Intermediated Investment Management

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Abstract

A majority of mutual funds are sold through brokers. This paper explores the effects of intermediation through brokers and financial advisers on portfolio performance, fund flows, fund sizes and welfare. We show that advisers facilitate the participation of small investors in an actively managed fund. Surprisingly, advisory services are utilized to a greater extent when advisers are subject to influence from the portfolio manager. Our model can explain underperformance of an actively managed portfolio relative to a passive benchmark, as well as the underperformance of funds sold through brokers relative to those sold directly. We show that advisory services improve overall social welfare although investors are harmed whenever influence activities are present.

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1 Introduction

Distributor play an integral role in the way many products and services are brought to market by serving as intermediaries between consumers and producers. Intermediation is also important in the investment management industry where advisers help their clients to find suitable assets and assist them in creating portfolios to achieve their goals. For example, Bergstresser, Chalmers and Tufano (2006) document for the US that 81% of all mutual fund share classes were sold through brokers in the year 2002. More generally, the Investment Counsel Association of America (ICA, 2004) has estimated the total amount of assets managed by SEC-registered investment advisers at $23.4 trillion in the year 2004.

Advisory services are offered through many organizational forms such as investment banks, commercial banks, partnerships and sole proprietorships. Services offered include asset allocation, financial planning, pension consulting, securities ratings, market timing and the selection of portfolio managers.

Investment advisory services facilitate the participation of investors in financial markets by economizing on information costs and providing expertise for portfolio choice. On the other hand, since advisers often have direct relationships with different portfolio managers, and frequently receive multiple forms of compensation, they may also face substantial conflicts of interest. Despite their crucial role, intermediaries in the investment management industry have largely been ignored by the existing literature. Most previous studies on delegated portfolio management consider only the bilateral relationship between investors and portfolio managers. The contribution of our paper is to model explicitly the effects of intermediation and the associated conflicts of interest on the equilibrium in the investment management industry.

Conflicts of interest result from the explicit and implicit compensation arrangements in the investment advisory relationship. In the case of mutual funds, for instance, brokers receive direct compensation by sharing front end loads, back end loads and 12b-1 fees with the management company. Industry estimates state that mutual fund investors paid $3.2 billion in front end loads, $2.8 billion in back end

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1 One reason why this figure is so large is that it reflects multiple layers of advisory services, including portfolio management services provided by fund management companies.

2 E.g., Bhattacharya and Pfleiderer (1983) and Stoughton (1993).

3 Front end loads are extra sales charges added onto the purchase price of the fund at the purchase date, back end loads are added at the redemption date and 12b-1 fees are paid on a continuing annual basis as long as the fund is held.
loads and $8.8 billion in 12b-1 fees in the year 2002 alone. Other forms of compensation are less explicit. As an example, John Hancock Funds makes the following disclosures in its statement of additional information. “Non-cash compensation may also take the form of occasional gifts, meals, tickets, or other entertainment as limited by NASD requirements… These payments may provide an incentive to a Selling Firm to actively promote the Funds… You should ask your Selling firm for more information about … marketing support payments” (John Hancock Funds, 2006).

The purpose of this paper is to study the investment management structure of advisers, investors and portfolio managers, and to determine the effect of influence activities on fund flows, fund returns, fees and investor welfare. By focusing on the distribution channels we are able to explain a number of important findings of the empirical literature. First, we rationalize the widespread use of investment advisers. We find that, surprisingly, when asset allocation is biased through kickbacks from the portfolio manager to advisers, the usage of advisory services increases. Therefore conflicts of interest, even when rationally anticipated, do not reduce the utilization of investment advisory services. Second, our model can explain the fact that actively managed funds underperform passive benchmarks after fees are deducted, a frequently documented finding (Pastor and Stambaugh, 2002). This arises in our model as a result of the kickbacks paid by the portfolio manager. Third, our paper explains the underperformance of load funds relative to no-load funds, as well as indirectly sold funds relative to directly sold funds (see, for example, Carhart (1997) and Bergstresser et al. (2006)). Fourth, our analysis predicts that differences in net performance can be entirely accounted for by differences in management fees. This is because higher management fees can be supported by aggressive marketing payments, i.e. kickbacks.

Our model consists of investors, a representative investment adviser, a passive (index) fund and an active fund run by a portfolio manager. Investors have heterogeneous wealth levels, and can go directly to the portfolio manager or through the indirect channel of using an adviser. In order to invest directly, investors would have to pay a cost to identify an active manager who can outperform a passive index. As a result of this, only high networth individuals invest directly. Moreover, they earn a surplus over their reservation opportunity, which is to invest in the passive fund. Portfolio managers have market power. They are able to charge fees increasing in their expected portfolio ‘alpha’. But as the active fund has

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4See Bergstresser et al. (2006).
diminishing returns to size, there is an optimal amount of assets invested actively. Investment advisers also charge a fee, which compensates them for their information in making optimal asset allocations on behalf of their investors.

We first derive an equilibrium assuming that the investment adviser's decisions are made to maximize investors' risk-adjusted returns. We then extend this model by allowing the portfolio manager to bias these decisions through kickbacks. We solve for the optimal amount of bias preferred by the portfolio manager and the kickback required to sustain it. Finally we also derive the form of an equilibrium without an adviser and compare outcomes to the cases with biased or unbiased advisers, respectively.

The seminal paper on the subject of investment management is the AFA Presidential address of Sharpe (1981). This paper was the first to recognize, in the context of the standard Markowitz (1952) paradigm, that there are more than two parties (the client and a single portfolio manager) in the practical problem of investment management. In this paper, Sharpe analyzes the coordination failure in the presence of multiple portfolio managers. By contrast, we explicitly consider the role of an intermediary between the client and the portfolio manager.

Empirical work on the subject of mutual funds and their distribution channels is very recent and still at an early stage. Bergstresser et al. (2006) look at the performance of mutual funds offered through the brokerage channel as compared to those offered directly to investors. They find that, even before fees are deducted, risk-adjusted returns are lower for funds offered through the brokerage channel as compared to those making direct offerings to investors. We show that this finding can be rationalized in a model where fund size negatively affects its performance.

A second important paper in this area is that of Christoffersen, Evans and Musto (2005). They use a new database of fund filings in which inflows and outflows are characterized by the category of intermediary that is involved. Looking only at the subset of funds that exclusively deal with captive or unaffiliated brokers, they find that average (as opposed to contractual maximum) loads rebated to brokers are significantly lower for the captive brokers as compared to the unaffiliated brokers. They find mixed performance benefits for captive versus unaffiliated brokers. The former add more value in the

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Some examples of mutual fund companies that utilize direct channels include Fidelity, Vanguard and Janus. Examples of companies that offer their products through brokers include Investment Company of America, Washington Mutual and Putnam.
case of inflows; the latter in the case of outflows. More pertinent to our paper, though, they find some evidence that the higher rebates to unaffiliated brokers buys favorable treatment in attracting inflows.

A recent paper, Chen, Yao and Yu (forthcoming), shows that mutual funds managed by insurance companies underperform their non-insurance counterparts by more than 1%. The authors find that this has to do with the fact that insurance funds are often cross-sold through the extensive broker/agent network of their parent firms. To the extent that the insurance agents are captive to the parent company, this is consistent with our model, which predicts that biased advisors lead to lower fund performance.

