order book market.

Keywords: limit order book, specialist, hybrid market

JEL Classification: G10, G24, D82

*Many of the ideas in this paper were developed in discussions with my Ph.D supervisor B. Biais. I am grateful to U. Axelson, K. Heen, T. Mariotti, T. Ramadorai, B. Rindi, P. Strömberg, and E. Thibault for helpful comments and discussions. I would like to thank seminar participants at LSE PhD seminar, SIFR, Stockholm School of Economics and Toulouse University. All omissions and errors are mine.

†GREMAQ, Toulouse University, and Swedish Institute for Financial Research. E-mail: sabrina.buti@sifr.org

1 Introduction

The battle between exchanges concerns liquidity and price: when investors come to the market they want to execute their orders at the best price. Which type of market structure will fulfill most investors' expectations? Glosten (1994) analyzes whether the electronic order book is inevitable. He shows that the open limit order book (LOB) does "as well as can be hoped at handling extreme adverse selection problems"[1] and he points out that no other anonymous exchange can improve on the LOB. However, in this new age of computerised trading, the existence of a floor market like the NYSE is not in line with Glosten's results. A possible explanation could be that people prefer to trade where they know other traders gather. This implies that an exchange is likely to attract more trading volume once it has a large amount of liquidity, even if there are more efficient but less liquid alternatives. However, an alternative explanation could be the advantage of the floor system in comparison to anonymous electronic markets: relationship trading. So, what if competition to the LOB comes from a non-anonymous market architecture where relationship trading is possible?

In order to answer this question, we compare two different market structures. The first market structure is an anonymous LOB with free entry in which the market promotes competition between liquidity suppliers. The second market structure is a specialist market. In this market structure one still finds competitive liquidity suppliers on the LOB, but liquidity can also be offered by a monopolist specialist. The specialist interacts repeatedly with the brokers on the floor, this allows him to build a relationship with the brokers based on trust and reputation. When brokers credibly certify to the specialist that their customers are uninformed, the specialist can offer better quotes than the LOB. As a result, in the hybrid market informed traders will go on the anonymous LOB to benefit from competition, while uninformed traders will go to the specialist, to benefit from relationship trading.

The first aim of this work is to analyze the trade-off involved by the coexistence of the specialist and the LOB. On the one hand, the specialist worsens adverse selection on the LOB: Ready (1999) finds that orders stopped and executed directly by the specialist are more profitable for liquidity suppliers than orders allowed to trade with the book.[2] On the other hand, the specialist lowers the asymmetric information problem via relationship trading. The idea that the unique relationship between the specialists and floor brokers leads to less anonymity is supported by Garfinkel and Nimalendran (2003). They analyze the change in spread measures on the NYSE and the NASDAQ on days when an insider trades, to assess the ability of specialists to detect and respond to insider trading. They find evidence consistent with less anonymity in the NYSE specialist system compared to the NASDAQ dealer system. A similar result is provided by Heidle and Huang (2002) and is supported by the analysis of a natural experiment on actual insider trades done by Fishe and Robe (2004).

When a specialist coexists with the LOB, there is competition in the offer of liquidity between principals with different information on the order flow. The specialist knows if orders are uninformed, and can offer better trading terms. The other liquidity suppliers are uninformed and can

[1] "Is the Electronic Open Limit Order Book Inevitable?", pag. 1129.

[2] Bessembinder and Kaufman (1997) also find evidence of "cream skimming" of uninformed trades on the NYSE.

not discriminate between uninformed and informed traders. So, anticipating the increase in adverse selection costs due to specialist "cream skimming", the spread widens on the LOB.[3]

This trade-off is particularly interesting given the controversial results of the empirical literature on the effectiveness of the specialist in increasing market liquidity. On the one hand, Venkataram (2001) shows that, other things equal, the NYSE (a specialist market) is more liquid than the Paris Bourse (a limit order market) since the automated system is not able to replicate the benefits of human intermediation on a trading floor. On the other hand, Ready (1999) shows that the specialist's own trading worsens adverse selection costs borne by limit order traders. The previous theoretical literature has considered just one of the two aspects: Benveniste et al. (1992) focuses on the positive side, the lower level of asymmetric information due to relationship trading, while Rock (1996) focuses on the negative side of the specialist, the higher adverse selection on the book. Our model instead provides a joint analysis of these two effects.

The second aim of this study is to endogenize both price and quantities quoted by the specialist when he is competing with the LOB. The crucial role of both price and depth in the specialist's interaction with the LOB has been shown in the empirical works of Kavajecz (1999) and Kavajecz and Odders-White (2001). However, the previous theoretical literature has focused on only one of the two aspects. Dupont (2000) and Caglio and Kavajecz (2006) incorporate the specialist's quoted depth as an endogenous variable but in the absence of a limit order book. On the contrary, Rock (1996), Seppi (1997) and Parlour and Seppi (2003) analyze the interaction between the specialist and the LOB, but their models provide no role for the specialist's quoted depth. In our model, we analyze both the depth issue and the specialist's interaction with the LOB.

The objective of our paper is threefold. First, we analyze how the adverse selection problem and the competition from the LOB influence the price schedule offered by the specialist to uninformed traders. Second, we show the conditions in which a LOB with specialist can offer better expected trading terms than a LOB without specialist. Finally, we compare the welfare gains due to the higher quantity offered to uninformed traders by the specialist with the welfare loss due to the worsening of the spread offered on the LOB.

The LOB is modelled following the contract theory approach of Biais et al. (2000), when the number of traders goes to infinity. It is a publicly visible screen that provides traders with bids and offers, each specifying a price and a quantity available at that price. Liquidity on the LOB is provided by a population of risk neutral liquidity suppliers. So, the equilibrium is characterized by a zero expected profit condition. Moreover, given the "discriminatory" nature of the book, asks and bids will be related to "upper tail" and "lower tail" expectations as in Glosten (1994).[4]

[3] Madhavan and Sofianos (1998) show that the specialist participates more in the market when the bid-ask spread is wide. We show that the widening of the spread could be simply motivated by the active participation of the specialist on the market, anticipated by liquidity suppliers on the LOB, and the consequent worsening of adverse selection costs on the LOB. So, instead of having the specialist attracted by the wide spread, the cause of the wide spread could be the specialist himself.

[4] In this framework, liquidity suppliers can't condition on total market order quantity. What the liquidity suppliers know, for example, is that if a limit order to sell is hit, the market order is at least as large as the cumulated depth of the book up to that price. Expectations of asset value given that this order has been hit are called "upper–tail

We model the specialist-broker relationship as an infinitely repeated game. The risk-neutral broker is trading on behalf of risk-averse traders who can be informed or not. We assume that the client's type is known by the broker but not by the specialist. So, when the broker reports his client as uninformed, the specialist offers better trading terms in comparison to the book. However, to avoid misreporting on clients' type by the broker, the specialist punishes ex-post the broker who lies by refusing to improve quotes for that broker in the future. This implies that if the client is uninformed, the broker goes to the specialist, while if the client is informed, the broker weighs the better trading terms currently offered by the specialist against the future discounted benefits of continuing a relationship with the specialist. Hence, cooperation will be easier for brokers who place a lot of weight on the future.

The specialist selects the price schedule that maximizes his profits. The inverse demand function he considers depends on the level of asymmetric information and on the discounting rate of the broker. The specialist compares the profits of offering a high quantity with the rents he has to leave to the uninformed trader in order to prevent broker's misreporting. For low levels of the discount rate, future trading opportunities are highly valuable for the broker and a large quantity can be offered by the specialist. However, as the discount rate, or as the asymmetric information problem increases, the value of keeping a good relationship with the specialist decreases and, to prevent misreporting, the specialist has to offer a less attractive price-quantity schedule.

If asymmetric information worsens or if the relationship with the broker is less stable, the specialist's reaction focuses more on depth than on the price. The quoted depth is a monotone decreasing function of adverse selection costs and a monotone increasing function of the probability of continuation of the relationship with the specialist, while price is not monotone in these two parameters.

In welfare terms, the specialist is Pareto improving for high levels of adverse selection, when the LOB quoted spread is already wide given the high risk of trading with informed traders. If this is the case, then there is no trade-off in the market: the introduction of the specialist only has a positive welfare effect since better liquidity is offered to uninformed traders and the specialist makes profits. The negative effect due to the worsening of the LOB is absent. For lower levels of adverse selection, the trade-off arises. However, the LOB with specialist can still improve on the LOB if the discount rate is low. If this is the case, the broker highly values the possibility of interacting with the specialist in the future. So, the specialist can improve on the trading terms offered by the LOB. He offers a price schedule to uninformed traders that is more attractive than that of the LOB's and guarantees him high profits. Finally, if the discount rate is high, the specialist offers uninformed traders a lower quantity than the one available on the LOB without specialist. Moreover, the necessity to control the broker lowers specialist's profit. As a result, the welfare is lower in a LOB with specialist.

The impact of adverse selection on depth is consistent with the empirical findings of Kavajecz (1999). He finds that specialists use depth as a strategic choice variable to regulate the amount of

expectations", and have been used first by Glosten (1994) for a LOB with an infinite number of competing market makers.

liquidity they provide.[5] For example, he shows how specialists and limit order traders reduce depth around informative events, reducing their exposure to adverse selection costs. In our analysis, both the specialist and the limit order traders reduce the quoted quantity if the asymmetric information problem worsens.

Our model is also consistent with the results obtained by Battalio et al. (2005). They claim that if relationships are important in attenuating adverse selection problems, changes of specialist's location on the floor should influence liquidity costs since they often imply a relationship ending. Indeed, they find that liquidity costs increase when a stock moves. Similarly, in our model the quantity offered by the specialist to uninformed traders is decreasing in the weakness of his relationship with floor brokers.

Our work also offers an explanation for the differential execution costs among specialist firms on the NYSE documented by Cao et al. (1997), Corwin (1999) and Coughenour and Deli (2002). Coughenour and Deli show that differences in liquidity provision arise from differences in specialist firm organizational form. They argue that specialists using their own capital have a greater ability to reduce adverse selection costs, since they can credibly bond information-sharing relationships and they have found evidence to support their hypothesis. If we interpret the broker's discount rate as the probability of continuing the relationship with the specialist, our paper shows that if brokers perceive the relationship as more stable, the specialist is able to offer better trading terms. In this way, our work provides a theoretical justification for the results obtained by Coughenour and Deli.

Our results suggest that a relationship trading system can improve on the trading terms offered by the LOB for stocks that are highly exposed to adverse selection problems. So, we would expect the specialist to be beneficial for thinly traded stocks or for stocks in their initial quotation phase. Notice also that if the relationship with the specialist is stable, then the specialist can be beneficial in welfare terms also for stocks with low adverse selection problems. Even if the broker is worse off, the high specialist's profit guarantees a higher welfare level. The actual system that imposes such an obligation on the specialist to guarantee a "fair and orderly market" could be a way to redistribute welfare gains due to relationship trading from the specialist to traders. However, our model suggests that once the specialist's system is in place, it can not be overcome by a competing pure LOB even when it is inefficient. The introduction of a specialist on a LOB should be carefully thought over.

The remainder of the paper is organized as follows. Section 2 discusses how the paper relates to the previous literature. Section 3 introduces the model structure and its hypotheses. Section 4 focuses on the pure LOB market structure. Section 5 analyzes the hybrid market, LOB and specialist, and in particular the price schedule offered by the specialist. Section 6 focuses on the comparison between the LOB with and without a specialist: we analyze quoted and effective spreads, and utilitarian welfare. Some empirical and policy implications are derived. Finally, Section 7 presents some concluding remarks. All the proofs are in the Appendix.

[5] Kavajecz shows that specialists change their quoted depth in 90% of all quote changes. Moreover, 50% of all quote changes are unaccompanied by changes in the quoted price.

2 Literature

This article is closely related to four lines of prior works. The first of these is the work of Benveniste et al. (1992) on the importance of relationship trading in mitigating the effects of asymmetric information. In their model, the specialist can impose sanctions (i.e. less improvement on quoted prices or less favorable future prices) on traders using private information. Since the traders bear the full cost of informed trading, they are less likely to impose adverse selection costs on others.[6] There are two crucial differences with our work. First, the specialist is not strategic: he is only regulated by the zero profit condition if passive or by the maximization of the broker's profits if active. This hypothesis underestimates the costs of giving an informational advantage due to relationship trading to a profit seeking agent. Second, in their approach liquidity is provided to the market by the specialist only, while in our model the specialist competes with the LOB. In this way we can jointly analyze the potential benefits of introducing a specialist on the LOB with the eventual costs due to the worsening of adverse selection on the LOB.

A second line of work involves strategic liquidity supply with adverse selection. In particular, we are interested in the screening game where first market makers post price schedules and then one strategic informed trader selects the quantity to trade. The adverse selection problem in a discriminatory auction has been analyzed by Glosten (1994).[7] We use the Glosten approach, and the contract theory framework of Biais et al. (2000) to model the LOB. However, we introduce in their setting a liquidity supplier with a comparative advantage: the capacity to build relationships due to repeated interaction.

