Design and Renegotiation of Debt Covenants

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Abstract

We analyze the design and renegotiation of covenants in debt contracts as a particular example of the contractual assignment of property rights under asymmetric information. In particular, we consider a setting where future firm investments are efficient in some states, but also involve a transfer from the lender(s) to shareholders. While there is symmetric information regarding investment efficiency, managers are better informed about any potential transfer than the lender. The lender can learn this information, but at a cost. In this setting, we show that the simple adverse selection problem leads to the allocation of greater ex-ante decision rights to the uninformed party than would follow under symmetric (in particular, full) information. Consequently, ex-post renegotiation is in turn biased towards the uninformed party giving up these excessive rights. In many settings, this result yields the opposite implication from standard Property Rights results regarding contracting under incomplete contracts and ex-ante investments, whereby rights should be allocated to minimize inefficiencies due to distortions in ex-ante investments. Indeed, for debt contracts as well as other settings, the uninformed party, who receives strong decision rights in our setting, is likely to have few significant ex-ante investments to undertake relative to the informed party.

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1 Introduction

In a world with complete contracts, renegotiation would be unnecessary. In practice, of course, contracts are incomplete and various terms and contingencies are often renegotiated as further information becomes available. One striking feature about the renegotiation of debt covenants, as well as many other types of contracts, however, is the one-sided direction of renegotiation that typically occurs. That is, provisions in debt contracts are almost always renegotiated to involve debtholders relinquishing rights – i.e., covenant waivers – often in exchange for some additional consideration for debtholders. One does not generally see the converse, where debtholders pay management more (or accept a lower interest rate) in return for additional constraints that they can impose on management.[1]

Our paper attempts to explain this observed asymmetry, by analyzing the assignment of decision rights in the presence of asymmetric information and information acquisition costs. We start with the presumption that debtholders are less informed than an entrepreneur about potential future transfers from debt to equity,[2] and explore the implications of this adverse selection on the allocation of decision rights in an initial contract and on subsequent renegotiation. Intuitively, we analyze the notion that debtholders may receive stronger decision rights than under symmetric information in order to "protect" them without necessitating too much inefficient information acquisition on their part. We analyze under what conditions this notion is correct, and what are the subsequent implications for information acquisition and renegotiation.

Most debt contracts include covenants that involve restrictions placed on firms in conjunction with the debt issue. For example, a firm might not be allowed to issue new debt if net working capital is below a specified level or if an interest coverage ratio is too low. Common covenants conditions are based on firm net worth, working capital, leverage, interest coverage, and cash flow; and involve restrictions on issuing debt and paying dividends, or impose conditions such as the acceleration of debt payments if the specified condition is binding.[3]

It is generally understood that such covenants serve to protect bondholders against activities that transfer wealth from debtholders to shareholders. Efficiency considerations, however, dictate

[1] Beneish and Press (1993) do find covenants that are tightened during renegotiation. However, as noted by Smith (1993), this is likely to involve replacing binding covenants with tighter nonbinding covenants on other variables. To the extent that this is the case, debtholders are relinquishing current control rights (as specified by the binding covenant) for other considerations (including future control rights) in the renegotiations.

[2] Throughout this paper we speak of an owner/entrepreneur holding equity; we could equivalently speak of a manager acting on behalf of shareholders.

[3] See, for example, Smith and Warner (1979) and Smith (1993).

that some activities that transfer wealth from debtholders to shareholders should be permitted, and some should be restricted. For example, the investment in both good (positive NPV) and bad (negative NPV) new projects are likely to involve a transfer from pre-existing debtholders to shareholders, by increasing the risk in firm returns.[4] While an owner/entrepreneur, will have an ex-post incentive to overinvest, any such anticipated inefficiency would be borne ex-ante by him, through the issue price of the bonds. Consequently, the owner/entrepreneur would like to commit in advance to only undertake positive NPV investments, and restrict himself from negative NPV investments.

While debt covenants serve this efficiency enhancing role, in practice, a vague contractual provision to "only undertake positive NPV projects" is likely to be unenforceable by the courts. Consequently, covenants are instead conditioned on more easily observable accounting variables, such as financial ratios, that are likely to be correlated with future project quality. Since this correlation is imperfect, there will at times be scope for renegotiation when a new project becomes available. Thus, it should not be surprising that debt covenants are commonly renegotiated. What is quite striking, however, is that such contracts seem to be written so that the renegotiation almost always involves the debtholders relaxing some restriction for some consideration in return.[5]

Simple considerations of renegotiation costs do not seem to predict such an asymmetry in the direction of renegotiation.[6] In a setting where renegotiation is costly, but one can only contract on variables imperfectly correlated with the quality of future projects, minimizing future renegotiation costs would involve trading off the cost of giving debtholders rights that will sometimes restrict good investments with the cost of giving shareholders rights that will sometimes allow bad investments. Typically, an interior solution to such a problem would obtain, with some renegotiation ensuing in each direction. That is, in some states, debtholders would allow efficient projects to be undertaken for a payment, and in others, shareholders would abstain from inefficient projects for a payment.

Notably, however, the former form of renegotiation seems to be much more prevalent than the

[4]We will take such "asset substitution activity" of Jensen and Meckling (1976) as a canonical case, but it should be understood that any other activity that transfers wealths from debtholders to shareholders would serve the same purpose in our model. There is a voluminous finance literature that focuses on the size, costs, and consequences of asset substitution and on other manners of wealth transfer.

[5]In a typical covenant waiver, the debtholder allows the manager to undertake some action prohibited in an initial covenant or relaxes the covenant that is in breach, in exchange for new additional restrictions and/or a higher interest rate. See, for example, Beneish and Press (1993), Sweeney (1994), and Beneish and Press (1995).

[6]Notably, almost all covenant violations and subsequent renegotiations occur with private debt. Consistent with this fact, Smith and Warner (1979) and Leftwich (1981) have argued that coordination problems with *public* debtholders will make renegotiation more difficult, and consequently public debt covenants will be set with more slack leading to less renegotiation. Coordination does not explain, however, the observed asymmetry in the *direction* of renegotiation of private debt.

latter. Debtholders seem to be granted *strong* decision rights; that is, more rights than what would follow from a simple criterion of minimizing future expected renegotiation costs. Furthermore, this asymmetry goes in the opposite direction of that which would typically be predicted from standard models of hold-up under incomplete contracting.[7] In particular, considerations of hold-up indicate that control rights should be allocated in order to minimize distortions in ex-ante investments due to the potential of future hold-up. Ex-ante "effort" by entrepreneurs or managers is likely to be more important to the success of investment projects than similar "effort" by outside debtholders (who frequently play a passive role in firm investments). Consequently, considerations of hold-up would imply that strong decision rights should be granted to owner/entrepreneurs rather than to lenders in debt contracts.