Grundy (2005) develops a model in which it is optimal to employ an intermediary, the investment bank, in order to achieve the optimal fund size. The bank can do this when its advantage in terms of resolving information asymmetry outweighs the additional contracting imperfections. By contrast, in our model, a competitive adviser resolves the information asymmetry problem completely given a known return on the passive asset.

An interesting paper exploring the role of kickbacks in the medical field is that of Pauly (1979). He considers a medical practitioner who is able to engage in 'fee-splitting' practices with a specialist. He finds that there is no point in prohibiting such practices in a fully competitive environment because services are provided at marginal cost. However, when there are market imperfections such as monopoly or incomplete pricing of insurance, fee-splitting can actually improve client welfare.

Several other papers have examined the structure of the investment management industry. Mamaysky and Spiegel (2002) and Gervais, Lynch and Musto (2005) provide a theory for the existence of fund families. Massa (1997) presents a model to explain the market segmentation and fund proliferation in the mutual fund industry. Chen, Hong and Kubik (2006) document that many fund families farm out a sizeable fraction of their funds to unaffiliated advisory firms and that funds managed externally significantly underperform those run internally. Furthermore, Massa, Gaspar and Matos (2006) demonstrate that agency issues within mutual fund families are of considerable relevance.

In the next section, we set up the basic model, with behavioral assumptions on the three participants in the game: the investors, the investment adviser and the portfolio manager. In section 3, we derive the form of the equilibrium without the possibility of influence activity by portfolio managers. Section 4 derives the impact of kickbacks from the portfolio manager to the advisers. Section 5 considers
the form of the equilibrium in a situation where advisory services are not available. Section 6 compares the outcome and welfare in the three alternative scenarios. Section 7 concludes the paper.

2 Model Setup

In this section we describe the various agents, their behavior and how they interact. There are three classes of agents in the model: (1) the active manager; (2) the set of investment advisers, modeled as a representative agent; (3) the pool of investors in the economy.

2.1 Assets and Managers

There are two types of assets in which investors can invest. First, there is a passive fund, such as an index fund, with expected gross return, $R_f$ (i.e., one plus the rate of return). If investors are risk averse and all use the same model of risk premia in pricing assets, then we could describe $R_f$ as a ‘risk-adjusted’ expected return. However to simplify description of the problem, we refer to this as an expected return, as if investors were risk-neutral. In order to invest in this benchmark asset directly, we assume that a ‘small’ amount of proportional transaction costs needs to be incurred, equal to $\tau$. This cost represents both the explicit management fee to be paid for index products as well as trading costs associated with tracking the index. We assume that this transaction cost can be reduced through the use of an investment adviser. For simplicity we assume that these costs are zero if the investment is made via an adviser.

The second type of asset is an active fund, whose expected return (once again risk-adjusted) is equal to $R_p$. The active portfolio manager utilizes her expertise in managing these assets. However, because of market impact or limited applicability of the portfolio manager’s expertise, we assume decreasing returns to scale in the amount of investment. Specifically, we assume that

$$R_p = \alpha - \gamma A,$$  

where $\alpha$ represents the initial expected return (assumed to be greater than the passive return) and $\gamma$ is a

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6 Another example is that some index products do not fully pass through the dividends to the investors.

7 Advisers may have access to more efficiently managed index products. For example, the enhanced index funds managed by Dimensional Fund Advisors emphasize that they reduce the effective bid/ask spread by negotiating large block trades. Those funds are sold exclusively through advisers.
coefficient representing the rate at which returns decline with respect to the aggregate amount of funds, *A*, that are placed with the portfolio manager. For a discussion of this assumption see Berk and Green (2004) and Dangl, Wu and Zechner (2006).

In addition to investing in the passive asset and obtaining returns net of transactions costs equal to $R_f - \tau$, the investors can choose to delegate their portfolio decisions to an investment adviser who advises multiple clients, or they can decide to invest directly with the portfolio manager. Because there is a large number of active managers with inferior returns relative to the passive fund, we assume that there is a fixed *ex ante* learning cost, $C_0$, that must be expended by an investor in order to identify potentially valuable active managers. Therefore a direct investor pays the cost $C_0$ and then is able to identify a manager whose fund returns on the first dollar under management are superior, i.e. for whom $\alpha > R_f$.

Investors decide optimally whether or not to pay this fixed cost. Alternatively they can avoid it by delegating their funds to the adviser in which case he makes the investment choice for them.

The investment adviser has access to the same set of two investment opportunities. At the beginning of the period, the adviser incurs a constant marginal cost based on the initial amount of assets under management. This cost represents, for instance, capital requirements and other costs, such as costs for human resources, office space, and operational risks which are tied to the amount of assets under management. For simplicity, we assume that the constant marginal cost is

$$c_A = \frac{\tau}{R_f},$$

i.e., variable costs are equal to the present value of investors’ transaction costs. This parametric specification implies that if the adviser’s portfolio has a return equal to $R_f$ and the adviser charges a fee equal to the marginal cost, then investors will be indifferent between investing via the adviser and investing in the passive fund directly. This not only simplifies our analysis significantly, but also makes sure that any potential welfare improvement due to the presence of the adviser does not simply come from any

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8Empirical evidence supporting this effect can be found in Chen, Hong, Huang and Kubik (2004) and Ang, Rhodes-Kropf and Zhao (2006).

9One interpretation of this cost is that it is the cost of paying for one’s own ‘private adviser’ whose sole function is to certify that the portfolio manager is not drawn from the bad pool.

10Our cost structure for the advisers and the investors is similar to Holmström and Tirole (1997) who derive a theory of financial intermediation from these assumptions.
exogenously assumed cost advantage of the adviser.

Finally we describe the nature of the fees that are charged by the active manager and the investment adviser. We assume that the adviser charges a proportional fee, $f_A$, based on the end-of-period value of all assets under management, including both assets invested with the passive fund and assets invested with the active manager. The portfolio manager also charges a proportional fee, $f_P$, on the end-of-period value of assets managed actively.

The investors therefore have to decide amongst three investing strategies. If they invest in the passive index themselves, they incur the transaction fee. If they invest directly with the portfolio manager, they pay their fixed information collection fee as well as the portfolio management fee. If they delegate their decision to the adviser, they can take advantage of his expertise in making the optimal asset allocation decision, but they have to pay two management fees for the actively invested portion of their holdings: both that of the adviser and that of the portfolio manager. The active manager can receive funds directly from the investors (the direct channel) or indirectly through the investment adviser (the indirect channel).

2.2 Investors Behavior

Assume that each investor has wealth $x + C_0$, where $x$ follows a Pareto distribution with the following probability density function:

$$f(x) = \frac{k A_m^k}{x^{k+1}}, \quad k > 1,$$

where $A_m > 0$ denotes the minimum wealth level (net of the searching cost $C_0$).\[11\] The Pareto distribution has been widely used to describe the distribution of wealth among individuals. Empirical studies have found that this distribution characterizes the real world fairly well, except for its properties at the lower end.\[12\] An important feature of this distribution is that the probability density $f(x)$ decreases monotonically in wealth, implying that the fraction of wealthy investors is relatively small while the fraction of

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11Our general conclusion does not depend on this specific assumption about the wealth distribution. In previous versions, we have utilized several different assumptions about the wealth distribution; our results are qualitatively similar. Including $C_0$ in the initial wealth simplifies the subsequent notation.