The third line of work concerns the comparison between different market structures. Seppi (1997) analyzes an hybrid market where a specialist competes with value traders in offering liquidity. Active traders who submit market orders are not strategic, while in our model investors optimally select the quantity they want to trade on the LOB depending on market conditions. Therefore, in Seppi (1997) the specialist cannot use prices to influence the submitted quantity, while in our work the specialist uses both quantity and price strategically. Moreover, adverse selection costs are not explicitly modelled and are just summarized in the decreasing submissions costs on the LOB. On the contrary, in our framework adverse selection costs are endogenous since there is asymmetric information on the asset value. Parlour and Seppi (2003) extend the work of Seppi (1997) and analyze two competing exchanges: a pure LOB and a hybrid market with both a specialist and a LOB. Again, the main differences with our framework are that both the quantity traded by the agents and adverse selection costs are exogenous.

Finally, other works have analyzed the joint determination of the optimal price and depth by the specialist. Dupont (2000) and Caglio and Kavajecz (2006) consider a monopolist specialist, while in our model the monopolistic market power of the specialist is limited by both the competition from the LOB and the necessity to induce truthtelling from the broker.

[6] A similar conclusion is reached by Chan and Weinstein (1993): in their model specialists reward floor brokers with tighter bid-ask spreads on future trades if they reveal private information on their orders.

[7] Glosten (1989) considers a monopolistic liquidity supplier, Bernhardt and Hughson (1997) focuses on the duopoly case, while Biais et al. (2000) shows that as the number of liquidity supplier goes to infinity, the oligopolistic equilibrium converges to the competitive equilibrium analyzed in Glosten (1994).

3 The Model

We analyze a financial market for a risky asset where liquidity is supplied by risk-neutral limit order traders to risk-averse and expected utility-maximizing traders. In each period $t \in [1, ..., \infty]$, a trader arrives on the market with a certain willingness to trade. His motivation to trade can be private information on the asset value, inventory rebalancing, or a mixture of the two. We compare two market structures: a competitive LOB and a competitive LOB with a specialist. In the first structure, liquidity is offered on the LOB by a large number of limit order traders posting quotes. In the second structure, liquidity is offered by both a large number of limit order traders posting quotes on the LOB and a specialist, who is allowed to improve LOB quotations by direct interaction with floor brokers.

3.1 The Information Structure

The value of the asset is given in each period by $v_t = s_t + \varepsilon_t$, where s_t is the private signal of the trader and ε_t is a noise on the signal, $\varepsilon_t \sim (0, \sigma^2)$. Notice that the informative signal lasts only one period. The informed trader also privately observes his endowment I_t in the risky asset. Information and endowment shocks are not correlated among different periods. We assume that the inventory shock takes the values $\{I, 0, -I\}$, where $I > 0$, with equal probability, while the informative signal takes the values $\{\underline{s}, \overline{s}, -\underline{s}, -\overline{s}\}$, where $\underline{s} < \overline{s}$, with probability η or zero with probability $1 - 4\eta$, where $\eta \in [0, 1/4]$. The parameter η represents the probability of informed trading: the lower η, the higher the probability to have an uninformed trader on the market.

3.2 Types of Agent

In the model there are four types of agent: traders, liquidity suppliers, brokers and a specialist.

3.2.1 Traders

As already mentioned, traders come on the market for a combination of inventory rebalancing and private information reasons. The trader's wealth in period t is the following:

$$W_t = (Q_t + I_t)v_t - P_t(Q_t)Q_t$$

where Q_t is the total traded quantity of the asset and $P_t(Q_t)$ is the average price paid for that quantity. We assume that the trader has mean variance preferences[8] and a risk aversion parameter γ. His objective function is stated as:

$$U_t = E\left[W_t \mid I_t, s_t\right] - \frac{\gamma}{2} V\left[W_t \mid I_t, s_t\right]$$

We define $\theta_t = s_t - \gamma\sigma^2 I_t$. This parameter represents the willingness to trade of the trader who arrives in period t and it reflects the trader's mix of risk sharing and informational motivations to

[8] We can obtain the same result by assuming normality in the error term and CARA utility function with absolute risk-aversion parameter γ.

trade. Therefore, it is increasing in trader's private signal and decreasing in trader's initial position due to risk aversion. Trader's utility can be rewritten as:

$$U_t = \left(I_t s_t - \frac{\gamma \sigma^2}{2} I_t^2 \right) + \left(\theta_t Q_t - \frac{\gamma \sigma^2}{2} Q_t^2 - P_t(Q_t) Q_t \right)$$

The first term represents the reservation utility of the trader if he decides not to participate in the market, while the second term represents his gains from trade.

Willingness to Trade Distribution. Given our assumptions about the distribution of s and I, θ_t is distributed in the following way:

	Inventory Prob.	1/3	1/3	1/3
Signal Prob.	Shock Values	$-I$	0	I
η	$\overline{s}$	$\overline{s} + \gamma\sigma^2 I$	$\overline{s}$	$\overline{s} - \gamma\sigma^2 I$
η	$\underline{s}$	$\underline{s} + \gamma\sigma^2 I$	$\underline{s}$	$\underline{s} - \gamma\sigma^2 I$
$1-4\eta$	0	$\gamma\sigma^2 I$	0	$-\gamma\sigma^2 I$
η	$-\underline{s}$	$-\underline{s} + \gamma\sigma^2 I$	$-\underline{s}$	$-\underline{s} - \gamma\sigma^2 I$
η	$-\overline{s}$	$-\overline{s} + \gamma\sigma^2 I$	$-\overline{s}$	$-\overline{s} - \gamma\sigma^2 I$

In order to have asymmetric information on the market, the specialist and the liquidity suppliers have to be ignorant as to whether orders are motivated by information or by inventory shocks, so we assume that: $\underline{s} = \overline{s} - \gamma\sigma^2 I = \gamma\sigma^2 I$, $\overline{s} = \underline{s} + \gamma\sigma^2 I$ and $\underline{s} - \gamma\sigma^2 I = -\underline{s} + \gamma\sigma^2 I = 0$. Given these equalities, we define: $\theta_i^l = \theta_{ni}^l = \underline{s} = \gamma\sigma^2 I$, $\theta^m = \overline{s} = 2\gamma\sigma^2 I$ and $\theta^h = 3\gamma\sigma^2 I$. So, θ_t is distributed as in the following table:

	Inventory Prob.	1/3	1/3	1/3
Signal Prob.	Shock Values	$-I$	0	I
η	$\overline{s} = 2\gamma\sigma^2 I$	$\theta^h = 3\gamma\sigma^2 I$	$\theta^m = 2\gamma\sigma^2 I$	$\theta_i^l = \gamma\sigma^2 I$
η	$\underline{s} = \gamma\sigma^2 I$	$\theta^m = 2\gamma\sigma^2 I$	$\theta_i^l = \gamma\sigma^2 I$	0
$1-4\eta$	0	$\theta_{ni}^l = \gamma\sigma^2 I$	0	$-\theta_{ni}^l = -\gamma\sigma^2 I$
η	$-\underline{s} = -\gamma\sigma^2 I$	0	$-\theta_i^l = -\gamma\sigma^2 I$	$-\theta^m = -2\gamma\sigma^2 I$
η	$-\overline{s} = -2\gamma\sigma^2 I$	$-\theta_i^l = -\gamma\sigma^2 I$	$-\theta^m = -2\gamma\sigma^2 I$	$-\theta^h = -3\gamma\sigma^2 I$

Even if $\theta_i^l = \theta_{ni}^l$, there is a crucial difference between the two types of traders due to their different motivation to trade: θ_{ni}^l is on the market only for inventory rebalancing, while θ_i^l trades for a combination of informative signals and inventory rebalancing. We denote by $\theta^l = \{\theta_{ni}^l, \theta_i^l\}$ the group of all types with a low evaluation of the asset and by p^h, p^m, p_i^l and p_{ni}^l the probabilities respectively of types θ^h, θ^m, θ_i^l and θ_{ni}^l. Notice that, by construction, the two sides of the market are perfectly symmetric. Hence, we analyze only the ask side.

3.2.2 Brokers

The risk-neutral broker is trading on behalf of risk-averse traders and represents, in different periods, clients who come to the market for different reasons. We assume that the broker knows the identity of his client. For example, he observes if his client is a passive fund manager, who is trading to rebalance his inventory, or a smart investor, like a hedge fund, who is trading for both information and inventory reasons. Hence, the broker knows whether his client is uninformed or not and he is able to differentiate between θ_{ni}^l (i.e. the mutual fund) and $\left\{\theta^h, \theta^m, \theta_i^l\right\}$ (i.e. the hedge fund). However, knowing the identity of the trader does not tell to the broker the informed trader's private signal. Therefore, the broker can not differentiate types $\left\{\theta^h, \theta^m, \theta_i^l\right\}$. The broker is evaluated at the end of each trading period by the single investor. He extracts a constant fraction of the investor's surplus, the gains from trade.[9] Hence, he maximizes his intertemporal utility given the LOB and the price-quantity schedule offered by the specialist. His objective function is:

$$V_t(\theta_t) = \begin{cases} GT_t(\theta^a) + \sum\limits_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} bE_\theta\left[GT_s(\theta)\right] & \text{if } \theta_t \in \left\{\theta^h, \theta^m, \theta_i^l\right\} \\ GT_t(\theta_{ni}^l) + \sum\limits_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} bE_\theta\left[GT_s(\theta)\right] & \text{if } \theta_t = \theta_{ni}^l \end{cases}$$

where $GT_t(\theta) = \left(\theta_t Q_t - \frac{\gamma\sigma^2}{2} Q_t^2 - P_t(Q_t)Q_t\right)$ represents the gains from trade, θ^a is the type announced by the client, $b \in [0,1]$ is the constant fraction of the gains from trade that is assigned to the broker and r is the broker's discount factor. The discount factor has two interpretations: it could be the T-bill rate and so the discounting would represent an opportunity loss, or $1/(1+r)$ could represent the probability of continuation of the relationship with the specialist. In this last case, an increase in r represents a weakening of the specialist-broker relationship.

We assume that if the broker is indifferent between trading on the LOB or with the specialist for his uninformed clients, he will go to the specialist.[10] Moreover, we assume that the client submits his order to the broker in both market structures.[11] In the LOB without a specialist, the broker executes the order directly on the book (passive broker). In the LOB with a specialist, the broker plays an active role: he considers LOB quotes, asks for specialist's improvement and selects the best offer (active broker). We assume that if the broker accepts the specialist's offer, he cannot also trade on the LOB.

[9] This hypothesis implies that the reward of the broker is proportional to the satisfaction of the client. Even if brokerage fees are generally fixed, on the NYSE a client goes to the broker expecting to obtain good trading terms. It is reasonable to assume that the reward of the broker depends on how much his client is satisfied by his work.

[10] This hypothesis influences the model's results only in the least interesting case from an economic point of view, where any positive value of the broker/specialist relationship worsens the broker's truthtelling constraint. In this case, the benefits related to the introduction of the specialist are clearly limited. An extensive discussion of the alternative hypotesis (i.e. the indifferent broker goes to the LOB) is presented in the Appendix B.

[11] When the broker trades on the LOB without a specialist, he plays a passive role. We could have assumed that in this market structure the client can trade directly on the book. However, we decided not to consider differentials in order execution fees in our analysis and have preferred to focus on relationship trading.

3.2.3 Liquidity Suppliers

Liquidity suppliers post quotes on the LOB. They are risk neutral and their number is large enough to drive profits on the LOB to zero due to the high price competition. Moreover, since the LOB is anonymous, liquidity suppliers can not build a relationship with brokers. For every level of the book, they satisfy the following zero profit condition:

$$\pi^L = E(s \mid \theta \geq \widetilde{\theta})q(\widetilde{\theta}) - P(\widetilde{\theta})q(\widetilde{\theta}) = 0 \quad \text{for} \quad \widetilde{\theta} \in \left\{\theta^h, \theta^m, \theta^l\right\}$$

where $q(\theta)$ and $P(\theta)$ are the marginal quantity and the relative price offered at each level of the book. Notice that liquidity suppliers are unable to differentiate between types θ_i^l and θ_{ni}^l, since both traders have the same asset evaluation when they arrive on the LOB. So, it is impossible ex-ante to discriminate between them.