We instead explore the notion that lenders might receive strong rights as a natural consequence of an adverse selection problem, where they are granted such rights to "protect" them without necessitating too much inefficient information acquisition on their part. Debtholders may be less informed than an entrepreneur, for example, about the scope of potential asset substitution activity, about the ability of an entrepreneur to divert assets to perk consumption or other private benefits, or about the value that an entrepreneur places on building a reputation not to engage in such activity.

While uninformed debtholders will value future decision rights, this, of course, does not necessarily imply they will be granted such rights in equilibrium. Such decision rights have a value to the entrepreneur as well, and the entrepreneur will only give up such rights for a price. Indeed, if ex-post renegotiation was costless and efficient and if information was symmetric, both parties would have the same value for the decision rights, and its initial assignment would be irrelevant; i.e., the Coase Theorem would hold. Our paper analyzes when in fact it will be the case that asymmetric information and information acquisition costs imply that the less informed party will be granted strong decision rights.

We consider a stylized model designed to analyze these issues in a simple manner. In our setting, an entrepreneur needs to raise money from a lender to undertake a project (at time 0). At a future date (time 1), a decision must be made whether to invest further and expand the project. The efficiency of this time 1 investment depends on the time 1 state of the world, which is not known by either party ex-ante (at time 0), but which is known by both parties prior to the investment. We

[7] See, for example, Klein, Crawford, and Alchian (1978), Grout (1984), Williamson (1985), and Grossman and Hart (1986) for the starting point of this large literature.

assume that while the time 1 state of the world is non-contractible at time 0, the initial contract can allocate the right to make the time 1 decision. The agency conflict we assume that makes our problem interesting is that it is always beneficial for entrepreneurs to invest further at time 1, and it is always detrimental for the lender.

Prior to the time 1 investment, the two parties can renegotiate the decision to be made (or equivalently, who gets to make this decision). This renegotiation, however, is complicated by asymmetric information regarding the division of surplus under the new investment: in particular, the new investment will transfer an uncertain quantity x from the lender to the entrepreneur, where the entrepreneur knows the realization of x, but the lender only knows the distribution from which x is drawn unless he pays a cost to learn the realization. One simple interpretation is that further time 1 investments increase risk, leading to an expected transfer from the lender to the entrepreneur through the familiar asset substitution effect.

One implication of this simple specification is that at time 1 it is always common knowledge whether or not renegotiation would be beneficial, as both parties know whether the investment is efficient at this time. Also, the entrepreneur will always choose to invest while the lender will always choose not to. Consequently, if renegotiation is not beneficial, and if the lender is uniformed about x at time 1, there is no need for him to become informed then. Consequently, there will be an efficiency gain if such information was never acquired by the lender. If instead, however, there is scope for renegotiation at time 1, then it may be beneficial for the lender to become informed, as the lender may fare better in the renegotiation, and the outcome of the renegotiation may be more efficient.

This setup allows us to analyze a natural trade-off between early and late acquisition of information. The early acquisition of information will allow the lender to negotiate the initial contract without an informational disadvantage, and may in turn lead to a more efficient contract. However, such information acquisition costs may be wasteful if the information relates to future states of the world that are never realized. Postponing information acquisition until the state of the world is realized will allow the lender to forgo these costs for states where the information is not relevant.

More importantly, the setting allows us to analyze the implications that the asymmetry of information has for the assignment of decision rights in the initial contract. Here we are asking the standard Grossman-Hart-Moore Property Rights question of optimal assignment of rights given contractual incompleteness.[8] However, we depart from much of this literature by considering a

[8]See Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995).

different form of contractual inefficiency. In particular, we analyze the implications of asymmetric information with costly information acquisition, rather than the standard inefficiencies of hold-up due to non-contractible ex-ante actions.[9]

In this setting, we find that strong rights are granted to the lender in the initial contract when early information acquisition is sufficiently inefficient that the lender chooses to remain ex-ante uninformed. Intuitively, lacking information about future asset substitution, the lender must form inferences about the entrepreneur's information based on the contract that is being offered. In equilibrium, the entrepreneur will have to compensate the lender, through the terms of the contract, for the inferred amount of asset substitution activity that will ensue. "Good types" – i.e., entrepreneurs who do not have as much such activity at their disposal – will prefer to give strong decision rights to lenders, even when this is informationally inefficient (i.e., even when this leads to excessive renegotiation and ex-post information acquisition costs), due to this adverse selection problem. Consequently, lenders on average will be given stronger decision rights than what would follow under symmetric information (where the allocation of decision rights is given by minimizing expected renegotiation costs). Ex-post renegotiation is in turn biased towards the uninformed party giving up these excessive rights: compared with symmetric information, the uninformed party will more frequently give up contractual rights (in exchange for a payment) and the informed party will do so less frequently.

While we focus on the interpretation of debt covenants for the sake of a concrete example, there is nothing in our analysis that is unique to this setting. Our model could be applied equally well to a number of other settings. For example, home mortgage contracts and fixed-price procurement contracts also seem to exhibit a similar asymmetry in renegotiations: banks often agree to relax restrictions on home mortgages[10]; and contractees often agree to pay contractors more to cover "unanticipated costs" or changes in design. Notably, in both of these cases, the party granted the initial right which is typically relaxed under renegotiations is the one that is likely to be more uninformed. In particular, the lender (who relaxes mortgage restrictions) is likely to be more uninformed about the potential for default on a given loan, and the contractee (who has the right to approve or veto any changes in initial plans) is likely to be more uninformed about potential

[9] Stole and Zwiebel (2002) have argued that while ex-ante non-contractible investments have received an enormous amount of attention in the literature, there are other likely manners of contractual incompleteness, yielding new implications for the allocation of decision rights and ownership that have been relatively unexplored, which merit further attention.