12See Persky (1992) for a brief review of this literature.
investors with low levels of wealth is relatively large. The parameter $k$ characterizes the extent of wealth equality. Complete equality of wealth is characterized by $k \to \infty$, while $k \to 1$ corresponds to complete inequality.\footnote{The Gini coefficient, a widely used measure of inequality, for the Pareto distribution is given by $\frac{1}{2k-1}$.} Pareto's original estimate of $k$ based on income data clustered around 1.5. Later estimates of this parameter ranged from 1.6 to 2.4 for income distributions (Champernowne (1952)) and from 1.3 to 2.0 for wealth distributions (Yntema (1933)).

We standardize the population to be 1. Therefore the total wealth available for investment is

$$W = \int_{A_m}^{\infty} x f(x) dx + C_0 = \frac{kA_m}{k-1} + C_0, \ k > 1. \quad (4)$$

Based on their wealth level, investors can choose whether to invest directly or indirectly. Let $A_D$ and $A_I$ denote the amounts of direct and indirect investment to the active fund. Therefore the total amount of money under active management is $A = A_D + A_I$ and the rate of return of the actively managed portfolio is $R_p = \alpha - \gamma(A_D + A_I)$.

Investors will invest either directly or indirectly in the actively managed portfolio if the expected return is not less than the reservation rate of return $R_f - \tau$. Suppose that the expected return of indirect investing is equal to the reservation rate, $R_f - \tau$, which will indeed be the case given our assumption on the advisory cost and the competitive nature of the advisory service industry. Then an investor with wealth $A^* + C_0$ will be indifferent between contracting directly with the portfolio manager and getting a net rate of return $R_p(1 - f_P)$ and investing via the adviser, where $A^*$ satisfies the following condition:

$$[\alpha - \gamma(A_D + A_I)](1 - f_P)A^* = (R_f - \tau)(A^* + C_0),$$

i.e.,

$$A^* = \frac{C_0(R_f - \tau)}{(1 - f_P)[\alpha - \gamma(A_D + A_I)] - (R_f - \tau)}. \quad (5)$$

Note that this relation assumes that investors behave in an ‘atomistic’ fashion; they do not take into account the diseconomy of scale in active portfolio management when they decide where to channel
their funds.

It is obvious that all investors whose wealth is smaller than \(A^* + C_0\) will prefer to invest via the adviser whereas those with wealth greater than \(A^* + C_0\) will prefer to contract directly with the portfolio manager. We therefore refer to this latter set of investors as ‘high networth individuals’. Indeed there is evidence to this effect. In [ICI (2004)] it is documented that customers who purchase funds through a brokered mutual funds sales force channel as opposed to the direct channel are less wealthy (with median income of $93,800 v. $101,300). They also have lower median financial assets ($363,700 v. $447,900).

Given that all investors with wealth levels greater than \(A^* + C_0\) invest directly, we can solve for the amount of money channeled directly to the portfolio manager:

\[
A_D = \int_{A^*}^{\infty} x f(x) dx = \frac{k A_m^k}{(k - 1)(A^*)^{k-1}}. \tag{6}
\]

Note that if \(A^* = A_m\), then \(A_D = W - C_0\), and all investors would contract with the portfolio manager directly. To make our analysis interesting, we assume that

\[
\tau < \frac{C_0(R_f - \tau)}{A_m}.
\]

This implies that there are always some investors preferring the indirect channel if the return of the active portfolio after the management fee equals \(R_f\).

We also assume that if all wealth net of the searching cost \(C_0\) is invested in the active portfolio, the return of the active portfolio will be lower than the reservation rate of the investors, i.e.,

\[
\alpha - \gamma(W - C_0) < R_f - \tau.
\]

Combining the two conditions above, we have

\[
\tau < \min(R_f - \alpha + \gamma(W - C_0), \frac{C_0(R_f - \tau)}{A_m}). \tag{7}
\]

This upper bound on the transaction costs is maintained throughout our analysis.
3 Investment Management Equilibrium

We now solve for the equilibrium in our model of investment management. First we discuss the behavior of the adviser and subsequently the behavior of the portfolio manager. In this section, we assume that there are no indirect ‘kickbacks’ to the adviser so that his decision-making is unbiased and only evaluates the respective asset returns. In the next section, we allow for influence activities between the portfolio manager and the advisers.

3.1 Advisers Behavior

Our model features competitive behavior on the part of investment advisers. It is a critical assumption that they behave atomistically and do not consider the impact on the rate of return on the active portfolio in making their asset allocation decisions. Suppose that a representative fund adviser charges a proportional advisory fee $f_A$, based on the total end-of period asset value of his client’s accounts. He can either put the client’s money in the passive portfolio or in the portfolio actively managed by the portfolio manager. Let $w$ represent the portfolio weight of the active asset and $1 - w$ the portfolio weight of the passive asset. Then the investment adviser will solve the following problem:

$$\max_w \quad w f_A [\alpha - \gamma(A_D + A_I)](1 - f_P) + (1 - w) f_A R_f.$$ \hfill (8)

Obviously the proportional fee, $f_A$, charged by the adviser will not affect his asset allocation. If the following condition representing equality of the respective rates of return is not satisfied, the adviser would put all of the funds in either the active or passive portfolio:

$$[\alpha - \gamma(A_D + A_I)](1 - f_P) = R_f.$$ \hfill (9)

Notice that the adviser behaves in a ‘returns-taking’ manner, i.e., he takes the return $R_p = \alpha - \gamma(A_D + A_I)$ as given. It cannot be an equilibrium consistent with our basic assumptions if equation (9) is not satisfied. In particular if the active portfolio had a higher net return than the passive, then all funds in the economy would be invested in active management, which by assumption would depreciate the rate
of return below the reservation value of the investors. On the other hand, the passive portfolio cannot
have a higher expected return than the active in equilibrium because the active portfolio always has a
higher marginal return on the first dollar of investment.

Since the adviser operates in a competitive environment, competition will drive down the advisory
fee, \( f_A \), to the marginal cost \( c_A = \tau/R_f \). Note that this advisory fee satisfies the participation constraint
of the indirect investors: indirect investors earn a return of \( R_f \) before the advisory fee since both the
active portfolio and the passive fund have a return of \( R_f \); as a result, the net return after the advisory fee,
\( (1 - \tau/R_f)R_f \), is equal to investors’ reservation return \( R_f - \tau \). The adviser just breaks even, which must
be the case in a competitive market.

Substituting equation (9) into equation (5), we have
\[
A^* = \frac{C_0(R_f - \tau)}{\tau}. \tag{10}
\]

Notice that this threshold level of wealth does not depend on \( \alpha \). Therefore the investors can optimally
decide whether or not to collect information in the equilibrium without knowing the portfolio managers'
potential ability, \( \alpha \).

Substituting equation (10) into equation (6) then gives the equilibrium amount of money invested
directly:
\[
A_D = \frac{kA_m^k \tau^{k-1}}{(k-1)(C_0(R_f - \tau))^{k-1}}. \tag{11}
\]

The most important property of this relationship is that the amount of money invested directly is inde-
dependent of the fees of the portfolio manager. This is because of the competitive nature of the adviser. If
the active manager attempts to increase her fees, for the same gross rate of return, the adviser reduces his
asset allocation to active investing. Therefore the same net rate of return is achieved by active investing
and thus there is no effect on the marginal net rate of return or the aggregate amount of money invested
directly. This property is critical to understanding the model and will be exploited below.
3.2 Portfolio Manager Behavior

Now we describe the problem faced by the portfolio manager. From equation (9) we know that total funds under active management satisfy

\[ A_D + A_I = (\alpha - \frac{R_f}{1 - f_P})^{1/\gamma}. \]  

(12)

That is, funds under management are related to the difference between \( \alpha \) and the grossed-up return on the passive asset, scaled by the depreciation factor, \( \gamma \). The manager now takes this into account and solves the following fee maximization problem:

\[
\max_{f_P} [\alpha - \gamma(A_D + A_I)](A_D + A_I)f_P
\]

subject to equation (12).