3.2.4 The Specialist

The specialist trades the asset with a representative broker. We assume that the specialist has no obligations imposed by the exchange authorities in order to concentrate on relationship trading. Hence, in our framework, the specialist is a profit seeking agent who owns a comparative advantage towards the other liquidity suppliers: the possibility to build a relationship with brokers due to non-anonymous repeated interaction. Thanks to this trust relationship, the specialist can offer better trading terms to traders after the broker's announcement. Notice that the specialist is able to improve on the trading terms offered by the LOB only for uninformed clients. In fact, we have assumed that the broker is able to differentiate only between uninformed and informed clients. So, if a client is informed, his type revelation must be incentivized and the specialist does not have any informative advantage compared to the LOB. As showed by Biais at al. (2000), in this case an extra strategic liquidity supplier can not improve on fully competitive LOB's quotations.[12]

We assume that the specialist knows ex-post the broker's informative signal[13] and he can commit to never improving on LOB trading terms for a broker who has misreported his client's trading motives. This hypothesis is based on the idea that the specialist is trading at the same time with many different brokers, so he can credibly commit not to trading anymore with anyone of them. If the client is uninformed, the broker will go to the specialist to ask for better trading terms. If the client is informed, then the broker will weigh the better trading terms offered by the specialist against the future benefits of continuing a relationship with the specialist. Hence, the broker could prefer to trade with the book and keep the possibility of interacting with the specialist in the future.

[12] In particular, Biais et al. (2000) show that a market with infinite strategic risk-neutral traders, competing in schedules to supply liquidity, is equivalent to a Glosten (1994) limit order book. In our setting, if the specialist has to induce truthtelling from broker's clients, than he is just an additional liquidity supplier in a Glosten (1994) LOB and can not improve on liquidity. Moreover, even if we consider the specialist's offers as an additional market, Glosten (1994) shows that no other anonymous exchange can compete with a LOB.

[13] We assume for simplicity as Benveniste et al. (1992) and Seppi (1990) that violations of no-informed trading agreements are observable ex-post. We refer to Desgranges and Foucault (2005) for an alternative modelization that shows how no-informed trading agreements can still be sustained when their violation cannot be observed.

Notice that the specialist solves the same problem each period, unless the broker has cheated before. The specialist's quoted quantity and price are therefore independent of time: $Q_t^S = Q^S$ and $P_t^S = P^S$. Moreover we assume that the specialist can commit to quote the same quantity and price in the future. The specialist's expected utility from trading with the broker is the following:

$$\pi^S = \begin{cases} p_{ni}^l \left(P^S - E[v \mid \theta_{ni}^l]\right) Q^S = p_{ni}^l P^S Q^S & \text{if no deviation} \\ 0 & \text{if deviation} \end{cases}$$

3.3 Gains from Trade

We can reformulate the gains from trade in terms of marginal quantities offered at each level of the book. We denote by $q_t^h = q_t(\theta^h)$, $q_t^m = q_t(\theta^m)$ and $q_t^l = q_t(\theta^l)$ the marginal quantities offered on the book for each agent's type, and by $P_t^h = P_t(\theta^h)$, $P_t^m = P_t(\theta^m)$ and $P_t^l = P_t(\theta^l)$ the corresponding marginal price.

Limit Order Book without a Specialist. The structure of the LOB implies that the broker does not announce his client's type. In fact, the broker hits quantities on the LOB ex-post, when liquidity suppliers have already posted limit orders. Notice also that clients have no incentives to misreport their type since they have the same objective function as brokers. Gains from trade, if we denote as $\widetilde{\theta}$ the agent's type, are given by:

$$GT_t^{LOB}(\widetilde{\theta}) = \left(\widetilde{\theta}_t \sum_{\theta \leq \widetilde{\theta}} q_t(\theta) - \frac{\gamma\sigma^2}{2}\left(\sum_{\theta \leq \widetilde{\theta}} q_t(\theta)\right)^2 - \sum_{\theta \leq \widetilde{\theta}} P_t(\theta) q_t(\theta)\right) \quad \text{for } \widetilde{\theta} \in \{\theta^h, \theta^m, \theta^l\}$$

Limit Order Book with a Specialist. Limit orders are still posted ex-ante by liquidity suppliers, but the specialist can now offer better trading terms to uninformed clients. Informed clients have no incentives to misreport their type to the broker as in the previous case. However, the broker could have an incentive to misreport his client's type to the specialist to obtain quote improvement. Gains from trade are stated as:

$$GT_t^S(\widetilde{\theta}, \widehat{\theta}) = \begin{cases} \left(\widetilde{\theta}_t \sum_{\theta \leq \widetilde{\theta}} q_t(\theta) - \frac{\gamma\sigma^2}{2}\left(\sum_{\theta \leq \widetilde{\theta}} q_t(\theta)\right)^2 - \sum_{\theta \leq \widetilde{\theta}} P_t(\theta) q_t(\theta)\right) & \text{if } \widehat{\theta} \in \{\theta^h, \theta^m, \theta_i^l\} \\ \left(\widetilde{\theta}_t Q_t^S - \frac{\gamma\sigma^2}{2}\left(Q_t^S\right)^2 - P_t^S Q_t^S\right) & \text{if } \widehat{\theta} = \theta_{ni}^l \end{cases}$$

where $\widetilde{\theta}$ is the agent's type and $\widehat{\theta}$ is the type announced by the broker.

Expected Gains from Trade. If the broker has never deviated, he will trade with the LOB if his client is informed and with the specialist if he is uninformed. Expected gains from trade are:

$$E(GT^S) = \left[p^h GT^S(\theta^h, \theta^h) + p^m GT^S(\theta^m, \theta^m) + p_i^l GT^S(\theta_i^l, \theta_i^l) + p_{ni}^l GT^S(\theta_{ni}^l, \theta_{ni}^l)\right]$$

If the broker has deviated, he will trade only on the LOB since the specialist is no longer going to improve LOB quotations for him. We assume that the LOB can not improve on the offered prices and quantities once a broker has been caught lying. In fact, the breakdown of the specialist/broker relationship should not be visible to third parties. Moreover, there is still the problem of order picking by the other brokers who have the possibility of asking the specialist for better quotes. Therefore, if we consider trader's gains from trade, nothing changes for broker's informed clients, since the broker can still go on the anonymous LOB to execute their orders. On the contrary, gains from trade for type θ^l_{ni} move from $GT^S(\theta^l_{ni}, \theta^l_{ni})$ to $GT^S(\theta^l_i, \theta^l_i)$, the gains from trade available on the LOB for type θ^l_i. Expected gains from trade after deviation are:

$$E(GT^S dev) = \left[p^h GT^S(\theta^h, \theta^h) + p^m GT^S(\theta^m, \theta^m) + (p^l_i + p^l_{ni}) GT^S(\theta^l_i, \theta^l_i) \right]$$

Broker's Punishment. Punishment for deviation is equal, in each period, to:

$$b\left[E(GT^S) - E(GT^S dev)\right] = b p^l_{ni} GT^S(\theta^l_{ni}, \theta^l_{ni})$$

The higher is the probability to have uninformed clients, the greater is the loss related to the breaking with the specialist. The better the trading terms offered to uninformed clients, the higher the costs of no longer trading with the specialist.

3.4 Timing

The timing is the following for each period t:

1. Competitive liquidity suppliers post quotes on the LOB anticipating, if there is a specialist, that their limit orders will be hit only by informed traders.

2. A trader arrives on the market and goes to a broker to perform his transaction. The broker decides, depending on his client's motivations to trade, if he goes to the specialist to ask for better trading terms, or if he executes the order directly on the book.

3. If the broker goes to the specialist and has never deviated before, then the specialist offers him better trading terms. Otherwise the specialist offers the same terms of trade as the LOB.

4. After the transaction, the specialist discovers the trader's informed/uninformed status.

4 The Limit Order Book Market

The LOB is a publicly visible screen that provides traders with bids and offers, each of which specify a price and a quantity available at that price. Liquidity suppliers compete on prices as in Glosten (1994). Reputation plays no role in this anonymous market structure and we can drop the time index. Liquidity suppliers' problem for $\widetilde{\theta} \in \{\theta^h, \theta^m, \theta^l\}$ is:

$$
\begin{aligned}
(\mathcal{P}_1) \quad & \max_{q(\widetilde{\theta}),P(\widetilde{\theta})} \quad GT^{LOB}(\widetilde{\theta}) \\
& s.t. \quad (ZP(\widetilde{\theta})) \quad \left[E(s \mid \theta \geq \widetilde{\theta}) - P(\widetilde{\theta})\right] q(\widetilde{\theta}) = 0 \\
& \qquad\quad (IR(\widetilde{\theta})) \quad GT^{LOB}(\widetilde{\theta}) \geq 0 \\
& \qquad\quad q(\theta),\ P(\theta) \ solve\ \mathcal{P}_1 \ for\ \theta \leq \widetilde{\theta}
\end{aligned}
$$

Liquidity suppliers offer the quantity that maximizes the gains from trade of the lowest investor's type who hits that limit order. Any larger quantity would be hit only by traders with higher asset evaluations and would imply negative profits.[14] Moreover, liquidity suppliers take into account that the trader has already hit the orders at the lower levels of the LOB.

Lemma 1 *In a LOB without a specialist, prices are equal to upper tail expectations:* $P(\widetilde{\theta}) = E(s \mid \theta \geq \widetilde{\theta})$. *The spread is increasing in the probability of informed trading,* η. *Marginal traded quantities are increasing in* η *for* θ^h *and* θ^m, *and decreasing for* θ^l.

The optimal marginal quantities offered on the LOB depend on the level of asymmetric information in the market. As η decreases, the offer of liquidity moves from the high to the low levels of the book since adverse selection costs are lower. Hence, the marginal quantity for θ^l is decreasing in η, while marginal quantities for θ^h and θ^m are increasing in η. As a result, the limit order book presents different shapes depending on the size of adverse selection costs as the table shows:

	Marginal Depth at Ask Prices		
Ask LOB Prices	$\eta < 1/11$	$\eta \in [1/11, 1/7)$	$\eta \in [1/7, 1/4]$
$P^h = 2\gamma\sigma^2 I$	$\frac{8\eta}{1+\eta}I$	$\frac{2}{3}I$	$\frac{2}{3}I$
$P^m = (5/3)\gamma\sigma^2 I$	0	$\frac{2}{3}\left(\frac{11\eta-1}{1+\eta}\right)I$	$\frac{1}{3}I$
$P^l = \frac{8\eta}{1+\eta}\gamma\sigma^2 I$	$\frac{1-7\eta}{1+\eta}I$	$\frac{1-7\eta}{1+\eta}I$	0

We denote the ex-ante gains from trade on the LOB without a specialist as $E(GT_a^{LOB})$:

$$E(GT_a^{LOB}) = \left[p^h GT^{LOB}(\theta^h) + p^m GT^{LOB}(\theta^m) + (p_i^l + p_{ni}^l) GT^{LOB}(\theta^l)\right]$$

where $a = \{1, 2, 3\}$ corresponds respectively to the LOB available for $\eta < 1/11$, $\eta \in [1/11, 1/7)$ and $\eta \in [1/7, 1/4]$.

[14] As an example, consider the quantity offered to client θ^l. This quantity is bought also by types θ^m and θ^h. Moreover, types θ^m and θ^h are eager to buy a bigger quantity than type θ^l at price $P(\theta^l)$. So, if liquidity suppliers offer a bigger quantity than the one demanded by type θ^l, they realize negative profits on the extra quantity. In fact, the expected value of the asset is no more $P(\theta^l) = E(s \mid \theta \geq \theta^l)$, but $E(s \mid \theta \geq \theta^m)$.

5 The Hybrid Market: The Limit Order Book and a Specialist

We now consider a market with competitive liquidity suppliers on the LOB and a specialist. The broker compares the LOB and the specialist's offer in order to obtain the better trading terms for his client. The specialist improves LOB quotes only for broker's uninformed clients and influences the liquidity available on the LOB. In fact, liquidity suppliers anticipate the specialist's "cream skimming" and update their expectations on the level of asymmetric information in the market.

5.1 The Limit Order Book with a Specialist

Liquidity suppliers solve the same problem as in the LOB without a specialist, but they take into account in computing upper tail expectations that the specialist could improve trading terms for uninformed clients.

Lemma 2 *Quantities and prices quoted on a LOB with a specialist are equal to the ones quoted on a LOB without a specialist for high levels of asymmetric information, i.e. for* $\eta \in [1/7, 1/4]$.