[10] A common example is for lenders to allow homeowners with loans that exceed the current home value to walk away from their home, without instigating foreclosure or going after further assets.

cost overruns.[11]

There are several papers in the incomplete contracting literature related to ours. Perhaps most closely related are Spier (1992) and Sridhar and Magee (1997). Spier (1992) shows that contractual incompleteness may arise from adverse selection, in a setting where the suggestion of certain contractual clauses may signal asymmetric information. Similarly, in our setting, the choice by the informed party of decision rights signals some information about transfers associated with future decisions. However, by assumption, the decision rights are always contracted on in our setting (i.e., the contract is not incomplete on this dimension). As such, our focus is on the effect of asymmetric information on the allocation of such rights and on subsequent information acquisition, rather than on contractual incompleteness.[12] Sridhar and Magee (1997), like our paper, considers the design of debt covenants under asymmetric information and incomplete contracts. They most significantly differ from us in assuming the asymmetry in renegotiations that we derive as a central result in our analysis; i.e., they assume that only the lender, and not the owner, can relinquish rights (through unilateral covenant waivers when it is beneficial for them to do so). They subsequently focus on different aspects affecting the assignment of covenant rights from us; notably, the informativeness of contractible variables and the scope for accounting misrepresentations.

Section 2 presents our model. Section 3 characterizes equilibrium in our model. Section 4 analyzes properties of this equilibrium and gives intuition. Section 5 concludes. Further extensions that more explicitly model several features specific to debt contracts, and proofs, are given in the Appendix.

2 The Model

We consider a wealth-constrained entrepreneur E who needs funding I at time 0 to undertake a project. Ex-ante E faces a competitive lending market and consequently offers a break-even contract to L in return for I. Both E and L are risk-neutral, and for simplicity, the interest rate is 0. If undertaken, this project yields a certain return of R at time 2 (provided that no further investments – described below – are taken). In return for this financing, E signs a contract

[11] Note that these two examples both typically involve an infrequent participant in the market (homeowner, contractee), and a repeat player (lender, contractor). In one case, however, the repeat player (the contractor) likely has more information, and in the other case, the infrequent participant (the homeowner) likely does. Given this difference, it is striking that in both these cases, the uninformed party seems to be granted strong initial rights, and that renegotiation typically involves the uninformed party giving up rights.

[12] See also Hermalin (1988). Huberman and Kahn (1988) notes that costly contractual contingencies should decrease with the ability to renegotiate contracts.

promising a payment D to L when returns are realized, and specifying time 1 decision rights, as described below. We assume that state-contingent contracts are not possible. If the parties enter into such an agreement, play proceeds as summarized in the timeline of Figure 1, which we now describe in detail.

Conditional on undertaking the project at time 0, there is an opportunity to undertake a further investment at time 1. One interpretation is simply an expansion of the initial project. This further investment requires no additional cash outlay. We denote the decision to take or not take this additional investment by A (action), NA (no action) respectively. The initial time-0 contract specifies who has the right to make this investment decision; if this right is assigned to L, we will call this a covenant.[13] However, the parties may choose to renegotiate prior to this investment decision. In particular, the party that does not have the decision right might pay the other party in order to take a different action (or, equivalently, to transfer the decision right). We let t represent the net payment from E to L in such a renegotiation.

For simplicity, we assume that in such renegotiations, L makes a take-it-or-leave-it offer, but observe here that this assumption does not play an important role in our results. As we discuss below, for the main case we consider (when L always buys information either at time 0 or time 1), the outcome is independent of the ex-post bargaining division specified here, as the time 0 agreement anticipates this division and adjusts accordingly. We further assume that renegotiation is costly, in that a fixed cost $c_r > 0$ has to be paid by L — L has all of the bargaining power — in order to renegotiate. We interpret this cost as an administrative cost, including, among others, legal expenses and the opportunity cost of time. The renegotiation cost gives the two parties an incentive to write an initial contract that minimizes the probability of renegotiating.[14]

After the time 0 project is undertaken, the state of the world is revealed to be either good (G) or bad (B) to both parties. Prior to this time, both E and I only know that the probability of

[13] In practice, debt covenants that allow debtholders the right to veto investments are generally contingent on a verifiable state of the world; e.g., a low capital ratio. Such contingent covenants would naturally follow in our model, to the extent that new investments primarily transfer wealth from lenders to shareholders only in certain verifiable states (i.e., when the firm is in or near financial distress). Such contingencies could easily be accounted for, albeit at the cost of needlessly complicating the model. In particular, consider the following addition to the model: Assume that there is an additional moment in time – say, time $\frac{1}{2}$ – between when the initial contract is signed at time 0 and time 1. Between time 0 and time $\frac{1}{2}$ L and E learn whether E is in "financial distress" or not, an event presumed to be contractible at time 0. If E is not in financial distress, then no asset substitution activities are possible (as debt will be safe and paid for sure). If E is in financial distress, the game proceeds as in our model, with the same payoffs. In such a setting, the contract that would be written at time 0 would always grant the entrepreneur control contingent on there being no financial distress at time $\frac{1}{2}$, while it would follow our equilibrium contingent on financial distress.

[14] The case of $c_r = 0$ presents some technical complications. We describe the issue, and derive results for this case, in Appendix C.

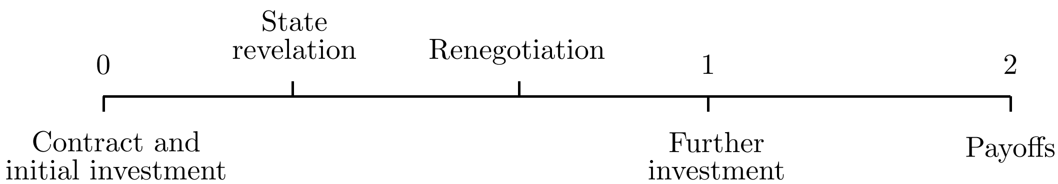

Figure 1: Time diagram

state G is p. The time-1 investment is efficient in state G and inefficient in state B. In particular, in state G, this added investment will yield an additional expected return of $y > 0$ for E at time 2, and in state B it will yield an additional expected return of $-y$ for L at time 2, where y is deterministic and known by all ex ante.[15]

Additionally, the time 1 investment will lead to an uncertain transfer x from L to E. This transfer x can be interpreted as an expected transfer due to increased risk (i.e., asset substitution) or, alternatively, it can be interpreted as an amount of assets that E can "pocket" for himself due to the added complexity of further investments. Ex-ante, E knows the realization of x, while L only knows that x is distributed with full support over the interval $[a, b]$, with $0 \leq a < b$, according to the cdf F. For simplicity we assume that F is atomless, and that the associated pdf f is strictly positive on $[a, b]$ almost everywhere. The structure of the information about x means that E may know more than L about future risks involved with a project, or future opportunities to pocket funds. We refer to x as E's type, and for exposition will adopt the asset-substitution interpretation of x.[16]

While initially uninformed about x, L can learn its realization for a cost of c_0 at time 0 prior to the signing of the initial contract, or at cost c_1 at time 1 prior to the investment decision.[17] We

[15]This loss of $-y$ to L in state B, which could be taken to be $-y'$ with $y' \neq y$ without changing the analysis, is discussed further in footnote 16 below.