Substituting the constraint into the objective function, the manager's problem is

\[
\max_{f_P} \left[ \frac{R_f}{1 - f_P} \right] \left[ \alpha - \frac{R_f}{1 - f_P} \right] \frac{1}{\gamma} f_P.
\]

It is easy to solve this for the optimal portfolio manager fees, which we record as a proposition.

**Proposition 1.** The optimal portfolio manager fee in the investment management equilibrium without influence is

\[
f_P^* = \frac{\alpha - R_f}{\alpha + R_f}.
\]

(14)

Now using this result, we can determine how the fee is impacted by \( \alpha \). Taking the partial derivative of equation (14),

\[
\frac{\partial f_P}{\partial \alpha} = \frac{2R_f}{(\alpha + R_f)^2} > 0.
\]

Hence the fees are increasing in the managerial ability. Further we can solve for the active manager's
profit:

\[ \Pi_p = \frac{(\alpha - R_f)^2}{4\gamma}, \]  

(15)

and the total assets allocated to the manager:

\[ A_I + A_D = \frac{\alpha - R_f}{2\gamma}. \]  

(16)

Substituting equation (11) into equation (16), we have

\[ A_I = \frac{\alpha - R_f}{2\gamma} - \frac{k A_n^k \tau^{k-1}}{(k - 1)(C_0(R_f - \tau))^{k-1}}. \]  

(17)

We assume that \( C_0 \) is big enough, or \( \gamma \) is small enough, to ensure that \( A_I \) in (17) is positive, i.e., not all investment into the active portfolio comes from the direct channel.\(^{14}\)

In summary the investment management equilibrium features positive profits of the portfolio manager and zero profit of the investment adviser. Returns on the actively managed portfolio net of management fees are equal to those of the passive fund. Rates of return earned by direct investors in the active fund exceed those earned by indirect investors. Nevertheless only the high net worth individuals find it optimal to invest directly.

4 Influence Activities

We now extend the model to consider the aspect of rebates or 'kickbacks' from the portfolio manager to the investment adviser. The idea here is that the portfolio manager desires to influence the decisions of the investment adviser. The purpose of this part of our model is to evaluate the impact of such behavior on the investors and their investment performance.

Rebates associated with influence activity can take various forms, either explicit or implicit. The most common type of a rebate involves the load charge of a mutual fund. In this case the fund charges a sales fee and then rebates a large part back to the sales agent (broker). In practice actual loads are based

\(^{14}\)If this condition would not hold, the adviser is not utilized at all. In this case, the analysis in section 5 would apply.
on a declining scale related to the magnitude of the investment in a fund. There are front end loads, back end loads and 12b-1 fees, which are incurred on a continuing basis. Bergstresser et al. (2006) document the nature of these charges for different investment channels, and find that the charges are much higher for brokered sales as compared to direct sales.

4.1 Impact on Fund Flows

We choose to model the behavior of such influence activity in the following way. The rebate or kickback is specified ex ante and paid at the end of the period by the portfolio manager to the adviser in the form of a lump sum transfer of monetary value, $K_A$. In return for this transfer committed by the portfolio manager, the adviser biases his investment decision in favor of the portfolio manager. We treat the commitment of the manager, whether explicit or implicit, as enforceable. The bias in the asset allocation decision is handled in the following way. Suppose that the expected (risk-adjusted) return of the portfolio manager is $R_p$ and the passive index return is $R_f$. Then, the adviser will still invest in the active fund as long as $R_p(1 - f_p) + \delta \geq R_f$. That is, $\delta$ represents the amount of bias in the form of a ‘ghost’ return that is effectively added to the active return when assets are allocated by the adviser. Of course this does not translate into any realized return at all on the part of the investors.

With the introduction of this bias, the equilibrium condition that will be satisfied by the adviser's asset allocation decision will be

$$[\alpha - \gamma (A_D + A_I)](1 - f_p) = R_f - \delta.$$  \hspace{1cm} (18)

This expression can be rewritten to provide the fund size as

$$A_D + A_I = (\alpha - \frac{R_f - \delta}{1 - f_p}) \frac{1}{\gamma}. \hspace{1cm} (19)$$

As in the case without bias, we can substitute equation (18) into (5) and we find that the threshold wealth level becomes

$$A^* = \frac{C_0(R_f - \tau)}{\tau - \delta}, \hspace{1cm} (20)$$
and the amount invested directly becomes

\[ A_D = \frac{kA_m^k(\tau - \delta)^{k-1}}{(k-1)(C_0(R_f - \tau))^{k-1}}. \] (21)

We see first by comparing respectively equations (20) with (10) and (21) with (11) that the impact of the bias increases the wealth level of the marginal investor who is indifferent between investing directly and indirectly. Consequently the amount of funds invested directly decreases due to the bias. This effect continues up to the point where the direct investment drops to zero, which occurs when the bias converges to \( \tau \). Hence a key result is that kickbacks shift investors from the direct investment channel to the indirect investment channel. This occurs because with biased investment advisers, the diseconomy of scale results in less attractive net returns. As investors are shifted into the indirect channel, we will see that they suffer a welfare loss because they are now pushed down to their reservation utility. As before, though, the portfolio manager’s fee is constrained by the asset allocation decision of the adviser and the amount of direct investment in the active portfolio is independent of the fee in equilibrium.

4.2 Kickback Required by the Adviser

To see what consideration must be provided, we first consider the equilibrium advisory fee in the presence of kickbacks. Since the money allocated to the passive fund earns \( R_f \), while the money allocated to the active portfolio earns only \( R_f - \delta \), an advisory fee equal to the marginal cost \( c_A = \tau / R_f \) will lead to a net return of indirect investing lower than \( R_f - \tau \). Therefore such a fee cannot satisfy investors’ participation constraints any longer.

The equilibrium advisory fee that satisfies investors’ participation constraints must be determined by setting the aggregate net return to investing through the adviser to that of investing in the passive fund. Assuming that investors who are indifferent between the passive asset and investing through an adviser choose to delegate all their capital to the adviser, we obtain\(^{15}\)

\[ (1 - f_A)[A_I(R_f - \delta) + (W - \theta C_0 - A_D - A_I)R_f] = (W - \theta C_0 - A_D)(R_f - \tau), \] (22)

\(^{15}\)This assumption can be relaxed as long as the total wealth intermediated by the adviser is larger than \( A_I \).
where \( \theta \equiv \int_{A^*}^{+\infty} f(x) dx = (A_m/A^*)^k \) denotes the fraction of investors choosing the direct channel, \( W - \theta C_0 - A_D \) represents the total amount of money handled by the adviser, of which \( A_I \) is allocated to the active fund and the rest is allocated to the passive asset\(^\text{16}\).