The specialist's cream skimming of broker's clients implies that adverse selection costs for liquidity suppliers increase. In fact, the adverse selection problem becomes severe and independent from the number of uninformed traders on the market since the introduction of the specialist "eliminates" these traders from the LOB.[15] Therefore, if $\eta \in [1/7, 1/4]$, nothing changes between a LOB with and without a specialist. The adverse selection problem is already so severe that any worsening of it, due to introduction of the specialist, does not produce any effect on the LOB and on trader's gains from trade: $GT_3^{LOB}(\widetilde{\theta}) = GT^S(\widetilde{\theta})$ for $\widetilde{\theta} \in \{\theta^h, \theta^m, \theta_i^l\}$. On the contrary, if $\eta < 1/7$, the introduction of the specialist lowers the liquidity offered on the LOB. The LOB with a specialist is summarized in the following table:

Ask LOB Prices	Marginal Depth
$P^h = 2\gamma\sigma^2 I$	$\frac{2}{3}I$
$P^m = (5/3)\gamma\sigma^2 I$	$\frac{1}{3}I$
$P^l = (8/5)\gamma\sigma^2 I$	0

5.2 The Specialist's Problem

In our model the specialist is a monopolist: profit maximization should determine the optimal price and quantity offered to uninformed traders. However, two effects bind specialist's monopoly power. Firstly, the specialist has to prevent the broker from reporting an informed client as an uninformed one. Secondly, the specialist has no control over the gains from trade available to informed traders

[15] A LOB with a specialist is equivalent to a LOB without a specialist where $\eta = 1/4$. In fact, no uninformed trader hits the LOB: they all go to the specialist to obtain better trading terms. This is equivalent to assuming that there are only informed traders (i.e. $\eta = 1/4$) in the LOB without a specialist.

on the LOB. The specialist solves the following optimization problem:

$$
\begin{aligned}
\max_{Q^S,P^S} \quad & p^l_{ni} P^S Q^S \\
s.t. \quad & (IC^h) \quad GT^S(\theta^h,\theta^h) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S) \geq GT^S(\theta^h,\theta^l_{ni}) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S dev) \\
& (IC^m) \quad GT^S(\theta^m,\theta^m) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S) \geq GT^S(\theta^m,\theta^l_{ni}) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S dev) \\
& (IC^l_i) \quad GT^S(\theta^l_i,\theta^l_i) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S) \geq GT^S(\theta^l_i,\theta^l_{ni}) + \sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^t E(GT^S dev) \\
& (IR^l_{ni}) \quad GT^S(\theta^l_{ni},\theta^l_{ni}) \geq GT^S(\theta^l_{ni},\theta^l_i)
\end{aligned}
$$

The (IR^l_{ni}) constraint states that for the broker the specialist's quote for an uninformed client is more attractive than that on the LOB. The incentive constraints prevent the broker from going to the specialist when his client is informed. They state that the actual gains from trade obtained by the broker on the LOB for his client plus the future commissions obtained by the broker if he maintains the possibility of interacting with the specialist in the future must be greater than the higher commissions obtained if the broker pretends his client is uninformed plus the future commissions from trading with the LOB only. We can rewrite the specialist's problem in the following way:[16]

$$
\begin{aligned}
\max_{Q^S,P^S} \quad & p^l_{ni} P^S Q^S \\
s.t. \quad & (IC^h) \quad GT^S(\theta^h,\theta^h) + \left(\tfrac{p^l_{ni}}{r} - 1\right) GT^S(\theta^l_{ni},\theta^l_{ni}) \geq (\theta^h - \theta^l)Q^S \\
& (IC^m) \quad GT^S(\theta^m,\theta^m) + \left(\tfrac{p^l_{ni}}{r} - 1\right) GT^S(\theta^l_{ni},\theta^l_{ni}) \geq (\theta^m - \theta^l)Q^S \\
& (IC^l_i) \quad GT^S(\theta^l_i,\theta^l_i) + \left(\tfrac{p^l_{ni}}{r} - 1\right) GT^S(\theta^l_{ni},\theta^l_{ni}) \geq 0 \\
& (IR^l_{ni}) \quad GT^S(\theta^l_{ni},\theta^l_{ni}) \geq GT^S(\theta^l_{ni},\theta^l_i)
\end{aligned}
$$

The restatement of the specialist's problem points out that the incentive constraints differ in two dimensions. First, they differ among traders since the gains from trade available on the LOB depend on the agent's type. Trading on the LOB is more attractive for high θ types, since they evaluate the asset more and they are offered bigger quantities in the book. Second, the incentive constraints differ since gains from cheating also depends on the agent's type. The specialist offers only one contract, so any cheating trader pays the same price for the specialist's quantity. However, high θ types benefit more from misreporting since they evaluate to a greater extent the quantity traded with the specialist. This implies that the specialist's quoted quantity determines the relevant incentive constraint. If a large quantity is offered, the relevant incentive constraint will be (IC^h): the specialist's offer is competitive with the one this agent gets on the LOB. If a small quantity is offered, then the high type can obtain better trading terms on the LOB. However, the offer still attracts informed clients with lower evaluations who get less on the LOB: (IC^m) or (IC^l_i) binds.

[16] We refer the reader to the proof of *Proposition 1* for a detailed explanation of the rewriting.

Notice also that the specialist's offer (both price and quantity) has a double influence on the incentive constraints: on the one hand an attractive offer increases the broker's payoff from misreporting, on the other it increases the benefits of the interaction with the specialist in the future. The prevailing effect depends on the ratio:

$$\frac{p_{ni}^l}{r} = \frac{(1-4\eta)}{3r}$$

If $\frac{p_{ni}^l}{r}$ is high, the broker will place a high value on the relationship with the specialist. The broker thinks that he will often have uninformed clients in the future or that he will continue his relationship with the specialist with a high probability. In this case, an attractive specialist's offer decreases incentives to misreport. On the contrary, as r increases or as the asymmetric information problem worsens, the value of keeping a good relationship with the specialist decreases. When $\frac{p_{ni}^l}{r}$ becomes lower than one, an attractive specialist's quote increases incentives to misreport.

5.3 Analysis of the Specialist's Price Schedule

We now wish to consider how the parameters r and η influence the specialist's price schedule. The following Proposition is obtained:

Proposition 1 *The quantity offered by the specialist is monotonically decreasing in r and η, while the price offered is a non monotone function of r and η. Explicit values for Q^S and P^S are in the Appendix.*

We analyse as an example how the specialist varies the price and quantity offered to uninformed traders to compensate an increase in r, and so a decrease in the stability of the relationship with the broker.[17] The specialist's choice is presented in Fig. 1, as a function of r. The picture shows that as the relationship with the specialist worsens, the specialist's quoted quantity decreases. On the contrary, the price reaction is not monotonic. These results depend on the crucial role of incentive constraints in the specialist's choice. First, as opposed to a standard monopoly problem, the binding incentive or participation constraint determines the specialist's inverse demand function, $P^S(Q^S)$. Therefore, the demand of the uninformed client can play no role in the specialist's choice. Second, the incentive constraints, as opposed to the uninformed client's participation constraint, become harder to satisfy when r increases. Hence, the specialist must react to the worsening of the relationship if such constraints are binding. We differentiate two cases depending on the ratio between asymmetric information and relationship trading.

Case 1: A low $\frac{p_{ni}^l}{r}$ ratio. If the ratio $\frac{p_{ni}^l}{r}$ is low, i.e. if $r > \frac{17}{105}(1-4\eta)$, the specialist faces high incentives for the broker to misreport. Hence, he prefers to quote a small quantity and to extract all gains from trade from the uninformed client in order to make cheating as unattractive as possible for the broker: the (IR_{ni}^l) constraint binds in this region. This implies that future

[17] A similar analysis applies to adverse selection costs (represented in our model by the probability of informed trading, η) and is omitted in the paper.

trading opportunities with the specialist have no value. The broker compares only actual gains in order to decide whether reporting truthfully his customer's type. Therefore, the stability of the relationship with the specialist, r, does not influence the broker's choice. The specialist can offer the same price-quantity pair for any value of r, as Fig. 1 shows.

Case 2: A high $\frac{p^l_{ni}}{r}$ ratio. If the ratio $\frac{p^l_{ni}}{r}$ is high, i.e. if $r \leq \frac{17}{105}(1-4\eta)$, the broker is interested in keeping an ongoing relationship with the specialist since he values it highly. Hence, the specialist can quote larger quantities and use the threat of relationship breaking to discipline the broker: the (IR^l_{ni}) constraint does not bind. This implies that in this case the specialist's quantity and price have to vary to compensate an increase in r and the consequent worsening of the incentive constraints. Notice that both a decrease in the price or in the quantity quoted by the specialist improve the incentive constraints: $\partial(IC)/\partial P^S < 0$ and $\partial(IC)/\partial Q^S < 0$.[18] In fact, a more attractive offer increases the value of keeping a good relationship with the specialist. But should the specialist's reaction to an increase in r focus on prices or on quantities?

Consider first the price-quantity pair determined when the incentive constraint of the high type, θ^h, binds. In Fig.1 this corresponds to the price and quantity quoted for $r \leq \frac{2}{33}(1-4\eta)$. The optimal quantity decreases as r increases: when the broker evaluates the relationship less, the specialist lowers the offered quantity in order to maintain truthtelling. In fact, the increase in the discount rate moves down the inverse demand function given by IC^h and the new intersection with the marginal cost line implies a lower quantity.

However, the optimal price is not monotone in r: as r increases, initially the price decreases and then it increases. This effect also depends on the fact that the inverse demand function is determined by the incentive constraint of the high type. Since this constraint depends on both the specialist's quantity and the discount rate, the inverse demand function depends on both variables as well. Formally:

$$\frac{\partial P^S(Q^S_*(r),r)}{\partial r} = \frac{\partial P^S(Q^S_*(r),r)}{\partial r} + \frac{\partial P^S(Q^S_*(r),r)}{\partial Q^S_*(r)}\frac{\partial Q^S_*(r)}{\partial r}$$

where $P^S(Q^S_*(r),r)$ is the inverse demand function and $Q^S_*(r)$ is the optimal quantity as a function of the discount rate. If r increases, two opposite effects are realized. On the one hand, the incentive constraint worsens and the price should decrease to compensate. The first negative part of the derivative accounts for this effect. On the other hand, the quoted quantity decreases, so the specialist can increase the price and still keep the incentive constraint satisfied. This effect is represented by the second positive part of the derivative. Hence, the net effect on price of an increase in the discount rate is ambiguous. As it is evident from the picture, the first effect dominates initially, while the second one increases in strength after a while.[19]

We now consider the price and quantity offered by the specialist when both (IC^h) and (IC^m) bind. In Fig.1 this corresponds to the price and quantity quoted for $r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$. The

[18] The derivatives of the incentive constraints with respect to the specialist's quantity are not always negative. However, each incentive constraint derivative is always negative in the relevant regions, where the constraint binds.

[19] A similar explanation applies when the incentive constraint of type θ^m binds. In Fig.1 this corresponds to the price and quantity quoted for $r \in \left[\frac{4}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$.

optimal quantity is constant with respect to r, while the price is decreasing. In fact, two incentive constraints are binding now and the specialist must select the quantity that solves: $(IC^h) = (IC^m)$. As r increases, the specialist has not anymore the option to decide between decreasing the price or the quantity. The quantity is fixed, so the only possibility to keep the incentive constraints satisfied is to decrease the price.

6 Market Structure Comparison

In this Section we compare the two presented market structures: the LOB and the LOB with a specialist. First, we analyze the spread on the two markets. Second, we compare the two market structures considering utilitarian social welfare. Finally, we present some empirical and policy implications of our work.

6.1 Market Spread

Different market structures are often compared in terms of quoted and effective spreads. The quoted spread is defined as the difference between the best bid and best ask offered on the LOB and the effective spread as twice the difference between the transaction price and the midquote for a given trade size. Clearly, the effective spread reflects savings due to trading inside the quotes. Consider first the quoted spread: the LOB without a specialist clearly dominates as *Lemma 2* shows. In fact, the quoted spread catches only the negative part of the specialist's introduction: the increase in adverse selection costs on the LOB. In order to catch also the positive effect, the decrease in asymmetric information thanks to relationship trading, we should also consider trades inside the quoted spread. In order to do this, it is necessary to focus on the average effective spread on the two markets. The following Proposition is obtained:

Proposition 2 *The hybrid market offers a lower average effective spread for high levels of asymmetric information, i.e. for $\eta \geq 1/7$, while the LOB without a specialist offers a lower average effective spread for low levels of asymmetric information, i.e. for $\eta < 1/12$. For average levels of asymmetric information, i.e. for $\eta \in [1/12, 1/7)$, the hybrid market or the pure LOB offer a lower average effective spread depending on the value of r.*

The hybrid market has a lower effective spread for high levels of asymmetric information: both markets offer the same liquidity on the LOB, but the hybrid one offers also the possibility to trade with the specialist inside the quotes. For average levels of asymmetric information, the strength of relationship trading becomes crucial to compare the effective spreads. Initially the hybrid market has a lower average effective spread than the pure LOB for low values of r, since the stable relationship with the broker allows the specialist to quote a low price. However, as η decreases further, the effective spread in the pure LOB market decreases as well. This implies that the hybrid market starts to offer a lower effective spread only for the specialist's best prices, and so for "average" values of r. Finally, for low levels of asymmetric information, the pure LOB offers such a big quantity on the lowest level of the book that the effective spread is always lower than in the hybrid market independently from the strength of relationship trading.

6.2 Welfare Analysis

We define utilitarian social welfare as the expected gains from trade available on the market. We compare the expected gains from trade on the LOB with a specialist, $E(GT^S)$, and on the LOB without a specialist, $E(GT_a^{LOB})$.