[16] With these interpretations, it would be natural for x to depend on the promised payment D. Similarly, in many settings, the inefficiency $-y$ associated with investing in the bad state of the world would be divided between E and L as a function of D. (For example, L might suffer the marginal loss of $-y$ only when E was illiquid or when verifiable cash was less than D.) For simplicity we take the realization of x to be independent of D, and similarly, we assume the inefficiency $-y$ is realized entirely by L, but demonstrate in Appendix A that all qualitative features of the model hold when these assumption are relaxed.

[17]One natural case would be $c_0 = c_1$; i.e., the cost of acquiring the information does not depend on the time at which the information is acquired.

Table 1: Game payoffs to (E, L)

	A	NA	Probability
G	$(R-D+x+y, D-x))$	$(R-D, D)$	p
B	$(R-D+x, D-x-y)$	$(R-D, D)$	$1-p$

assume that L's decision to acquire information is observable to E. We also assume that E cannot hold up the cost c_0 if incurred by L.[18]

Finally, at time 2, all returns are realized and payoffs are made. We will assume that R is sufficiently high so that D (which is determined in equilibrium), is less than or equal to R. Table 1 describes the Period 2 payoffs to E and L conditional on the state of the world and action.

Several remarks on the payoff matrix are in order here. First, when NA is chosen, the payoffs are insensitive to both the state (this feature plays no role in the analysis) and, more importantly, the private information x. In the context of covenants, it is natural to interpret NA as not taking a further investment; more generally, NA can be interpreted as a "neutral decision" not impacted by E's private information. Second, absent renegotiation, L would always prefer NA to A, and E would prefer A to NA, while the socially optimal choice is A in state G, and NA in state B. Thus, if E has the decision right, it is efficient to renegotiate in state B but not state G, and the converse is true if L has the decision right. And third, we note that the constant R plays no role in the play of the game or in our analysis, save but to ensure that required payments by E are feasible under the interpretation of an ex-ante wealth constrained entrepreneur.[19]

3 Equilibrium

We consider Pure-Strategy Perfect Bayesian Equilibria (PSPBE). Since we are primarily interested in the interplay between costly information acquisition and the assignment of control rights, we will

[18]Otherwise, given our assumption of an ex-ante competitive lending market, L will never expend these time 0 costs, as they are sunk, and he will be held to 0-profit not including these costs when the contract is signed. This assumption can be justified in a number of manners: We could assume that E can commit with L prior to signing a contract to reimburse these costs; or that E could, at a cost c_0 make his private information verifiable; or that undertaking these costs raises L's outside option by an identical amount (perhaps because the information learned is equally valuable in alternative relationships). Alternatively, at the cost of some further analysis, we could forgo this assumption altogether by allowing L to have some ex-ante bargaining power. We make this assumption simply because we want to explore the trade-off between early and late information acquisition, and our focus is not on the hold-up of ex-ante investment costs.

[19]For other interpretations of our model, where E and L have some joint benefit to cooperation, but where E is not ex-ante wealth constrained, this term is unnecessary.

concentrate on parameters such that in equilibrium, L acquires information at least in some states. Specifically, we will consider parameters such that if L has chosen not to become informed at time 0, and if there is scope for renegotiation at time 1, L will choose to become informed then. We will give conditions which ensure that this holds and discuss it in some detail below. Additionally, to ensure that ex-post renegotiation ensues when the inefficient action would otherwise be taken, we assume that $c_1 + c_r < y$. That is, we assume the efficiency gains from renegotiating exceed the total cost of information acquisition and renegotiation.

First, consider play at time 1, under this presumption that one of the costs c_0 and c_1 is small enough such that if there is scope for renegotiation at this time, L will be informed. There are four cases to consider, depending on the contractual assignment of the decision right and the state of the world. In two of the cases, when $(r = E, G)$ and when $(r = L, B)$, there is no scope for renegotiation, as the owner of the right already prefers the optimal decision. Consequently, the net payoffs (that is, payoffs including transfers but not including information acquisition costs) to E and L are $(R - D + x + y, D - x)$ when $(r = E, G)$, and $(R - D, D)$ when $(r = L, B)$. Note that if L is not informed entering time 1 in one of these two states, there is no need for him to become informed at this time as such information will yield no benefit. This captures the notion that sometimes information that is acquired at time 0 is unnecessary.

If, instead, the state is G when $r = L$, or if the state is B when $r = E$, there will be scope for renegotiation. As noted above, we are presuming for now that at the time of this renegotiation L has become informed and therefore knows x. In the case $(r = E, B)$, absent renegotiation, E would choose the inefficient action A, since that would yield him $R - D + x > R - D$, despite a combined payoff less than under action NA, i.e., $R - y < R$. Given this, L will offer an additional payment x to E in exchange for action NA to be taken. Thus, the final net payoffs are $(R - D + x, D - x)$. Similarly, in the case $(r = L, G)$, absent renegotiation L would choose NA; thus L will ask for E's entire benefit $x + y$ from taking action A instead. This yields payoffs of $(R - D, y + D)$. The final net payoffs in the four cases are summarized in Table 2. Note that Table 2 differs from Table 1 only in that in both inefficient states, renegotiation yields added benefits of y, which are realized entirely by L since L is assumed to have all of the bargaining power.

Table 2: Final net payoffs after renegotiation to (E, L)

	$r = E$	$r = L$	Probability
G	$(R - D + x + y, D - x))$	$(R - D, y + D)$	p
B	$(R - D + x, D - x)$	$(R - D, D)$	$1 - p$

Turning to the time-0 contract, note that in any pure strategy equilibrium, there can be at most one contract associated with each choice of the decision right r. That is, if a contract (r, D) is accepted in equilibrium, no type of E would offer the contract (r, D'), $D' > D$. Consequently, there are at most two contracts offered in equilibrium, one with $r = E$ and one with $r = L$. Let $S_E \in [a, b]$ denote the set of types who offer, in equilibrium, a contract of the form $(r = E, t)$, i.e., the types who keep the decision right.