Solving for \( f_A \) gives the following fee relationship:

\[
f_A = \frac{A_I \delta - (W - \theta C_0 - A_D) \tau}{A_I \delta - (W - \theta C_0 - A_D) R_f}.
\] (23)

It can be seen that for fixed \( A_I \), as \( \delta \) increases, the advisory fee is reduced. From equation (22), we can also see that the adviser’s total fee income is \( (W - \theta C_0 - A_D) \tau - A_I \delta \). Since \( (W - \theta C_0 - A_D) \tau \) is just enough to cover the advisory cost, the adviser has a net loss of \( A_I \delta \).\(^\text{17}\) Therefore he is effectively bearing all the cost of the distortions in his asset allocation. It follows that, in order to be fully compensated for administering such distortions, the adviser must get a kickback, \( K_A \), such that

\[
K_A = A_I \delta.
\] (24)

Another way to understand the amount of kickback required by the adviser is as follows. For any positive \( \delta \), an individual adviser can always increase his portfolio return by shifting \( A_I \) from the active portfolio to the passive. The gain from such a deviation, on the condition that other advisers do not deviate, is \( A_I \delta \). Since indirect investors always get their reservation rate of return, all the benefits from such a deviation are captured by the adviser. Therefore, to prevent the adviser from such a deviation, the portfolio manager must provide a kickback equal to \( A_I \delta \).

### 4.3 Optimal Influence Activity

We now endogenize the amount of influence activity by allowing the portfolio manager to choose her optimal level. In this respect she must decide on the amount of compensation that must be rebated to the adviser in order to convince him to adopt the degree of distortion requested by her.

The portfolio manager will try to maximize her profit net of the kickback payments. Therefore, her

\(^{16}\) \( \theta C_0 \) is the total wealth expended by the direct investors for search costs.

\(^{17}\) Note that the adviser incurs a cost of \( (W - \theta C_0 - A_D) \tau / R_f \) at the beginning of the period. This is equivalent to a cost of \( (W - \theta C_0 - A_D) \tau \) at the end of the period.
problem is

$$\max_{f_p, \delta} \Pi_p = [\alpha - \gamma(AD + A_I)](AD + A_I)f_P - A_I \delta$$

subject to the constraints (18) and (21). We can now solve for the optimal portfolio manager fees, kickback payments and bias imparted to the investment adviser.

**Proposition 2.** When influence activity is feasible, the optimal fee charged by the portfolio manager is

$$f^*_P = \frac{\alpha - R_f + 2\tau/k}{\alpha + R_f}. \quad (25)$$

The optimal amount of bias introduced through the portfolio manager’s kickback to the investment adviser is

$$\delta^* = \frac{\tau}{k}. \quad (26)$$

**Proof.** Substituting the constraints into the objective function, we see that the first order condition for the optimal fee is

$$f_P = \frac{\alpha + 2\delta - R_f}{\alpha + R_f}.$$

Substituting this expression back into the objective function, we get

$$\Pi_P = \frac{(\alpha - R_f)^2}{4\gamma} + AD \delta.$$ 

Therefore we have

$$\frac{\partial \Pi_P}{\partial \delta} = \frac{kA_m^k(\tau - \delta)^{k-2}}{(k - 1)(C_0 R_f - \tau))^{k-1}(\tau - k\delta)}.$$
Setting this expression equal to zero implies that the first order condition for the optimal bias is 18

\[ \delta = \frac{\tau}{k}. \]

The second order conditions can be verified in a straightforward fashion.

Proposition 2 shows that there is an interior optimal amount of bias. The intuition for this result is as follows. The bias allows the manager to increase her fees. However, the manager’s benefits due to a higher fee are fully offset in the case of indirect investors by the compensating kickbacks to the advisor. By contrast, the benefits of a higher fee are fully captured by the portfolio manager in the case of a direct investor. Thus, biasing the advisor via kickbacks is profitable overall. However, as increasing fees shift investors from the direct to the indirect channel, this marginal benefit is only realized on a decreasing asset base, thus motivating an interior optimal amount of bias.

From equation (26), we can easily see that the optimal bias is increasing in the transaction cost \( \tau \) and decreases in \( k \), which measures the degree of equality of the wealth distribution. The transaction cost \( \tau \) represents the maximum bias that can be induced before all investors leave the direct channel. Therefore, not surprisingly, the optimal bias is increasing in \( \tau \).

The relation between the optimal bias and \( k \) is also intuitively appealing. When \( k \) is large, there are fewer high networth investors, therefore, the portfolio manager does not extract much surplus by inducing a bias. By contrast, when \( k \) is close to 1, the fraction of high networth investors is relatively large. As a result, the portfolio manager has a stronger incentive to bias more to be able to extract their surplus.

To further analyze the impact of influence activity, we compute the equilibrium size of the fund. Given that the fee is determined by (25), from equation (19) the fund size in the equilibrium with the kickback is

\[ A_D + A_I = \frac{a - R_f}{2\gamma}. \]

18If \( k > 2 \), \( \delta = \tau \) also satisfies the first order condition. However, in this case, \( A_D = 0 \); the portfolio manager’s profit is not maximized. Therefore \( \delta = \tau \) is not optimal.
Note that the fund size is exactly the same as in the investment management equilibrium without the kickback, equation (12). Intuitively this occurs because both the reservation return of the marginal investor of the active fund, who must be an indirect investor, and the net return from investing indirectly are the same with or without influence activity. Since the size of the active fund does not change, its gross return is the same as when the portfolio manager is precluded from influencing the adviser. However the net return after management fee is lower since $f_P$ is greater.

Influence activity, therefore, allows the portfolio manager to extract some surplus of the large investors. In this process, some investors will be switching from the direct to the indirect investment channel. Indeed, substituting the optimal $\delta^*$ in equation (26) into (21), we see that the active investment through the direct channel in the equilibrium with influence activity is

$$A_D = \frac{k A^k_{m}(\tau - \tau / k)^{k-1}}{(k-1)(C_0(R_f - \tau))^k - 1},$$

(27)

which is strictly smaller than $A_D$ without influence activity as embodied in equation (11).

We record the observations from this section in the form of a proposition.

**Proposition 3.** In the investment management equilibrium where advisers are subject to influence activity, the active fund size is the same as when advisers are unbiased. However, more investors utilize advisory services by investing indirectly and the net return of the active fund is lower. The portfolio manager charges a higher fee, while advisers charge a lower fee but receive a compensatory kickback from the portfolio manager.

A striking result of our analysis is that when advisers are more subject to influence, they are actually used to a greater extent in equilibrium than when they are not biased. The reason is that the portfolio manager optimally raises her fees, which makes direct investment less attractive. The adviser is forced to lower his own fees in order to remain competitive with the alternative asset. Hence, the adviser’s asset management business becomes larger. These results are in accord with the fact that some advisors differentiate themselves as ‘fee-only’ advisers and advertise their independence and lack of influence, which allows them to charge higher advisory fees. At the other extreme, there exist brokers who receive their entire compensation from load rebates and charge no other advisory fees.
5 Equilibrium Without Advisers

In order to investigate the role of investment advisers in delegated portfolio management, we now examine an equilibrium in which investment advisers do not exist. When there is no investment adviser, the only vehicle for active investing is directly through the portfolio manager. As a result, \( A_I = 0 \); the fund size is determined solely by \( A_D \). The portfolio manager maximizes her profit by choosing an optimal fee and fund size. Therefore, the manager’s problem can be written as:

\[
\max_{A_D, f_P} \Pi_P = (\alpha - \gamma A_D) A_D f_P
\]  

(28)

subject to

\[
A_D = \frac{k A_m^k}{(k-1)(A^*)^{k-1}},
\]

\[
(\alpha - \gamma A_D)(1 - f_P)A^* = (R_f - \tau)(A^* + C_0),
\]

(29)

where the second constraint ensures that the marginal investor is indifferent between investing with the portfolio manager and the passive fund. We can see from equation (29) that there is an inverse relation between fund size and fees charged by the portfolio manager. Therefore either variable can be optimized by the manager equivalently.