Proposition 3 *Utilitarian social welfare is higher on a LOB with a specialist for high levels of adverse selection (η high) and on a LOB without a specialist for low levels of adverse selection (η low). For average levels of adverse selection, a LOB with a specialist offers a higher utilitarian social welfare for low values of the discount rate, r.*

Expected gains from trade depend on the trading terms offered to informed and uninformed traders by the two market structures. Informed traders trade only on the LOB. Hence, they are indifferent in terms of a LOB with or without a specialist if $\eta \in [1/7, 1/4]$, since the two LOBs are identical. The situation changes for $\eta \in [0, 1/7)$. Now informed traders strictly prefer a LOB without a specialist since they can obtain higher gains from trade. In fact, this market structure offers the same or larger quantities than the LOB with a specialist and at a lower price. If we consider the uninformed traders, they prefer a LOB with a specialist if $\eta \in [1/7, 1/4]$. A pure LOB offers zero gains from trade, while a hybrid market allows for positive gains from trade since uninformed traders can exchange a positive quantity with the specialist. Notice that if r is greater, the specialist will extract all rents from the uninformed traders who become indifferent between the two market structures. For $\eta \in [0, 1/7)$ the uninformed traders prefer a pure LOB or a LOB with a specialist depending on the attractiveness of the specialist's offer, and so on the values of r and η. Traders' preferences are summarized in the following table:

	Trader's Preferred Market Structure	
Trader's Type	$\eta < 1/7$	$\eta \in [1/7, 1/4]$
$\theta^h, \theta^m, \theta_i^l$	LOB only	Indiff.
θ_{ni}^l	Depends on r, η	LOB with Specialist or Indiff.

Expected gains from trade are higher in a LOB with a specialist if adverse selection costs are high, while they are higher in a LOB without a specialist if adverse selection costs are low. In the first case the introduction of the specialist on the market is a Pareto improvement since the specialist provides liquidity to traders out of the market without a worsened LOB. In the second case the necessity to induce truthtelling limits specialist's offer of liquidity and the trading terms offered on the pure LOB are better than the ones offered on the LOB with a specialist even for uninformed traders. Finally, for average levels of adverse selection, the discount rate determines which market presents higher expected gains from trade. For low levels of the discount rate, the specialist can offer attractive quotes to uninformed traders and in this way compensate the lower gains from trade available on the LOB to informed traders. On the contrary, for high levels of the discount rate, the specialist's offer is not competitive enough to counterbalance the lower gains from trade on the LOB. Results are summarized in Fig. 2.

A second possible specification of utilitarian social welfare is to consider the sum of expected gains from trade and liquidity suppliers' profits. This second specification reinforces the results obtained when considering only gains from trade.[20] In this case utilitarian social welfare still coincides with the gains from trade in a LOB without a specialist, since liquidity suppliers are competitive and $\pi^L = 0$, while it is given by the sum of gains from trade and specialist's profits in a LOB with a specialist. This implies that the region where the hybrid market is optimal is larger, since we now include specialist's profits in the analysis. However, when asymmetric information is low or relationship trading is weak, the LOB without a specialist still offers a higher utilitarian welfare than the hybrid market. The necessity to induce truthtelling limits specialist's profits. Hence, they are not high enough to compensate the social loss due to the better trading terms offered on the LOB without a specialist.

6.3 Empirical Implications

A first empirical implication of our work is related to the comparison of quoted and effective spreads between a pure LOB and a specialist market. The quoted spread should be better, on average, in the pure LOB, since adverse selection is higher on the specialist market. The effective spread should be lower, on average, on the hybrid market for stocks thinly traded, stocks that have been recently quoted and stocks with high levels of insider trading, since these stocks are more exposed to asymmetric information problems. On the contrary, the effective spread should be lower on average on the pure LOB market for largely traded stocks and for stocks traded on the market from a long time, less exposed to adverse selection problems.

A second empirical implication concerns the specialist's quoted quantities and prices. The specialist's activity should concentrate on large trades when relationship trading works, and on small trades when it does not. Moreover, the specialist's price improvement should be small when the relation with the broker is not stable or the risk of being picked-off is high, and large in the opposite case. Notice also that the specialist uses both price and quantity to adjust for the increasing asymmetric information or the worsening of relationship trading. Specialist's reaction seems to focus more on depth than on prices.

Third, we consider the effective spread for different trade sizes. In our model, the specialist does not improve the effective spread for all trade sizes since the offered quantity depends on the stability of his relationship with the broker.[21] Therefore, if the specialist market offers a lower effective spread on average than a pure LOB, for which trade sizes does the specialist improve on the quotes? Our analysis suggests that when the broker/specialist relationship is stable, the effective spread is lower for big trades. The specialist can offer big quantities to uninformed traders without being afraid of incentivizing misreporting since reputation concerns matter a lot for brokers. On the contrary, when the relationship is unstable, the effective spread should be lower for small trades. The specialist improves prices within the quotes only for small quantities to decrease

[20] A complete analysis of this second specification of utilitarian social welfare is available upon the author.

[21] Since our model is discrete and quantities are endogenously determined, specific quantity sizes can be traded on one market structure but not on the other. So, a direct comparison is not possible. However, dividing trades in size groups, some empirical implications can be derived.

broker's incentives to misreport.

Finally, we consider the average number of trades. When asymmetric information problems are high, we should expect more trades on the hybrid market: the specialist opens the market to uninformed traders that would not trade in a pure LOB. Moreover, the greater intensity of trading compared to a pure LOB should concentrate on small quantities if the broker/specialist relationship is unstable and on big quantities if it is stable. The opposite situation occurs when asymmetric information is low: in this case, the hybrid market closes the door to informed traders with a low evaluation of the asset that would trade in a pure LOB. So, we should expect a lower number of trades on average in a LOB with a specialist. Notice that we could still see on this market small trades if the broker/specialist relationship is unstable, otherwise we should expect bigger traded quantities on average.

6.4 Policy Implications

There is an ongoing debate about the advantages and disadvantages of different market structures. In particular, the role of the specialist on the NYSE has been questioned: is it beneficial or not to give a monopolist position in a stock to a profit seeking agent, even if his market power is bounded by competition from the LOB and regulatory requirements?

We compare a pure limit order book with a hybrid specialist market and we focus our analysis on one important difference between the two market structures: relationship trading. Our work shows that the hybrid market can improve on a pure LOB if adverse selection costs are high or if the specialist/broker relationship is stable. In this case, the hybrid market offers the same quoted spread than a pure LOB, but a better effective spread. Moreover, it guarantees a higher welfare level. However, if adverse selection costs are low or if the specialist/broker relationship is unstable, then the hybrid market is worse than a pure LOB. In fact, both the quoted and the effective spread are wider in the hybrid market, the specialist's profits are too low to compensate the loss in the gains from trade due to the wider spread and social welfare is lower.

A crucial question arises: can competition among markets assure that the best market structure is going to dominate? Introducing a specialist can be beneficial for stocks thinly traded or highly exposed to asymmetric information, but detrimental for other stocks that do not have high adverse selection costs. Moreover, market conditions can change: a specialist system could be optimal in the initial quotation phase of a stock, when investors are less informed about the asset and the risk of exposure to insider trading is higher, but not optimal after a certain period, when the stock starts to be better known by the general public. Therefore, it is important to understand if free competition among markets can exclude the specialist market when it is suboptimal.

Our work suggests that the specialist hybrid market, once in place, cannot be overcome by a competing pure limit order book even when the hybrid structure is suboptimal. In fact, the specialist can always improve on the trading terms offered to uninformed traders, given that he owns an informative advantage due to relationship trading. Brokers, after looking at the pure LOB quotations, can always go to the hybrid market to ask the specialist for price improvement. Liquidity suppliers quoting on the pure LOB will anticipate the specialist's cream skimming and offer trading terms identical to the ones offered by the LOB on the hybrid market. So, no price improvement is provided by the competing pure LOB. However, the advantages of the hybrid market should increase in time. In fact, once the specialist's system is in place, the stability of the broker/specialist relationship should increase and trading terms improve. This effect should partially compensate the competition proofness of this market structure.

In our setting, as in Glosten (1994), trading and liquidity should concentrate in one market structure. However, differently from Glosten (1994), this market structure is not the pure LOB: competition by a non-anonymous market architecture as the hybrid market seems to be successful. If we compare our findings to Parlour and Seppi (2003), there are some differences. Even if theoretically a pure LOB and a LOB with a specialist can coexist, no real increase in competition comes from the pure LOB. Hence, competition seems not to be able to select the best market structure: even if the pure LOB could be potentially more liquid than the hybrid market, the monopolistic position of the specialist in relationship trading constrains market competition. Differently from Parlour and Seppi (2003), we do not consider the possibility of allowing more traders to supply liquidity ex-post. However, the benefits of the specialist's provision of liquidity ex-post are positively related to the stability of his relationship with the broker. We think that multiple ex-post liquidity suppliers would drastically reduce this stability and so the benefits of relationship trading.

To summarize, our work shows that the introduction of a specialist on a LOB must be carefully thought over. On the one hand, relationship trading can improve effective spreads and social welfare for thinly traded stocks or stocks with high adverse selection costs. On the other hand, the hybrid market structure is suboptimal for largely traded stocks not exposed to high adverse selection. Since our study suggests that competition among market structures can not be sufficient to select the optimal one, a hybrid market could lead to wider spreads and lower welfare levels.

7 Conclusions

In this paper we have analyzed the benefits of relationship trading by comparing two market structures. The first is an anonymous LOB with free entry, the second is a LOB with a specialist. In the LOB without a specialist, the market promotes competition among limit order traders in order to supply liquidity. In the LOB with a specialist, the specialist can offer better trading terms to broker's uninformed clients due to relationship trading. A trade-off arises: on the one hand, the specialist worsens adverse selection on the book, but on the other, he lowers the asymmetric information problem via relationship trading. So, in the hybrid market informed traders will go on the anonymous LOB to benefit from competition, while uninformed traders will go to the specialist, to benefit from relationship trading. Our analysis is divided in two parts. First, we have focused on the influence of adverse selection costs and of LOB competition on the price schedule offered by the specialist to uninformed traders. Second, we have compared markets' spreads and utilitarian social welfare across the two market structures.

The specialist's quoted depth is a monotone decreasing function of adverse selection costs and a monotone increasing function of the probability of continuation of the relationship with the specialist. On the contrary, the specialist's quoted price is not monotone in these two parameters.

If asymmetric information worsens or if the relationship with the broker is less stable, the specialist's reaction will focus more on depth than on the price. This result is consistent with empirical findings about the importance of the quantity aspect of a specialist's price schedule. In terms of social welfare, we show that the specialist is Pareto improving for high levels of adverse selection. In fact, in this case the LOB is already offering a wide spread and the introduction of the specialist does not have any effect on it. On the contrary, for low levels of adverse selection, the specialist lowers the liquidity offered on the LOB by competitive liquidity suppliers and can reduce social welfare.

Our results suggest that a relationship trading system can improve on the trading terms offered by the LOB for stocks that are highly exposed to adverse selection problems. On the contrary, for stocks that are not exposed to high levels of asymmetric information, the introduction of the specialist lowers social welfare. So, we expect that the specialist could induce a positive effect on market liquidity for stocks that are not frequently traded, for which asymmetric information problems are more relevant, or for stocks more exposed to insider trading. It is important to notice that the specialist can be beneficial in welfare terms also for stocks with low adverse selection problems if the relationship with the specialist is stable. In fact, in this last case the lower broker's gains from trade are compensated by the higher specialist's profit.

Appendix A

Proof of Lemma 1

Limit traders are competitive, so the posted marginal price must be equal to the expected value of the asset given that the order has been hit. This implies that, when computing the price for agent $\widetilde{\theta}$, limit traders anticipate that the order will be hit by any trader with an asset evaluation $\theta \geq \widetilde{\theta}$. Hence, prices are equal to upper tail expectations. A lower price would imply negative profits, while a higher price would be undercut by the other limit traders. Remember also that the LOB is unable to discriminate between types θ^l_i and θ^l_{ni}. Quoted prices are the following:

$$P^h_{LOB} = E(s_t \mid \theta \geq \theta^h) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)}{\Pr(\theta=\theta^h)} = 2\gamma\sigma^2 I$$

$$P^m_{LOB} = E(s_t \mid \theta \geq \theta^m) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)+E(s|\theta^m)\Pr(\theta=\theta^m)}{\Pr(\theta=\theta^h)+\Pr(\theta=\theta^m)} = \frac{5}{3}\gamma\sigma^2 I$$

$$P^l_{LOB} = E(s_t \mid \theta \geq \theta^l) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)+E(s|\theta^m)\Pr(\theta=\theta^m)+E(s|\theta^l)\Pr(\theta=\theta^l)}{\Pr(\theta=\theta^h)+\Pr(\theta=\theta^m)+\Pr(\theta=\theta^l)} = \frac{8\eta}{1+\eta}\gamma\sigma^2 I$$

The investor trades only if the price offered on the LOB is lower than his evaluation of the asset. Given that $\theta^h = 3\gamma\sigma^2 I$, $\theta^m = 2\gamma\sigma^2 I$ and $\theta^l = \gamma\sigma^2 I$, the price on the LOB can be too high for the low type. We differentiate two cases:

CASE 1: $\eta \geq 1/7$

Since $P^l_{LOB} \geq \theta^l$, limit traders anticipate that θ^l never hits an order at that price and offer a zero quantity: $q^{l*} = 0$. The optimal LOB's quantities are determined by solving the following two problems:

$$\max_{q^m} \quad E\left(2\gamma\sigma^2 I q^m - \frac{\gamma\sigma^2}{2}(q^m)^2 - \frac{5}{3}\gamma\sigma^2 I q^m\right)$$

$$\max_{q^h} \quad E\left(3\gamma\sigma^2 I\left(q^h + q^{m*}\right) - \frac{\gamma\sigma^2}{2}\left(q^h + q^{m*}\right)^2 - 2\gamma\sigma^2 I q^h - \frac{5}{3}\gamma\sigma^2 I q^{m*}\right)$$

where q^{m*} is the solution of the first maximization problem. So, $q^{l*} = 0$, $q^{m*} = \frac{1}{3}I$ and $q^{h*} = \frac{2}{3}I$.