Now consider the case in which L has not acquired information at time 0, but will acquire information at time 1 if there is scope for renegotiation. (Later, we shall consider time-0 information acquisition.) With ex-ante competition between lenders, L must break even for any equilibrium contract. These contracts of course anticipate correctly that renegotiation will occur in states $(r = E, B)$ and $(r = L, G)$ and not in states $(r = E, G)$, $(r = L, B)$. Given the final payoffs in Table 2, the ex-ante indifference conditions for L, for contracts with $r = E$ and $r = L$, are, respectively,

$$I = D - p\mathrm{E}\left[x \mid x \in S_E\right] - (1-p)\left(c + \mathrm{E}\left[x \mid x \in S_E\right]\right) \tag{1}$$

and

$$I = D + p(y - c), \tag{2}$$

where $c \equiv c_1 + c_r$. With D satisfying these conditions, the final payoffs to E for the contracts with $r = E$ and $r = L$, are in turn, respectively,

$$U^E_{r=E} = R - I - \mathrm{E}\left[x \mid x \in S_E\right] - (1-p)c + p(x + y) + (1-p)x, \tag{3}$$

and

$$U^E_{r=L} = R - I + py - pc. \tag{4}$$

Note that, when $r = E$, E's payoff increases with x, and that E's payoff is independent of x when $r = L$. Consequently, if type x weakly prefers $(r = E, D)$ to $(r = L, D')$, then all higher types $x' > x$ would strictly prefer the former contract. And likewise, if type x weakly prefers $(r = L, D')$ to $(r = E, D)$, then all lower types $x' < x$ would strictly prefer $(r = L, D')$. It follows that in any PSPBE there will be a cutoff level $\hat{x}$ (possibly equal to a or b) where all types below $\hat{x}$ pool together by offering the same contract with $r = L$, and all types above $\hat{x}$ pool on a single contract with $r = E$. Thus, the set S_E is of the form $[\hat{x}, b]$.[20]

[20] Strictly speaking, type $\hat{x}$ is of course indifferent, and could choose either contract. Our inclusion of this 0-measure

In equilibrium, the cutoff type $\hat{x}$ must be indifferent between keeping and giving up the right. Defining $G(\hat{x}') = \mathrm{E}\left[x \mid x \geq \hat{x}'\right] - \hat{x}'$, equating expressions (3) and (4) then implies that $\hat{x}$ is given by:

$$G(\hat{x}) = (2p-1)c. \tag{5}$$

With this in hand, the following proposition characterizes all PSPBEs.

Proposition 1 *Assume that L pays c_1 and learns x at time 1 if there is scope for renegotiation. Then, a PSPBE always exists, and takes the following form:*[21]

(i) If $G(x) > (2p-1)c$ for all $x \in [a,b]$, then all types offer $r = L$. The promised payment is $D = I - p(y-c)$.

(ii) If $G(x) < (2p-1)c$ for all $x \in [a,b]$, then all types offer $r = E$. The promised payment is $D = I + E[x] + (1-p)c$.

(iii) If there exists $\hat{x} \in [a,b]$ such that $G(\hat{x}) = (2p-1)c$, then types $x \geq \hat{x}$ offer $r = E$, while types $x < \hat{x}$ offer $r = L$. The promised payments are: $D_E = I + E\left[x \mid x \geq \hat{x}\right] + (1-p)c$ when $r = E$ and $D_L = I - p(y-c)$ when $r = L$.

We discuss and interpret the equilibrium of Proposition 1 in the next section, after we first indicate what conditions are necessary for its assumptions to be satisfied. Note that since F is atomless, G is continuous, and consequently, the condition for case (iii) will be satisfied if we are not in case (i) or case (ii) (i.e., the three cases form a complete partition of the space of parameters). Before turning to these conditions, we illustrate this Proposition with a simple example.

Example 1 *Let x be distributed uniformly on $[a,b]$. Then, $G(\hat{x}') = \frac{b-\hat{x}'}{2}$. Note that $G(\hat{x}')$ is monotonically decreasing in $\hat{x}'$. (Proposition 7 below indicates that this will hold whenever the hazard rate of the distribution of x is increasing.) Proposition 1 then indicates that:*

(i) If $p \leq \frac{1}{2}$, then all types of E give the control right to the lender ($r = L$).

(ii) If $p \geq \frac{1}{2} + \frac{b-a}{4c}$, then all types of E retain the control right ($r = E$).

(iii) If $\frac{1}{2} < p < \frac{1}{2} + \frac{b-a}{4c}$, then $\hat{x} = b - 2(2p-1)c$. E retains the control rights if $x \geq b - 2(2p-1)c$; otherwise L is granted the control right.

type in the set $r = E$ is inconsequential.

[21] All proofs are in Appendix B.

For Proposition 1, we have assumed that L does indeed prefer acquiring information at time 1 to not acquiring any information when $(r = E, B)$ and $(r = L, G)$ (that is, when there is scope for renegotiation). Proposition 2 indicates that provided that c_1 is small enough, L prefers acquiring information to bargaining with asymmetric information. Intuitively, the proof shows that the losses to L from bargaining with asymmetric information are bounded away from 0.

Proposition 2 *Assume that L does not acquire information at time 0. Then, there exists $\bar{c}_1 > 0$ such that, for $c_1 \leq \bar{c}_1$, L acquires information at time 1 in both case $(r = E, B)$ and case $(r = L, G)$.*

We now turn to the case where L acquires information at time 0. As noted above, we assume that E can commit to reimburse L for this information acquisition cost c_0, or alternatively, can pay it herself. Thus, information will be acquired at time 0 rather than at time 1 if the ensuing break-even contract for L yields higher expected profits for E (since ex-ante lenders compete with one another to contract with E). Proposition 3 follows immediately.

Proposition 3 *If L learns x at time 0, then*

(i) if $p > \frac{1}{2}$, $r = E$ and $D = x + I + c_0 + (1 - p)c_r$, and the net profits of E are $R - I + py - c_0 - (1 - p)c_r$;

(ii) if $p < \frac{1}{2}$, $r = L$ and $D = -py + I + c_0 + pc_r$, and the net profits of E are $R - I + py - c_0 - pc_r$.

(The two parties are indifferent between the two types of contracts when $p = \frac{1}{2}$.)

When L acquires information at time 0, the total information cost is c_0, while the expected renegotiation cost is the smaller of pc_r and $(1 - p)c_r$. Since we are assuming that information will either be acquired at time 0 or at time 1 if there is scope for renegotiation, the time 1 action taken will be efficient. Given risk neutrality, it follows that the contract that maximizes E's utility subject to the lender breaking even is the contract that minimizes the sum of expected information-acquisition costs and expected renegotiation costs. Hence, the choice between acquiring information at time 0 and acquiring it at time 1 follows from simply comparing these costs, as the following proposition states.