Note that the informational assumptions made here are somewhat stronger than those needed previously with the investment adviser. Recall that the decision whether to invest directly did not depend in equilibrium on the potential value of active management, \( \alpha \) when the investment adviser is present. Now we must assume that the direct investor knows in advance which \( \alpha \) will obtain, after the cost, \( C_0 \), is expended. This, of course does not violate our earlier justification for the cost, since without paying it, there would be an adverse selection problem borne by the investor.

Problem (28) is now solved in the next proposition.

**Proposition 4.** In the portfolio management equilibrium without advisers, there exists a unique interior optimal fund size and management fee, which are the solutions to the following set of equations:

\[
\alpha - R_f + \tau - 2\gamma A_D - \frac{\lambda k}{k-1} A_D^{1/(k-1)} = 0,
\]

(30)
and

\[ f_p = \frac{\alpha - \gamma A_D - (R_f - \tau + \lambda A_D^{1/(k-1)})}{\alpha - \gamma A_D}, \]  
\[ (31) \]

where

\[ \lambda \equiv C_0(R_f - \tau)(\frac{k-1}{k A_{m}^{1/(k-1)}}). \]  
\[ (32) \]

Proof. See Appendix A.1.

A general analytical solution to the equation system in Proposition 4 for an arbitrary \( k \) is not available. Appendix A.2 provides the solutions for two special values of \( k \) which are close to the empirically observed values: \( k = 1.5 \) and \( k = 2 \). For general values of \( k \) the solutions can be found using numerical methods.

In the following section we compare outcomes for the three scenarios: no advisers, unbiased advisers, and advisers subject to influence activity. In particular we address the question whose interests investment advisers really serve: the investors’ or the portfolio manager’s. The key question is how investors are impacted by the presence of the adviser. We also consider the consequence of influence activity on aggregate social welfare.

6 Comparison of Equilibria

The comparison between the equilibria with unbiased and biased advisers is relatively easy to make. However, it is less obvious how equilibrium outcomes change when there is no adviser. To illustrate the differences, we construct a numerical example and solve for the equilibria in the three cases: (1) an unbiased investment adviser; (2) an investment adviser with influence; and (3) no investment adviser.

6.1 Outcomes

In this subsection we analyze the equilibrium fund size, the amount of direct investment, the management fee and net returns as a function of \( k \). The results are illustrated in figure 1.
Figure 1: Comparison between equilibria

This figure compares the outcomes of three different equilibria. Solid lines correspond to the equilibrium with unbiased advisers (in panel (b), it also corresponds to the equilibrium with biased advisers), dashed lines correspond to the equilibrium with biased advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05, \alpha = 1.1, \gamma = 10^{-9}, A_m = 5 \times 10^7, C_0 = 5 \times 10^7, \tau = 0.02$. 

23
We plot the fund size in the three equilibria in figure 1a. Here the solid line represents the fund size in both the biased- and unbiased-adviser equilibria since they are the same. As we can see from the graph, the size of the active portfolio is substantially larger in the presence of investment advisers, especially when \( k \) is large, i.e., when the fraction of high networth investors in the economy is small. This is because investment advisers assist small investors to invest in the active portfolio.

Note that before-fee returns are inversely related to fund size. Figure 1a therefore implies that the before-fee performance is higher for funds that are sold only through the direct channel as compared to funds that are sold through both channels. This is supported by empirical evidence in Bergstresser et al. (2006), who show that returns are higher for direct channel funds as compared to brokered funds even before fees are deducted.

Figure 1b compares the amount of investment through the direct channel. In the case without advisers, this is equivalent to the total fund size. While the fund size is important for the portfolio manager's profit, the amount of direct-channel investment is critical for the investor welfare since only the direct investors earn a surplus over their reservation return. In all scenarios direct investment decreases with \( k \), i.e. when there are fewer high-networth individuals. Not surprisingly, the direct-channel investment is smallest in the presence of biased advisers because some investors are shifted to the indirect channel due to the kickback.

We now turn to the effect of the existence of unbiased advisers on the amount of direct investment in the active fund. There are two effects to consider. First, for given management fees, the absence of advisers increases the amount of direct investment, as there is no substitute. However, there is a second effect due to the endogeneity of the management fee. In the absence of advisers, it is optimal for the portfolio manager to charge higher fees since demand is more inelastic. The magnitude of this effect increases with the number of high-networth investors, which corresponds to a small \( k \). This second effect dominates the first in this region.

Figure 1c illustrates the portfolio manager's fees as a function of \( k \). In the case with unbiased advisers, the fee is constant. This is because the indirect investors are the marginal ones, and their reservation return is independent of their wealth. Moreover the fee charged in the case of unbiased advisers is by far the lowest of the three scenarios. The reason for the low fee is that the presence of the adviser effectively
makes the demand more elastic and thus fee reductions are more profitable for the portfolio manager.

If the adviser can be biased, then price discrimination between high networth and low networth investors becomes possible, since part of the management fee can be kicked back to the indirect investors via the adviser. Thus the higher fee charged by the portfolio manager only impacts the direct investors. As \( k \) becomes larger, i.e. as the relative number of high networth investors becomes smaller, the potential benefit of price discrimination decreases. Therefore the optimal fee is smaller.

Figure 1d compares the return of the active portfolio after management fee. Consistent with the results on the difference in fees, the net return is always lower in the case of kickbacks as compared to the situation with unbiased advisers. The behavior of net returns in the absence of the adviser is related to the direct investment decision of high networth investors. When there are many of them (\( k \) small) the portfolio manager can capitalize by increasing her fees to a greater extent. As a result the net returns are reduced below the unbiased case. When there are fewer of them (\( k \) large) the portfolio manager is only able to sell to lower networth investors by reducing her fees so that the net returns are more attractive than in the unbiased case.

6.2 Welfare Analysis

We now analyze how the aggregate social welfare is affected by the presence of advisers, as well as by influence activity. Since investment advisers, when in existence, as well as the indirect investors, always have zero surplus, we can define the social welfare as the sum of the portfolio manager's profit and the surplus of the direct investors, where investor surplus is defined relative to the default of not paying the search cost, investing in the passive asset and earning net return \( R_f - \tau \).

The effect of influence activity on investor welfare has already been described in section 4. Recall that influence activity shifts some investors from the direct channel to the indirect channel. Investors who are forced to switch to the indirect channel lose their surplus, while those who remain in the direct channel get a lower net return as the portfolio manager raises her fee. The total effect on investor surplus is therefore unambiguously negative.