CASE 2: $\eta < 1/7$

The optimal quantities offered are determined by solving the following problems:

$$\max_{q^l} \quad E\left(\gamma\sigma^2 I q^l - \frac{\gamma\sigma^2}{2}\left(q^l\right)^2 - \frac{8\eta}{1+\eta}\gamma\sigma^2 I q^l\right)$$

$$\max_{q^m} \quad E\left(2\gamma\sigma^2 I\left(q^m + q^{l*}\right) - \frac{\gamma\sigma^2}{2}\left(q^m + q^{l*}\right)^2 - \frac{5}{3}\gamma\sigma^2 I q^m - \frac{8\eta}{1+\eta}\gamma\sigma^2 I q^{l*}\right)$$

$$\max_{q^h} \quad E\left(3\gamma\sigma^2 I\left(q^h + q^{m*} + q^{l*}\right) - \frac{\gamma\sigma^2}{2}\left(q^h + q^{m*} + q^{l*}\right)^2 - 2\gamma\sigma^2 I q^h - \frac{5}{3}\gamma\sigma^2 I q^{m*} - \frac{8\eta}{1+\eta}\gamma\sigma^2 I q^{l*}\right)$$

where q^{l*} is the solution of the first maximization problem and q^{m*} the solution of the second one. The final solution is: $q^{l*} = \frac{1-7\eta}{1+\eta}I$, $q^{m*} = \frac{2}{3}\left(\frac{11\eta-1}{1+\eta}\right)I$ and $q^{h*} = \frac{2}{3}I$.
Notice that q^{m*} is not always positive. For $\eta < 1/11$ the limit order traders prefer to pool the low and average type at price $P^l_{LOB} = \frac{8\eta}{1+\eta}\gamma\sigma^2 I$. The liquidity suppliers' problem becomes:

$$\max_{q^l} \quad E\left(\gamma\sigma^2 I q^l - \frac{\gamma\sigma^2}{2}\left(q^l\right)^2 - \frac{8\eta}{1+\eta}\gamma\sigma^2 I q^l\right)$$

$$\max_{q^h} \quad E\left(3\gamma\sigma^2 I\left(q^h + q^{l*}\right) - \frac{\gamma\sigma^2}{2}\left(q^h + q^{l*}\right)^2 - 2\gamma\sigma^2 I q^h - \frac{8\eta}{1+\eta}\gamma\sigma^2 I q^{l*}\right)$$

The solution is $q^{l*} = \frac{1-7\eta}{1+\eta} I$ and $q^{h*} = \frac{8\eta}{1+\eta} I$.

Proof of Lemma 2

We first show that the specialist can always improve on pure LOB quotes. We must check that the specialist can both offer a better quote and induce broker's truthtelling. Notice that the specialist can always offer a better price since his asset evaluation is lower than the one of liquidity suppliers: $E(s \mid \theta = \theta_i^l) = 0$. Consider a pure LOB and suppose that the specialist offers to uninformed clients a lower price than the LOB and a quantity such that $GT^S(\theta_{ni}^l, \theta_{ni}^l) = GT^{LOB}(\theta^l) + \varepsilon$, with $\varepsilon \geq 0$. Given that the price can be as small as zero, the specialist can always make such an offer. We differentiate two cases:

CASE 1: $\eta \in [1/7, 1/4]$

In this case $GT^{LOB}(\theta^l) = 0$ and $q^l = 0$. We focus on the broker's truthtelling constraint when he has a client of type θ_i^l:

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \left(p_{ni}^l \varepsilon\right) \geq \varepsilon$$

We can rewrite it as:

$$\left(\frac{(1-4\eta)}{3r} - 1\right)\varepsilon \geq 0$$

If $r < \frac{1}{3}(1-4\eta)$ the constraint is satisfied for $\varepsilon > 0$, while if $r \geq \frac{1}{3}(1-4\eta)$ the constraint is satisfied for $\varepsilon = 0$. If the low type constraint is satisfied, the other two constraints are satisfied as well for $Q^S < \frac{1}{18} I$. So, the specialist can always offer a price-quantity pair that induces truthtelling and gives to uninformed clients gains from trade at least equal to the ones they can obtain in a pure LOB. A hybrid market always exists.

CASE 2: $\eta \in [1/11, 1/7)$

In this case $GT^{LOB}(\theta^l) = 0$ and $q^l > 0$. Broker's truthtelling constraint when he has a client of type θ_i^l is the following:

$$GT^{LOB}(\theta^l) + \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \left(p_{ni}^l \varepsilon\right) \geq GT^{LOB}(\theta^l) + \varepsilon$$

We can rewrite it as:

$$\left(\frac{(1-4\eta)}{3r} - 1\right)\varepsilon \geq 0$$

The same analysis of Case 1 applies. If the low type constraint is satisfied, the other two constraints are satisfied as well for $Q^S \leq q^l$. A hybrid market always exists.

CASE 3: $\eta < 1/11$

Also in this case $GT^{LOB}(\theta^l) = 0$ and $q^l > 0$, the same analysis of Case 2 applies. Again, a hybrid market always exists and the constraints of types θ^m and θ^h are satisfied if $Q^S < q^l$.

We now determine the optimal price and quantity offered on the LOB. Marginal prices are still equal to upper tail expectations of the asset value, but liquidity suppliers anticipate that the specialist is going to improve LOB quotations for uninformed clients:

$P^h_{SPEC} = E(s \mid \theta \geq \theta^h) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)}{\Pr(\theta=\theta^h)} = 2\gamma\sigma^2 I$

$P^m_{SPEC} = E(s \mid \theta \geq \theta^m) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)+E(s|\theta^m)\Pr(\theta=\theta^m)}{\Pr(\theta=\theta^h)+\Pr(\theta=\theta^m)} = \frac{5}{3}\gamma\sigma^2 I$

$P^l_{SPEC} = E(s \mid \theta \geq \theta^l) = \frac{E(s|\theta^h)*\Pr(\theta=\theta^h)+E(s|\theta^m)\Pr(\theta=\theta^m)+E(s|\theta^l_i)\Pr(\theta=\theta^l_i)}{\Pr(\theta=\theta^h)+\Pr(\theta=\theta^m)+\Pr(\theta=\theta^l_i)} = \frac{8}{5}\gamma\sigma^2 I$

The price offered on the LOB is too high for the low type investor. Competitive limit traders anticipate this and offer a zero quantity at that price. The limit traders' quoted quantities are equal to the ones in *Lemma 1* for $\eta \in [1/7, 1/4]$: $q^{l*} = 0$, $q^{m*} = \frac{1}{3}I$ and $q^{h*} = \frac{2}{3}I$. Notice also that $P_{SPEC} = P_{LOB}$ for both θ^h and θ^m.

Proof of Proposition 1

Notice that $GT^S(\theta^h, \theta^l_{ni}) = (\theta^h - \theta^l)Q^S + GT^S(\theta^l_{ni}, \theta^l_{ni})$, $GT^S(\theta^m, \theta^l_{ni}) = (\theta^m - \theta^l)Q^S + GT^S(\theta^l_{ni}, \theta^l_{ni})$ and that $GT^S(\theta^l_i, \theta^l_{ni}) = GT^S(\theta^l_{ni}, \theta^l_{ni})$, since $\theta^l_{ni} = \theta^l_i$. From the quantities determined in *Lemma 2* for the LOB with a specialist, we compute the LOB's gains from trade: $GT^S(\theta^h, \theta^h) = \frac{11}{18}\gamma\sigma^2 I^2$, $GT^S(\theta^m, \theta^m) = \frac{1}{18}\gamma\sigma^2 I^2$ and $GT^S(\theta^l_i, \theta^l_i) = GT^S(\theta^l_{ni}, \theta^l_i) = 0$. The specialist's problem can be rewritten in the following way:

$$\begin{array}{lll}
\max\limits_{Q^S, P^S} & & p^l_{ni} P^S Q^S \\
s.t. & (IC^h) & \dfrac{11}{18}\gamma\sigma^2 I^2 + \left(\frac{(1-4\eta)}{3r} - 1\right) GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq 2\gamma\sigma^2 I Q^S \\
& (IC^m) & \dfrac{1}{18}\gamma\sigma^2 I^2 + \left(\frac{(1-4\eta)}{3r} - 1\right) GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq \gamma\sigma^2 I Q^S \\
& (IC^l_i) & \left(\frac{(1-4\eta)}{3r} - 1\right) GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq 0 \\
& (IR^l_{ni}) & GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq 0
\end{array}$$

Notice that if $r \leq (1-4\eta)/3$, then (IC^l_i) and (IR^l_{ni}) are both satisfied if $GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq 0$. However, if $r > (1-4\eta)/3$, then (IC^l_i) is satisfied for $GT^S(\theta^l_{ni}, \theta^l_{ni}) \leq 0$ and (IR^l_{ni}) for $GT^S(\theta^l_{ni}, \theta^l_{ni}) \geq 0$. The only possible solution is $GT^S(\theta^l_{ni}, \theta^l_{ni}) = 0$.

CASE 1: $r > \frac{1}{3}(1-4\eta)$

(IR^l_{ni}) binds in order to have (IC^l_i) satisfied. There are three possible cases:

a) (IR^l_{ni}) binding

b) (IR^l_{ni}) and (IC^m) binding

c) (IR^l_{ni}) and (IC^h) binding

Case 1a: (IR^l_{ni}) binding

From (IR^l_{ni}) we obtain:

$$P^S_a = \gamma\sigma^2 I - \frac{\gamma\sigma^2}{2} Q^S_a$$

The specialist solves the following problem:

$$\max_{Q^S_a} \left(\gamma\sigma^2 I - \frac{\gamma\sigma^2}{2} Q^S_a\right) Q^S_a$$

The optimal quantity and price are: $Q^S_{a*} = I$, $P^S_{a*} = \frac{1}{2}\gamma\sigma^2 I$. However, the quantity and price so determined do not satisfy (IC^h) and (IC^m) and can not be a solution.

Case 1b: (IR^l_{ni}) and (IC^h) binding

From (IR^l_{ni}) and (IC^h) we obtain: $Q^S_{b*} = \frac{11}{36} I$, $P^S_{b*} = \frac{61}{72}\gamma\sigma^2 I$. However, the quantity and price so determined do not satisfy (IC^m) and can not be a solution.

Case 1c: (IR^l_{ni}) and (IC^m) binding

From (IR^l_{ni}) and (IC^m) we obtain: $Q^S_{c*} = \frac{1}{18} I$, $P^S_{c*} = \frac{35}{36}\gamma\sigma^2 I$. Given that the price and quantity so determined satisfy (IC^h), this will be the specialist's choice.

CASE 2: $r \leq \frac{1}{3}(1 - 4\eta)$

There are six possible cases:

a) (IC^l_i) binding

b) (IC^l_i) and (IC^m) binding

c) (IC^l_i) and (IC^h) binding

d) (IC^h) binding

e) (IC^m) binding

f) (IC^m) and (IC^h) binding

From the previous analysis, we already know that cases (a) and (b) can not be a solution, while (c) can be a solution where $Q^S_{c*} = \frac{1}{18} I$, $P^S_{c*} = \frac{35}{36}\gamma\sigma^2 I$. We analyze the remaining cases.