Proposition 4 *Assume that c_1 is small enough such that L would acquire information at time 1 when there is scope for renegotiation if he had not already done so at time 0. Then, L would acquire* *information at time 0 if and only if*

$$c_0 + \min(p, 1-p)c_r \leq (c_1 + c_r)\left[pF(\hat{x}) + (1-p)(1-F(\hat{x})\right]. \tag{6}$$

Early information acquisition imposes the inefficiency of paying the information cost when not necessary. More precisely, late information acquisition results in an information-spending reduction of $c_0 - c_1\left(pF(\hat{x}) + (1-p)(1-F(\hat{x}))\right)$, which is a positive quantity as long as c_0 is not too much smaller than c_1 (and is always positive for $c_0 = c_1$). On the other hand, early information acquisition reduces the expected renegotiation costs by minimizing the probability of renegotiating, yielding a cost saving of $\max[c_r(2p-1), 0] + F(\hat{x})$. (As we will note in the following Section, this savings is zero whenever $p \leq \frac{1}{2}$, as $p \leq \frac{1}{2}$ will imply we are in case (i) of Proposition 1, and consequently, there is no inefficient renegotiation.) Generally, whether early information acquisition is preferred to late information acquisition depends on how the additional information cost compares to the saved renegotiation cost.

Propositions 2 and 4 jointly give conditions under which information will be acquired at time 1 if there is scope for renegotiation. The former indicates when this will be preferred to no information acquisition, while the latter indicates when this will be preferred to information acquisition at time 0. Under these conditions, Proposition 1 holds. In the following section we will analyze Proposition 1 under the maintained assumption that these conditions are satisfied.

4 Equilibrium Properties and Discussion

In this section we analyze the properties of the equilibria with information acquisition at time 1. We discuss and interpret Proposition 1, derive several comparative-statics results, and analyze the relative frequency of the uninformed versus the informed selling rights during renegotiation. We end by analyzing the decision to acquire information early or late.

Before turning to the discussion of the equilibria with information acquisition at time 1, we state a simple benchmark with which to compare our results.[22]

Proposition 5 *If the two parties are symmetrically informed about x at time 0, then L receives control rights whenever $p < \frac{1}{2}$, while E receives control rights whenever $p > \frac{1}{2}$.*

[22]Note that if both parties know x, this result simply restates part of Proposition 3.

This benchmark follows immediately from observing that under symmetric information, E simply offers the break-even contract to L that leads to the least future expected inefficiency (i.e., renegotiation costs). When $r = L$, costly renegotiation (and the possible associated costly information acquisition) is averted in the bad state, whereas when $r = E$ renegotiation and the associated information acquisition are instead averted in the good state. When the bad state is more likely ($p < \frac{1}{2}$) than the good state, the expected costs are smallest when $r = L$, and vice-versa. We refer to this benchmark as the symmetric information outcome, or the constrained efficient outcome (since it minimizes transaction costs, subject to the constraint that contracts cannot be written on the time 1 state of the world G or B).

Returning to Proposition 1, a simple interpretation can be given to the equilibrium condition comparing $G(x)$ and $(2p-1)c$. First note that the latter term, $(2p-1)c$, measures the excessive amount of renegotiation and information-acquisition costs that must be undertaken when $r = L$ instead of $r = E$: this cost is given by $pc - (1-p)c = (2p-1)c$. (The cost is negative if $p > \frac{1}{2}$, signifying a benefit.)

Turning to $G(x)$, suppose that types $[\hat{x}', b]$ for E retained the decision right.[23] Then, L would expect asset-substitution activity given by $E[x \mid x \geq \hat{x}']$ when $r = E$, for which L must be ex-ante reimbursed.[24] The lowest type choosing $r = E$, type $\hat{x}'$, would only benefit from asset substitution activity in the amount $\hat{x}'$. The difference between these two, $\mathrm{E}\left[x \mid x \geq \hat{x}'\right] - \hat{x}' \equiv G(\hat{x}')$, measures the adverse-selection cost faced by the lowest type $\hat{x}'$ choosing $r = E$.

In equilibrium, all types must compare the adverse selection cost from choosing $r = E$ with the excess renegotiation cost from choosing $r = L$. The adverse selection cost is always greatest for the lowest type choosing $r = E$, and this is always positive. In contrast, when $p < \frac{1}{2}$ the excess renegotiation "cost" of choosing $r = L$ is negative; that is, renegotiation is less frequent when $r = L$ than when $r = E$. Consequently, there can not be an equilibrium with some types choosing $r = E$; the lowest type choosing $r = E$ will always benefit by defecting to $r = L$. Hence, when $p < \frac{1}{2}$ case (i) of Proposition 1 obtains, all types choosing $r = L$. This is the constrained efficient symmetric information outcome, as stated in Proposition 5.

In contrast, when $p > \frac{1}{2}$, the allocation of the decision right diverges from the constrained

[23]Recall that we previously argued that in any PSPBE, there must be a cutoff level such that types retain the decision right if and only if they exceed this level.

[24]Absent renegotiation, E would always choose A if she had the right, and L would always choose NA. Hence, in all states, E imposes an added transfer of x if she has the right instead of L. This transfer alters threat points, and is maintained through our renegotiation. In contrast, any efficiency gain y is captured entirely by L in our renegotiation due to our endowing L with the bargaining power through a take-it-or-leave-it offer.

efficient outcome of Proposition 5 when $(2p-1)c$ is not too large. With $p > \frac{1}{2}$, the constrained efficient outcome involves all types E retaining the decision right. Provided that $(2p-1)c$ does not exceed $G^M \equiv \max_x G(x)$, however, case (iii) obtains in Proposition 1, whereby low types will prefer to give the decision right to L despite the associated inefficiency in renegotiation and information acquisition costs. Intuitively, the excessive amount of renegotiation and information-acquisition costs that must be undertaken when $r = L$ instead of $r = E$ is less for these types than the adverse-selection costs they incur by retaining the decision right together with all the high types who choose to do so. In equilibrium, the lowest type choosing $r = E$ is indifferent: she must face adverse selection costs equal to the information and renegotiation inefficiency costs of instead choosing $r = L$; that is, the cutoff type is given by $G(\hat{x}) = (2p-1)c$.