Not surprisingly, the portfolio manager's profit changes in an opposite direction. Substituting the optimal fee \( f^*_P \) and the optimal bias \( \delta^* \) in Proposition 2 into the objective function of the portfolio manager,
we get the portfolio manager's profit in equilibrium with influence activity as

\[
\Pi_P = \frac{(\alpha - R_f + 2\tau/k)(\alpha - R_f)}{4\gamma} - \left(\frac{\alpha - R_f}{2\gamma} - A_D\right)\frac{\tau}{k} = \frac{(\alpha - R_f)^2}{4\gamma} + \frac{A_D\tau}{k},
\]

where \(A_D\) is given by equation (27). One can see that compared to profit in the zero bias equilibrium (equation (15)), the portfolio manager's profit increases by \(A_D\tau/k\).

Combining the profit for the portfolio manager with the surplus earned by the investors we compute the social welfare in the three scenarios. The details of these computations are carried out in appendix A.3. We are able to prove the following proposition:

**Proposition 5.** The levels of social welfare in equilibria with no advisers, \(U^0\), unbiased advisers, \(U^1\), and biased advisers, \(U^2\), are given respectively by

\[
U^0 = (\alpha - \gamma A^0_0 - R_f + \tau)A^0_0 - \theta^0 C_0(R_f - \tau),
\]

\[
U^1 = A^1_1 \tau - \theta^1 C_0(R_f - \tau) + \Pi^1_P,
\]

\[
U^2 = A^2_2 (\tau - \delta) - \theta^2 C_0(R_f - \tau) + \Pi^2_P,
\]

where \(\theta^i \equiv \int_{A^*}^{+\infty} f(x)dx\) denotes the fraction of investors choosing the direct channel, \(i = 0\) (no adviser), \(i = 1\) (unbiased adviser) and \(i = 2\) (biased adviser). Furthermore, we have

\[
U^1 - U^2 = (A^1_1 - A^2_2)\tau - (\theta^1 - \theta^2)C_0(R_f - \tau) > 0,
\]

i.e., the social welfare is strictly higher in the equilibrium with unbiased advisers than in the equilibrium with biased advisers.

**Proof.** See Appendix A.3

Figure 2a plots the social welfare in the three equilibria under the same parameter values used to plot figure 1. Consistent with Proposition 5, the social welfare in the unbiased-adviser equilibrium (the solid line) is always higher than in the equilibrium with kickbacks (the dashed line). More interestingly, the figure also shows that both of these equilibria dominate the no-adviser equilibrium: the social welfare in
This figure compares welfare in three different equilibria. Solid lines correspond to the equilibrium with unbiased advisers, dashed lines correspond to the equilibrium with biased advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05, \alpha = 1.1, \gamma = 10^{-9}, A_m = 5 \times 10^7, C_0 = 5 \times 10^7, \tau = 0.02$. 
the equilibrium without advisers (the dotted line) is lower than in the other two equilibria for all different values of $k$ we consider.

Figure 2b compares the profits of the portfolio manager in three equilibria for various values of $k$. With the presence of unbiased investment advisers, the profit of the portfolio manager is independent of the wealth distribution parameter $k$, therefore it is a horizontal line in the diagram. The portfolio manager is strictly better-off in the equilibrium with kickbacks. She benefits more from the advisers' biased allocation when $k$ is smaller. This is because she extracts more rents from the high networth investors when there are many such investors in the economy.

For most reasonable values of $k$, the portfolio manager also benefits from the presence of investment advisers, even when kickbacks are forbidden. This is not surprising because the existence of investment advisers allows the portfolio manager to provides her services to small investors, who will otherwise not participate in the active portfolio. Interestingly, this is not always the case. When $k$ is small, the portfolio manager's profit is higher in the no-adviser equilibrium than in the equilibrium with unbiased advisers, indicating that when there are many wealthy investors, the portfolio manager may be better off by declining investment through an indirect channel.

Figure 2c plots the investor surplus in different equilibria, as derived in Appendix A.3. Consistent with our analytical results, investor surplus is higher when advisers are unbiased than when they are biased. The figure also shows that from the investors' point of view, biased advisers are always worse than no advisers at all. More interestingly, when $k$ is large, even unbiased advisers can reduce investor welfare. Only when $k$ is small, investors are better off with unbiased advisers than without advisers. This is consistent with the shape of the net return of the active portfolio plotted in figure 1d.

In summary, our analysis shows that investment advisers, even if they can be influenced by the portfolio manager, improve social welfare. However, although investment advisers claim to serve investors, their presence mainly benefits the portfolio manager. Even when investment advisers are unbiased, they improve investor welfare only if the fraction of high networth investors in the economy is large. If they can be influenced by the portfolio manager, then investors as a whole are better off without them.
7 Conclusions

The market for financial products and services is expanding rapidly as corporations and financial institutions package cash flows and contingent claims in different ways. As the number of alternatives placed before investors has increased, investment advisers play a critical role in allocating assets. Investment advisory services are employed by many types and categories of investors. For instance, the fastest growing segment of mutual fund inflows comes through brokers, not through direct investment (with the notable exception of Vanguards Index 500 fund). Investment advisers are employed by corporate pension funds, university endowment committees. The purpose of this paper has been to look at such financial intermediaries in terms of the overall structure for investment management services.

In our model advisers facilitate the participation of small investors in an actively managed portfolio by achieving economies of scale in identifying skilled portfolio managers. In addition, our analysis shows that unbiased investment advisers expose the portfolio manager to increased competition from other investment alternatives. Specifically, we find that the presence of investment advisers increases the size of the active fund and reduces the fee optimally charged by the portfolio manager. These two effects on the net returns act in opposite directions. Which of these effects dominates depends on the distribution of investors’ wealth in the economy. If the economy is largely made up of small investors, then unbiased advisers lead to lower net returns; in the opposite case advisers lead to an increase in net returns.

We also extend our analysis to welfare implications for the investors and the portfolio manager. While the small investors are not affected by the presence of advisers, the large investors may either experience an increase or a drop in their welfare. If there are only few large investors, then the active fund experiences a significant diseconomy of scale once indirect sales via the adviser become feasible. This lowers the large investors’ welfare. By contrast, if there are many large investors, then the diseconomy of scale mentioned above is smaller and thus dominated by the increased competitive pressure on the portfolio manager. Thus the welfare of large investors is improved. The portfolio manager’s welfare is impacted in the opposite manner. Considering aggregate social welfare, we find that unbiased investment advisers benefit society, although most of the benefits are extracted by the portfolio manager.

We also show that portfolio managers wish to expend resources to influence the advisers’ decision making. The result of these activities will unambiguously lower the aggregate welfare of investors, al-
though the effects are concentrated on investors with higher wealth levels. We show that payments to influence the advisers allow the portfolio managers to charge higher fees. However, fund size is unaffected. As a result, the gross return of actively managed portfolios is unaffected, but the net return decreases. Thus, there is a positive relationship between the management fee and influence activities. Interestingly, influence actually increases the magnitude of investing through the adviser. Thus, under rational expectations, biased advisers are actually used to a greater extent than if they were unbiased. Influence activities are also shown to be positively related to the cost of individual investors associated with tracking the benchmark index.

The presence of investment advisers creates an externality for investors who do not use their services since they ensure competitive returns to the actively managed portfolio. As a result, the amount of direct investment is independent of fund characteristics, such as managerial ability and the degree of the diseconomy of scale. These characteristics, however, do influence the quantities sold via advisory services. For example, the importance of indirect sales through investment advisers increases with the skill of the active portfolio manager. Ceteris paribus, large funds are sold primarily indirectly whereas small funds feature proportionally more direct investors.