Case 2d: (IC^h) binding

From (IC^h) we obtain:

$$P^S_d = \frac{(9r+4\eta-1)}{(3r+4\eta-1)}\gamma\sigma^2 I - \frac{\gamma\sigma^2}{2} Q^S_d - \frac{11 r\gamma\sigma^2 I^2}{6(3r+4\eta-1)Q^S_d}$$

The specialist solves the following problem:

$$\max_{Q_d^S} \left(\frac{(9r+4\eta-1)}{(3r+4\eta-1)} \gamma\sigma^2 I - \frac{\gamma\sigma^2}{2} Q_d^S - \frac{11r\gamma\sigma^2 I^2}{6(3r+4\eta-1)Q_d^S} \right) Q_d^S$$

The optimal quantity and price are:

$$Q_{d*}^S = \frac{(9r+4\eta-1)}{(3r+4\eta-1)} I$$

$$P_{d*}^S = \left[\frac{(9r+4\eta-1)}{(3r+4\eta-1)} \frac{1}{2} - \frac{11r}{6(9r+4\eta-1)} \right] \gamma\sigma^2 I$$

First, we check if the offered quantity is positive: Q_{d*}^S is positive for $r < \frac{1}{9}(1-4\eta)$ and for $r > \frac{1}{3}(1-4\eta)$. Given that we are considering the case $r \leq \frac{1}{3}(1-4\eta)$, the only relevant region in which Q_{d*}^S can be a solution is $r < \frac{1}{9}(1-4\eta)$. In this case, we also have that $P_{d*}^S \geq 0$. Moreover, we check if Q_{d*}^S and P_{d*}^S satisfy the omitted constraints : in the relevant region, (IC_i^l) is satisfied for $r < \frac{25}{291}(1-4\eta)$ and (IC^m) for $r < \frac{2}{33}(1-4\eta)$. Therefore, Q_{d*}^S and P_{d*}^S can be a solution only for $r < \frac{2}{33}(1-4\eta)$.

Case 2e: (IC^m) binding

From (IC^m) we obtain:

$$P_e^S = \frac{(6r+4\eta-1)}{(3r+4\eta-1)} \gamma\sigma^2 I - \frac{\gamma\sigma^2}{2} Q_e^S - \frac{r\gamma\sigma^2 I^2}{6(3r+4\eta-1)Q_e^S}$$

The specialist solves the following problem:

$$\max_{Q_e^S} \left(\frac{(6r+4\eta-1)}{(3r+4\eta-1)} k - \frac{\gamma\sigma^2}{2} Q_e^S - \frac{r\gamma\sigma^2 I^2}{6(3r+4\eta-1)Q_e^S} \right) Q_e^S$$

The optimal quantity and price are:

$$Q_{e*}^S = \frac{(6r+4\eta-1)}{(3r+4\eta-1)} I$$

$$P_{e*}^S = \left[\frac{(6r+4\eta-1)}{(3r+4\eta-1)} \frac{1}{2} - \frac{r}{6(6r+4\eta-1)} \right] \gamma\sigma^2 I$$

First, we check if the offered quantity is positive: Q_{e*}^S is positive for $r < \frac{1}{6}(1-4\eta)$ and for $r > \frac{1}{3}(1-4\eta)$. Given that we are considering the case $r \leq \frac{1}{3}(1-4\eta)$, the only relevant region in which Q_{e*}^S can be a solution is $r < \frac{1}{6}(1-4\eta)$. In this case, we also have that $P_{e*}^S \geq 0$. Moreover, we check if the omitted constraints are satisfied by Q_{e*}^S and P_{e*}^S: in the relevant region, (IC_i^l) is satisfied for $r < \frac{17}{105}(1-4\eta)$ and (IC^m) for $r > \frac{2}{39}(1-4\eta)$. So, Q_{e*}^S and P_{e*}^S can be a solution only for $r \in \left[\frac{2}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$.

Case 2f: (IC^m) and (IC^h) binding

From (IC^m) and (IC^h) we obtain:

$$Q_{f*}^S = \frac{5}{9} I$$

$$P_{f*}^S = \frac{(438r+260\eta-65)}{90(3r+4\eta-1)} \gamma\sigma^2 I$$

The price and quantity so determined satisfy (IC_i^l). We also check that the price P_{f*}^S is positive. This is true for $r < \frac{65}{438}(1-4\eta)$.

We now consider different regions depending on the value of r:

- $r < \frac{2}{33}(1-4\eta)$

  In this region $\left(Q_{c*}^S, P_{c*}^S\right)$, $\left(Q_{d*}^S, P_{d*}^S\right)$ and $\left(Q_{f*}^S, P_{f*}^S\right)$ are feasible. The specialist selects the quantity-price pair that maximizes his profits. Given that $\pi_{d*}^S \geq \pi_{f*}^S > \pi_{c*}^S$ in this interval, the specialist's choice is $\left(Q_{d*}^S, P_{d*}^S\right)$.

- $r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$

  In this region $\left(Q_{c*}^S, P_{c*}^S\right)$ and $\left(Q_{f*}^S, P_{f*}^S\right)$ are feasible. Given that $\pi_{f*}^S > \pi_{c*}^S$ in the region considered, the specialist's choice is $\left(Q_{f*}^S, P_{f*}^S\right)$.

- $r \in \left[\frac{4}{39}(1-4\eta), \frac{65}{438}(1-4\eta)\right)$

  In this region $\left(Q_{c*}^S, P_{c*}^S\right)$, $\left(Q_{e*}^S, P_{e*}^S\right)$ and $\left(Q_{f*}^S, P_{f*}^S\right)$ are feasible. Given that $\pi_{e*}^S > \pi_{f*}^S$ and $\pi_{e*}^S > \pi_{c*}^S$ in the region considered, the specialist's choice is $\left(Q_{e*}^S, P_{e*}^S\right)$.

- $r \in \left[\frac{65}{438}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$

  Only $\left(Q_{c*}^S, P_{c*}^S\right)$ and $\left(Q_{e*}^S, P_{e*}^S\right)$ are feasible. Given that $\pi_{e*}^S \geq \pi_{c*}^S$ in the region considered, the specialist's choice is $\left(Q_{e*}^S, P_{e*}^S\right)$.

- $r \in \left[\frac{17}{105}(1-4\eta), \frac{1}{3}(1-4\eta)\right]$

  Only $\left(Q_{c*}^S, P_{c*}^S\right)$ is feasible and so is selected by the specialist.

To summarize, the specialist's optimal choice is described in the following table:

Value of r	Specialist's Quantity	Specialist's Price
$r < \frac{2}{33}(1-4\eta)$	$Q_{d*}^S = \frac{(9r+4\eta-1)}{(3r+4\eta-1)}I$	$P_{d*}^S = \left[\frac{(9r+4\eta-1)}{(3r+4\eta-1)}\frac{1}{2} - \frac{11r}{6(9r+4\eta-1)}\right]\gamma\sigma^2 I$
$r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$	$Q_{f*}^S = \frac{5}{9}I$	$P_{f*}^S = \frac{(438r+260\eta-65)}{90(3r+4\eta-1)}\gamma\sigma^2 I$
$r \in \left[\frac{4}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$	$Q_{e*}^S = \frac{(6r+4\eta-1)}{(3r+4\eta-1)}I$	$P_{e*}^S = \left[\frac{(6r+4\eta-1)}{(3r+4\eta-1)}\frac{1}{2} - \frac{r}{6(6r+4\eta-1)}\right]\gamma\sigma^2 I$
$r \geq \frac{17}{105}(1-4\eta)$	$Q_{c*}^S = \frac{1}{18}I$	$P_{c*}^S = \frac{35}{36}\gamma\sigma^2 I$

From the price and quantity determined, it is straightforward to show that the specialist's quantity is monotonically decreasing in r and η, while the specialist's price is not monotone in r and η. Moreover, the level of adverse selection on the market, η, does not influence the qualitative shape of the price and quantity offered by the specialist as a function of r. The parameter η only influences the size of the intervals for which $\left(Q_{d*}^S, P_{d*}^S\right)$, $\left(Q_{e*}^S, P_{e*}^S\right)$, $\left(Q_{f*}^S, P_{f*}^S\right)$ and $\left(Q_{c*}^S, P_{c*}^S\right)$ are offered. As η increases, the regions for which $\left(Q_{d*}^S, P_{d*}^S\right)$, $\left(Q_{e*}^S, P_{e*}^S\right)$ and $\left(Q_{f*}^S, P_{f*}^S\right)$ are optimal decrease, while the region for which $\left(Q_{c*}^S, P_{c*}^S\right)$ is optimal increases. A similar reasoning applies to r and the price and quantity offered by the specialist as a function of η.

Proof of Proposition 2

Given that the model is symmetric we compute half spreads. The effective half spread is the average price paid for the transaction by the agent:

$$\text{effective half spread } (\widetilde{\theta}) = \frac{\sum\limits_{\theta \leq \widetilde{\theta}} P_t(\theta) q_t(\theta)}{\sum\limits_{\theta \leq \widetilde{\theta}} q_t(\theta)}$$

If a quantity is not traded in a market, we assume that the effective spread is not available ($n.a.$) for that quantity. The effective half spread on a LOB without a specialist is the following:

LOB WITHOUT A SPECIALIST	$Q = \frac{1-7\eta}{1+\eta} I$	$Q = \frac{1}{3} I,$	$Q = I$
LOB 1: $\eta < 1/11$	$\frac{8\eta}{(1+\eta)} \gamma\sigma^2 I$	$n.a.$	$\frac{8\eta(3-5\eta)}{(1+\eta)^2} \gamma\sigma^2 I$
LOB 2: $\eta \in [1/11, 1/7[$	$\frac{8\eta}{(1+\eta)} \gamma\sigma^2$	$\frac{2(86\eta-5-197\eta^2)}{3(1+\eta)^2} \gamma\sigma^2$	$\frac{2(1+98\eta-191\eta^2)}{9(1+\eta)^2} \gamma\sigma^2 I$
LOB 3: $\eta \in [1/7, 1/4]$	$n.a.$	$\frac{5}{3} \gamma\sigma^2 I$	$\frac{17}{9} \gamma\sigma^2 I$

The effective half spread on a LOB with a specialist is represented in the following table:

LOB WITH A SPECIALIST	$Q = \frac{1}{18} I$	$Q = Q^S_{e*}$	$Q = \frac{1}{3} I$	$Q = \frac{5}{9} I$	$Q = Q^S_{d*}$	$Q = I$
$r < \frac{2}{33}(1-4\eta)$	$n.a.$	$n.a.$	$\frac{5}{3}\gamma\sigma^2 I$	$n.a.$	P^S_{d*}	$\frac{17}{9}\gamma\sigma^2 I$
$r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$	$n.a.$	$n.a.$	$\frac{5}{3}\gamma\sigma^2 I$	P^S_{f*}	$n.a.$	$\frac{17}{9}\gamma\sigma^2 I$
$r \in \left[\frac{4}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$	$n.a.$	P^S_{e*}	$\frac{5}{3}\gamma\sigma^2 I$	$n.a.$	$n.a.$	$\frac{17}{9}\gamma\sigma^2 I$
$r \geq \frac{17}{105}(1-4\eta)$	$\frac{35}{36}\gamma\sigma^2 I$	$n.a.$	$\frac{5}{3}\gamma\sigma^2 I$	$n.a.$	$n.a.$	$\frac{17}{9}\gamma\sigma^2 I$

Average effective spreads are computed as a weighted average of the effective spreads on a given market, were weights are given by the probabilities that a specific trade takes place. The Proposition is derived from the comparison of these values.

Proof of Proposition 3

We compute the expected gains from trade considering the optimal quantities for the LOB and the specialist derived in *Lemma 1*, *Lemma 2* and *Proposition 1*. The Proposition is obtained from the comparison between the gains from trade in the two tables.