Finally, if it is the case that $(2p-1)c > G^M$, then the adverse selection cost for all types choosing $r = E$ will always be less than the excess information and renegotiation costs from choosing $r = L$. In such an event, case (ii) of Proposition 1 obtains, whereby all types choose $r = E$. Note that since $G(0) > 0$, $(2p-1)c > G^M$ can only occur when $p > \frac{1}{2}$. Consequently, when this case occurs, the allocation of the decision right again matches the constrained efficient outcome of Proposition 5.

We summarize the allocation of the control right that follows from Proposition 1 as compared with the constrained efficient benchmark of Proposition 5 in the following Proposition.

Proposition 6 *Under asymmetric information, when $p > \frac{1}{2}$ and $G^M > (2p-1)c$, the uninformed party L receives the decision right more frequently than under the constrained efficient symmetric information outcome. When these conditions do not hold, the allocation of the decision right coincides with the constrained efficient symmetric information outcome.*

Specific properties of the equilibrium depend on the behavior of the function G. G is weakly positive and equals 0 at b. Its maximum value, G^M, is at least as large as $G(0) = \mathrm{E}[x] - a$. G need not, however, be monotonically decreasing, which makes the local dependence of $\hat{x}$ on parameters such as p or c, for instance, ambiguous. The following Proposition, however, indicates that G is monotonic under the large class of increasing-hazard-rate distributions for x, a class that includes the (truncated) normal, (truncated) exponential, and uniform, among other distributions.[25]

Proposition 7 *If the hazard rate of the distribution of x is increasing then $G(t)$ is monotonically decreasing.*

[25] If F is the cdf of x, the hazard rate of x at t is defined as $\frac{\partial}{\partial t}[F(t)]/(1-F(t))$.

Under the assumption of G monotonically decreasing, we can derive a number of simple comparative-static results. One of the comparative statics we are interested in is how the contract varies with the amount of asymmetric information. To this end, we employ the following partial ordering for the dispersion of a distribution.[26]

Definition 1 A distribution F is said to be *more dispersed* than a distribution F' if $F^{-1}(p) - F^{-1}(q) \geq F'^{-1}(p) - F'^{-1}(q)$ whenever $0 < q \leq p < 1$.[27]

With this definition in hand, we now state several comparative-statics results.

Proposition 8 *Let $p > \frac{1}{2}$. If G is strictly decreasing over the range $[a, b]$, then:*

(i) $\hat{x}$ is unique.

(ii) $\hat{x}$ decreases with both p and $c = c_1 + c_r$.

(iii) If F' is less disperse than F (i.e., there is less asymmetric information), then $F'(\hat{x}') \leq F(\hat{x})$. That is, the allocation of the decision right is closer to the constrained efficient outcome with less dispersion.

This proposition indicates that the proportion of types who give away the control rights inefficiently due to asymmetric information is smaller when the renegotiation and information-acquisition cost c or the probability of the good state p is large, and when the distribution of x has less dispersion. As the costs of renegotiation and informational inefficiency $(2p - 1)c$ grow large enough, or as differences in types shrink sufficiently, the expected information-acquisition and renegotiation costs may eventually exceed the maximum adverse selection cost G^M and case (ii) of Proposition 1 obtains, where the allocation coincides with the constrained efficient allocation.

The intuition for why results in the Proposition follow when G is monotone is straightforward. For any equilibrium with some types E choosing to retain control and other types choosing to give up control, the marginal type must be indifferent. As noted above, $G(z)$ represents the adverse

[26] See Shaked and Shanthikumar (1994). Under this definition, a random variable X is less dispersed than Y if and only if Y has the same distribution as $X + \phi(X)$ for some increasing function ϕ. If the logarithm of the pdf of X is concave, which is the case for the uniform, (truncated) normal, (truncated) exponential, etc., distributions, and which also implies that the hazard rate of X is increasing, then, for any random variable Z independent of X, $X + Z$ is more dispersed than X. We also note that the convolution of two random variables with increasing hazard rates has an increasing hazard rate, thus ensuring that G remains monotonic.

[27] Our assumptions on the distribution of x imply that F is continuous and strictly increasing, which makes F^{-1} unambiguously defined on $[0, 1]$.

selection costs to z of choosing $r = E$ if he is pooled together with all higher types $[z, b]$ in choosing E. If G decreases monotonically, this implies that by expanding the interval of types $[z, b]$ choosing $r = E$ by lowering the cutoff z, the adverse selection cost to the lowest type z in this pool increases. This condition would not, for example, hold at z if there was a large mass of types around z: if this mass was large enough, lowering the cutoff by ϵ to include z would lower the mean of the pool by more than ϵ. However, provided that this condition is satisfied, it follows that if the cost of choosing $r = L$ increases by either increasing p (making renegotiation more likely) or increasing c (increasing the cost of renegotiation), then the adverse selection cost for the marginal type choosing $r = E$ must increase as well. This means that the equilibrium cut-off $\hat{x}$ falls, i.e., more types choose $r = E$. Similarly, less dispersion in asymmetric information implies that the adverse selection costs for the lowest type in any given pool decrease. Consequently, more types can join this pool (i.e., $\hat{x}$ falls), until the cost for the marginal type increases to equal cost of inefficient renegotiation. We illustrate Proposition 8 with the following example.

Example 2 *Consider again the uniform distribution of Example 1. As noted in this example, provided that* $\frac{1}{2} < p < \frac{1}{2} + \frac{b-a}{4c}$, *the marginal type choosing* $r = E$ *is given by* $\hat{x} = b - 2(2p-1)c$. *(If the condition on* p *is not met, all types choose the same contract.) Note that, as indicated in Proposition 8, this cutoff* $\hat{x}$ *is clearly decreasing in* p *and* c.

Now suppose that instead x *is distributed uniformly over* $[a - \epsilon, b + \epsilon]$, *with* $\epsilon > 0$. *Dispersion in* x *increases in* ϵ. *And provided that* p *is such that an interior solution obtains, it follows that* $\hat{x} = b + \epsilon - 2(2p-1)c$, *and therefore, that* $F(\hat{x}) = \frac{b-a+2\epsilon-2(2p-1)c}{b-a+2\epsilon}$, *which is increasing in* ϵ. *Thus, more asymmetric information increases the proportion of types who choose to relinquish the right to* L *even though this is inefficient.*

Proposition 6 indicated that under asymmetric information L obtains the decision right more often than under the constrained efficient outcome at time 0, which in turn implies that L renegotiates to sell the right more frequently than under the constrained efficient outcome at time 1. Empirically, however, one is likely to only observe absolute magnitudes of renegotiation in each direction. The following Proposition lists conditions under which the frequency of renegotiations where the uninformed party gives up rights exceeds that where the informed party gives up rights.