Our results point out that without kickbacks advisers improve welfare, although the main beneficiaries are the portfolio managers and not the investors. When advisers can be biased through kickbacks, then investors’ welfare is unambiguously reduced. However, apparent policy implications for regulating the investment advisory services industry must be interpreted very cautiously. Even though investors are worse off with biased advisers, the portfolio manager and society as a whole is better off compared to not having investment advisers. Furthermore, in a more general model the value created by active managers would be endogenous. If their potential profit is curtailed by regulation, they are less likely to make the investment necessary to attain high levels of expertise.
A Appendix

A.1 Proof of Proposition

Proof. Combining the two constraints in problem (28) we immediately obtain equation (31). Substituting this expression back into the objective function and differentiating, we have

\[
\frac{\partial \Pi_P}{\partial A_D} = (\alpha - R_f + \tau) - 2\gamma A_D - \frac{\lambda k}{k-1} A_D^{\frac{1}{k-1}}
\]

\[
\frac{\partial^2 \Pi_P}{\partial A_D^2} = 2\gamma - \frac{\lambda k}{(k-1)^2} A_D^{\frac{2-k}{k-1}} < 0.
\]

Equation (30) is given by setting the first order condition above equal to zero. Since \( \Pi_P \) is strictly concave when \( A_D > 0 \), the first order condition is both necessary and sufficient condition for the solution to this maximization problem; furthermore, the optimal \( A_D \) is unique. To prove the existence of an interior solution, \( 0 < A_D < W - C_0 \), to the first order condition, note that \( \frac{\partial \Pi_P}{\partial A_D} > 0 \) if \( A_D = 0 \). Due to the monotonicity of the first derivative, it suffices to show this derivative becomes negative as \( A_D \to W - C_0 \), i.e., as \( A_D \) converges to the aggregate wealth of the economy net of the searching cost \( C_0 \). This is guaranteed by the boundary condition (7), which implies that

\[
\alpha - R_f - \gamma(W - C_0) + \tau < 0.
\]

A.2 Two Special Cases

An analytical solution to the first order condition for the no-adviser equilibrium, equation (30), is not available for an arbitrary \( k \). We provide the solutions for two special values of \( k \) which are close to the empirically observed values: \( k = 1.5 \) and \( k = 2 \). We can show that the optimal fund size chosen by the portfolio manager (via the optimal fee) for \( k = 1.5 \) and \( k = 2 \), respectively is

\[
A_D = \begin{cases} 
\frac{\alpha - R_f + \tau}{2\gamma + C_0(R_f - \tau)/A_D^2} & \text{if } k = 2, \\
\left(\sqrt{\gamma^2 + C_0(R_f - \tau)(\alpha - R_f + \tau)/(3A_D^3) - \gamma}\right)^{\frac{3A_D^3}{C_0(R_f - \tau)}} & \text{if } k = 1.5.
\end{cases}
\]
Accordingly, the optimal fee is

\[
f_P = \begin{cases} 
1 - \frac{R_f - \tau + C_0(R_f - \tau)A_D/2A_m}{a - f} & \text{if } k = 2, \\
1 - \frac{R_f - \tau + C_0(R_f - \tau)A_D^2/9A_m^3}{a - f} & \text{if } k = 1.5;
\end{cases}
\]  

(39)

and the return of the active portfolio net of the management fee is

\[
R_P(1 - f_P) = \begin{cases} 
R_f - \tau + \frac{C_0(R_f - \tau)A_D}{2A_m} & \text{if } k = 2, \\
R_f - \tau + \frac{C_0(R_f - \tau)A_D^2}{9A_m^3} & \text{if } k = 1.5.
\end{cases}
\]  

(40)

The optimal fund size can be derived easily from the first order condition, while the optimal fee and the net return of the active portfolio follow from the two constraints in problem (28).

### A.3 Proof of Proposition 5

**Proof.** In the case without investment advisers, the number of direct investors is the same as the number of investors investing in the active portfolio. Denote the total surplus of direct investors, relative to the default of passive investment, by 

\[
S^0_D = \int_{A^*_0}^{+\infty} [x(\alpha - \gamma A^*_0)(1 - f_P) - (x + C_0)(R_f - \tau)] f(x) dx
\]

where 

\[
A^*_0 = \int_{A^*_0}^{+\infty} x f(x) dx = \frac{A^*_0}{\theta^0},
\]

and 

\[
\theta^0 = \int_{A^*_0}^{+\infty} f(x) dx = \left(\frac{A^*_0}{\theta^0}\right)^k.
\]

Correspondingly, the total social welfare can be written as:

\[
U^0 = S^0_D + \Pi^0_P
\]

\[
= S^0_D + (\alpha - \gamma A^*_0)A_D f_P
\]

\[
= (\alpha - \gamma A^*_0 - R_f + \tau)A_D - \theta^0 C_0(R_f - \tau).
\]

In the equilibrium with unbiased advisers, the after-fee return of the active index is equated to 

\[R_f\] by
the advisers actions. Therefore the direct investor’s surplus, \( S^1_D \), is given by

\[
S^1_D = \int_{C_0(R_f - \tau)}^{+\infty} [x R_f - (x + C_0)(R_f - \tau)] f(x) dx
\]

\[
= A^1_D \tau - \theta^1 C_0(R_f - \tau),
\]

where \( A^1_D \) is derived in equation (11), and \( \theta^1 \equiv \left( \frac{\tau A_m}{C_0(R_f - \tau)} \right)^k \).

Similarly, since the after-fee return of the active portfolio in the case with kickback equals \( R_f - \delta \), the total surplus of the direct investors in the kickback equilibrium is given by

\[
S^2_D = A^2_D (\tau - \delta) - \theta^2 C_0(R_f - \tau),
\]

where \( A^2_D \) is derived in equation (27), and \( \theta^2 \equiv \left( \frac{(\tau - \delta) A_m}{C_0(R_f - \tau)} \right)^k \).

Adding the manager’s profit to the investor surplus, we get the social welfare \( U^1 \) and \( U^2 \) in Proposition 5.

To compare the social welfare in equilibria with and without kickback, first note that allowing kickbacks reduces the investors’ surplus by

\[
S^1_D - S^2_D = A^2_D \delta + (A^1_D - A^2_D) \tau - (\theta^1 - \theta^2) C_0(R_f - \tau).
\]

Furthermore, recall that from equation (33), we know that allowing kickbacks increases the portfolio manager’s profit by \( A^2_D \delta \). Combining these two welfare effects, we have

\[
U^1 - U^2 = (A^1_D - A^2_D) \tau - (\theta^1 - \theta^2) C_0(R_f - \tau)
\]

\[
= \int_{C_0(R_f - \tau)}^{C_0(R_f - \tau)} \frac{x f(x) \left[ \tau - \frac{C_0(R_f - \tau)}{x} \right] dx}{x} > 0.
\]

The above expression is positive for any \( \delta > 0 \), since \( \tau - C_0(R_f - \tau)/x \) is positive for any \( x \) not less than \( C_0(R_f - \tau)/\tau \). Therefore kickbacks always decrease social welfare. Our proof also makes it clear that in the equilibrium with kickbacks, the welfare loss of the investors staying in the direct channel is exactly offset by the gain of the portfolio manager. The loss of investors who would originally choose the direct channel, but are forced to switch to the indirect channel because of the kickback, is the deadweight loss of social welfare.
References