LOB WITHOUT A SPECIALIST	Expected Gains from Trade
LOB 1: $\eta < 1/11$	$\frac{(1-5\eta-13\eta^2+57\eta^3)}{6(1+\eta)^2} \gamma\sigma^2 I$
LOB 2: $\eta \in [1/11, 1/7[$	$\frac{(9-41\eta-253\eta^2+1525\eta^3)}{54(1+\eta)^2} \gamma\sigma^2 I$
LOB 3: $\eta \in [1/7, 1/4]$	$\frac{13\eta}{54} \gamma\sigma^2 I$

LOB WITH A SPECIALIST	Expected Gains from Trade
$r < \frac{2}{33}(1-4\eta)$	$\frac{13\eta}{54}\gamma\sigma^2 I + \frac{3(1-4\eta)(25-100\eta-291r)r}{(1-3r-4\eta)^2}\gamma\sigma^2 I$
$r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$	$\frac{13\eta}{54}\gamma\sigma^2 I + \frac{(1-4\eta)r}{2(1-3r-4\eta)}\gamma\sigma^2 I$
$r \in \left[\frac{4}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$	$\frac{13\eta}{54}\gamma\sigma^2 I + \frac{3(1-4\eta)(17-68\eta-105r)r}{(1-3r-4\eta)^2}\gamma\sigma^2 I$
$r \geq \frac{17}{105}(1-4\eta)$	$\frac{13\eta}{54}\gamma\sigma^2 I$

Appendix B

We consider the case where the broker who has an uninformed client goes to the LOB when he is indifferent in terms of trading on the LOB or with the specialist. The only difference with the case presented in the paper is that when $r \geq \frac{1}{3}(1-4\eta)$ an hybrid market does not exist. In fact, in this case the specialist can only offer the same gains from trade than the LOB to uninformed traders as the proof of *Lemma 2* shows. The Lemma is modified as follows:

Lemma 3 *If $r \geq \frac{1}{3}(1-4\eta)$ a hybrid market does not exist, while if $r < \frac{1}{3}(1-4\eta)$ a hybrid market exists. When the hybrid market exists, quantities and prices quoted on a LOB with a specialist are equal to the ones quoted on a LOB without a specialist for high levels of asymmetric information, i.e. for $\eta \in [1/7, 1/4]$.*[22]

The specialist's price and quantity become:

Value of r	Specialist's Quantity	Specialist's Price
$r < \frac{2}{33}(1-4\eta)$	$Q^S_{d*} = \frac{(9r+4\eta-1)}{(3r+4\eta-1)} I$	$P^S_{d*} = \left[\frac{(9r+4\eta-1)}{(3r+4\eta-1)} \frac{1}{2} - \frac{11r}{6(9r+4\eta-1)} \right] \gamma\sigma^2 I$
$r \in \left[\frac{2}{33}(1-4\eta), \frac{4}{39}(1-4\eta)\right)$	$Q^S_{f*} = \frac{5}{9} I$	$P^S_{f*} = \frac{(438r+260\eta-65)}{90(3r+4\eta-1)} \gamma\sigma^2 I$
$r \in \left[\frac{4}{39}(1-4\eta), \frac{17}{105}(1-4\eta)\right)$	$Q^S_{e*} = \frac{(6r+4\eta-1)}{(3r+4\eta-1)} I$	$P^S_{e*} = \left[\frac{(6r+4\eta-1)}{(3r+4\eta-1)} \frac{1}{2} - \frac{r}{6(6r+4\eta-1)} \right] \gamma\sigma^2 I$
$r \in \left[\frac{17}{105}(1-4\eta), \frac{1}{3}(1-4\eta)\right)$	$Q^S_{c*} = \frac{1}{18} I$	$P^S_{c*} = \frac{35}{36} \gamma\sigma^2 I$
$r \geq \frac{1}{3}(1-4\eta)$	$Q^S_{*} = 0$	$P^S_{*} = P^l_{LOB}$

If we compare this table with the one derived in the proof of *Proposition 1*, the only difference is that now, if $r \geq \frac{1}{3}(1-4\eta)$, the specialist cannot improve anymore on LOB trading terms. The relationship with the specialist is too unstable compared to the probability of having uninformed clients on the market in the future. Hence, the specialist cannot offer higher gains from trade to them than the LOB. Given our assumption, uninformed traders prefer then to go to the LOB.
As far as the utilitarian welfare comparison is concerned, all the previous results hold. However, we cannot compare anymore the two markets for $r \geq \frac{1}{3}(1-4\eta)$ since only the pure LOB exists. If we consider empirical and policy implications, the only relevant difference is about competition proofness. In this setting, the specialist system still stays in place even if inefficient. However, now an exogenous worsening of relationship trading or asymmetric information could drive the specialist out of the market. Given that usually the stability of the relationship improves with time and that asymmetric information decreases when a stock is traded from a long time, such changes are unlikely to happen once the specialist system is in place.

[22] The proof is closely related to the one of *Lemma 2* and is available upon the author.

References

[1] Battalio, R., Ellul A., and R. Jennings, 2005, Reputation Effects in Trading on the New York Stock Exchange, *Journal of Finance* forthcoming.

[2] Benveniste, L., Marcus, A., and W. Wilhelm, 1992, What's Special about the Specialist?, *Journal of Financial Economics*, 32, 61-86.

[3] Bernhardt, B. and E. Hughson, 1997, Splitting Orders, *Review of Financial Studies*, 10, 69-102.

[4] Bessembinder, H., and H. M. Kaufman, 1997, A Cross-Exchange Comparison of Execution Costs and Information Flow for NYSE-listed Stocks, *Journal of Financial Economics*, 46, 293-319.

[5] Biais, B., Martimort, D., and J. C. Rochet, 2000, Competing Mechanism in a Common Value Environment, *Econometrica*, 68, 799-837.

[6] Caglio, C. and K. A. Kavajecz, 2006, A Specialist's Quoted Depth as a Strategic Choice Variable: An Application to Spread Decomposition Models, *Journal of Financial Research*, 29, 383-398.

[7] Cao, C., Choe, H., and F. Hatheway, 1997, Does the Specialist Matter? Differential Execution Costs and Intersecurity Subsidization on the New York Stock Exchange, *Journal of Finance*, 52, 1615-1640.

[8] Chan, Y., and M. Weinstein, 1993, Bid-Ask Spread and Market Structure, *Financial Analysts Journal*, 57-62.

[9] Corwin, S. A., 1999, Differences in Trading Behavior across NYSE Specialist Firms, *Journal of Finance*, 54, 721-745.

[10] Coughenour, J. F., and D. N. Deli, 2002, Liquidity Provision and the Organizational Form of NYSE Specialist Firms, *Journal of Finance*, 57, 841-869.

[11] Desgranges, G., and T. Foucault, 2005, Reputation-based Pricing and Price Improvements, *Journal of Economics and Business*, 57, 493-527.

[12] Dupont, D., 2000, Market Making, Prices, and Quantity Limit, *Review of Financial Studies*, 13, 1129-1151.

[13] Fishe, R. P. H., and M. A. Robe, 2004, The impact of illegal insider trading in dealer and specialist markets: evidence from a natural experiment, *Journal of Financial Economics*, 71, 461-488.

[14] Garfinkel, J. A., and M. Nimalendran, 2003, Market Structure and Trader Anonymity: An Analysis of Insider Trading, *Journal of Financial and Quantitative Analysis*, 38, 591-610.

[15] Glosten, H., 1989, Insider Trading, Liquidity and the Role of the Monopolist Specialist, *Journal of Business*, 62, 211-235.

[16] Glosten, H., 1994, Is the Electronic Open Limit Order Book Inevitable?, *Journal of Finance*, 49, 1127-1161.

[17] Heidle, H. G., and R. D. Huang, 2002, Information-Based Trading in Dealer and Auction Markets: An Analysis of Exchange Listings, *Journal of Financial and Quantitative Analysis*, 37, 391-424.

[18] Madhavan, A. and G. Sofianos, 1998, An empirical analysis of NYSE specialist trading, *Journal of Financial Economics*, 48, 189-210.

[19] Kavajecz, K. A., 1999, A Specialist's Quoted Depth and the Limit Order Book, *Journal of Finance*, 54, 747-771.

[20] Kavajecz, K. A., and E. R. Odders-White, 2001, An Examination of Changes in Specialists' Posted Price Schedules, *Review of Financial Studies*, 14, 681-704.

[21] Parlour, C. A., and D. J. Seppi, 2003, Liquidity-Based Competition for Order Flow, *Review of Financial Studies*, 14, 681-704.

[22] Ready, M., 1999, The Specialist's Discretion: Stopped Orders and Price Improvement, *Review of Financial Studies*, 16, 301-343.

[23] Rock, K., 1996, The Specialist's Order Book and Price Anomalies, working paper, Harvard University.

[24] Seppi, D., 1990, Equilibrium Block Trading and Asymmetric Information, *Journal of Finance*, 45, 73-94.

[25] Seppi, D., 1997, Liquidity Provision with Limit Orders and Strategic Specialist, *Review of Financial Studies*, 10, 103-150.

[26] Venkataram, K., 2001, Automated versus Floor Trading: An Analysis of Execution Costs on the Paris and New York Stock Exchanges, *Journal of Finance*, 56, 1445-1485.

- Figures -

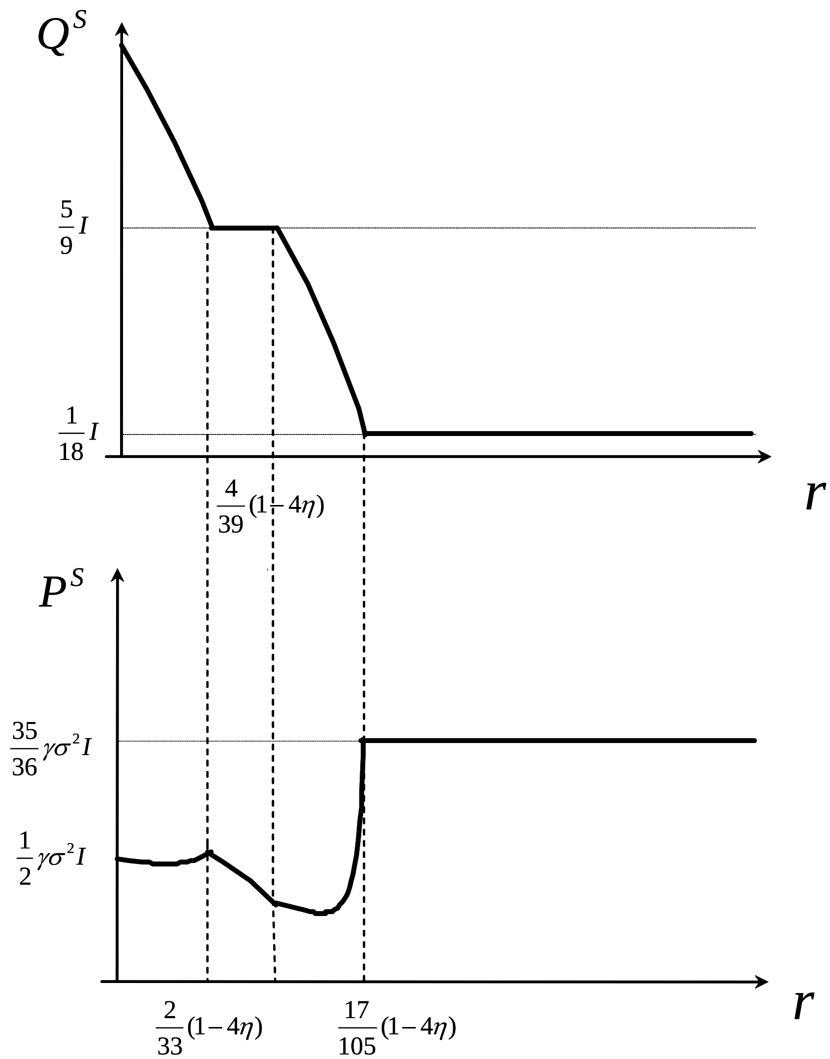

Figure 1: *QUANTITY AND PRICE QUOTED BY THE SPECIALIST.*

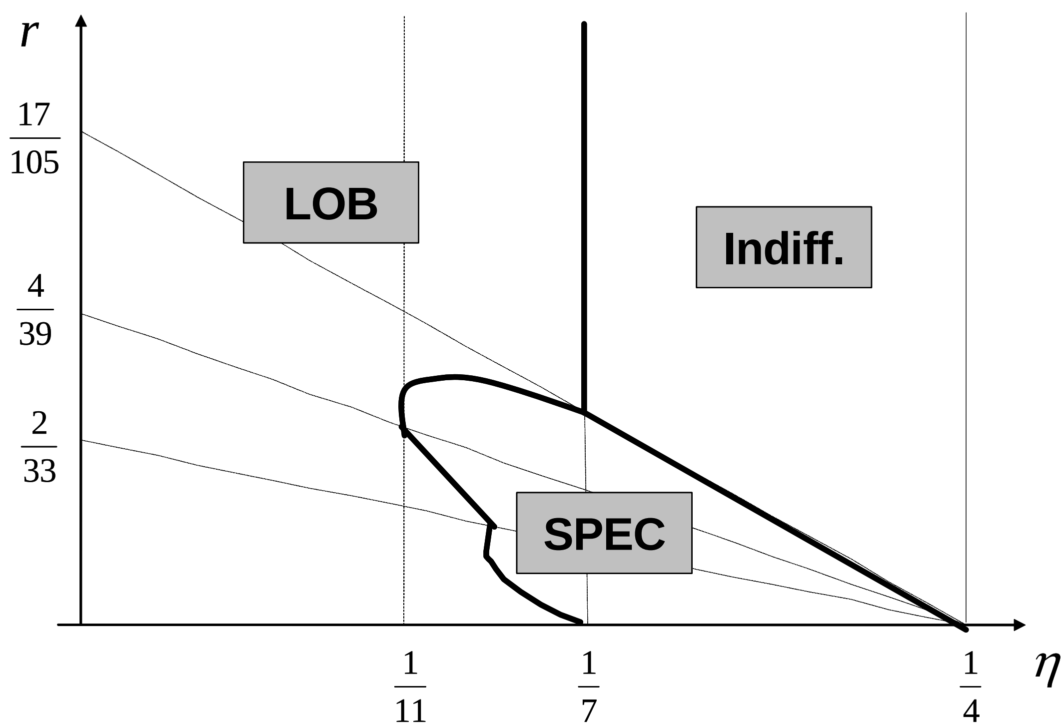

Figure 2: *GAINS FROM TRADE COMPARISON.*