In particular, in our equilibrium, L renegotiates to sell the rights with probability $p^s = pF(\hat{x})$ if $p > \frac{1}{2}$ and $p^s = p$ if $p < \frac{1}{2}$, while L renegotiates to obtain further rights with probability $p^b = (1-p)(1-F(\hat{x}))$ if $p > \frac{1}{2}$ and $p^s = 0$ if $p < \frac{1}{2}$. Note that, when $p < \frac{1}{2}$, all renegotiation involves transferring rights from L to E. When instead $p > \frac{1}{2}$, the following obtains.

Proposition 9 *Let $p > \frac{1}{2}$.*

(i) A necessary and sufficient condition for $p^s > p^b$ is $p > 1 - F(\hat{x})$.

(ii) If G is monotonic, the fraction $\frac{p^s}{p^s+p^b}$ of all renegotiations that involve L giving up rights (as opposed to E giving up rights) decreases with $c = c_1 + c_r$, and decreases if F becomes less disperse.

(iii) The fraction $\frac{p^s}{p^s+p^b}$ of all renegotiations that involve L giving up rights is close to 1 if and only if either p is close to 1 and $F(\hat{x})$ bounded away from 0, or $F(\hat{x})$ is close to 1.

If $p \leq \frac{1}{2}$, all renegotiations involve L transferring rights for considerations to E. If instead $p > \frac{1}{2}$, Proposition 9 indicates that the fraction of renegations that involve L instead of E giving up rights for considerations increases in asymmetric information and decreases in the costs of renegotiation and information acquisition.

Finally, we return to the question of the timing of the information acquisition of L. Recall from Proposition 4 that information is acquired late if and only if

$$c_0 + \min(p, 1-p)c_r - (c_1 + c_r)\left[pF(\hat{x}) + (1-p)(1 - F(\hat{x})\right] \geq 0. \tag{7}$$

The left hand side of this inequality gives the difference between expected information acquisition and renegotiation costs when information is acquired at time 0 instead of at time 1; we will refer to this as the *relative benefit of late information acquisition.* Trivially, it follows that an increase in c_0 increases the relative benefit of late information acquisition. General statements about c_1 and c_r do not follow, however, as $\hat{x}$ is a function of c_1 and c_r. For example, if G is monotonically decreasing, Proposition 8 indicates that $F(\hat{x})$ decreases with c_1 and c_r, making the effect of a change in either of these costs on inequality (7) ambiguous. Under the further assumption that early and late information acquisition costs are the same, however, we obtain the following results.

Proposition 10 *Assume that G is strictly decreasing over $[a, b]$. Then, (i) if $c_0 = c_1 \equiv c_I$, then an increase in c_I increases the relative benefit of late information acquisition; and (ii) an increase in the dispersion of the distribution F decreases the relative benefit of late information acquisition.*

Intuitively, for Part (i), if both c_0 and c_1 increase by the same amount, this yields a direct increase in the relative benefit of late information acquisition (since in some states of the world late information need not be acquired), and also yields an indirect increase through the decrease in $F(\hat{x})$ (less inefficient late information acquisition). Part (ii) follows directly from the increase in $F(\hat{x})$.

5 Conclusion

We analyze the design and renegotiation of covenants in debt contracts under asymmetric information. Specifically, we consider a setting where future firm investments are efficient in some states, but also involve a transfer from the lender(s) to shareholders. While there is symmetric information regarding investment efficiency, entrepreneur/owners are better informed about any potential transfer than the lender. The lender can learn this information, but at a cost. In this setting, we show that the simple adverse selection problem leads to the allocation of greater ex-ante decision rights to the uninformed party than would follow under symmetric (or full) information. This in turn implies that subsequent ex-post renegotiation is in turn biased towards the uninformed party giving up these excessive rights.

While our results stand in contrast to standard incomplete contracting results that indicate that rights should be allocated to parties with important ex-ante investments (which frequently would be the informed party), it should be emphasized that our approach is consistent with a general Property Rights approach to ownership and control rights. In particular, following the Property Rights notion, we derive implications for the ex-ante allocation of control rights based on contractual incompleteness. We part from Grossman and Hart (1986) and much of the following literature only in the specific contractual incompleteness on which we focus. Specifically, rather than considering ex-ante investments in light of incomplete contracts, we instead analyze asymmetric information and renegotiation costs when contracts are incomplete.

We have attempted to focus on one particular class of contracts for concreteness (i.e., debt contracts) while at the same time presenting the model with sufficient generality to suggest other applications. Taking this middle line runs the risk of falling short in both regards: the model may not seem tailored sufficiently to specific important features of debt contracts while at the same time its general applicability to other contracting settings may not be clear. Given this, a few words may be in order both on debt contracts and more general applications.

In the Appendix, we indicate how one can extend the model to capture several features of debt contracts not present in the text. In particular, Appendix A considers payoffs to the contracting parties that more directly follow from a standard asset substitution problem, and indicates how such a setting yields results qualitatively similar to our analysis. Also, in Footnote 13 above we additionally outlined how one could simply alter the model to account for contractible contingencies typically found in debt covenants.

More generally, our model and analysis seem appropriate for a wide range of contractual settings. What is crucial to apply our model is: a) the presence of ex-ante non-contractible future actions for which decision rights can be assigned; and b) asymmetric information between the contracting parties regarding the relative consequences of such future actions on the two parties. While clearly not central to all contracts, these two features seem to be prominent in many cases. Our analysis suggests that in such settings, there will be a bias to assigning decision rights to the uninformed party, and a corresponding bias in ex-post renegotiations where these strong rights given to the uninformed party are subsequently exchanged for other considerations.

A An Explicit Model of Debt and Asset Substitution

In this Appendix we alter several features of the model to tailor it more explicitly to a number of institutional features of debt contracts. First, we allow the transfer from D to E that is associated with action A to depend in a general manner on the promised debt payment and the mean and variance of returns. We additionally allow the inefficiencies under A in state B and under NA in state G to be asymmetric, and we allow for noisy outcomes to projects. We show that these features all can be simply incorporated into the model without qualitatively altering results. (Recall that in Footnote 13 we additionally outlined how one could simply alter the model to account for contractible contingencies typically found in debt covenants.)

Consider the following changes to the model. Let the total payoff from the project be z if NA is taken, $x = z + y_1 + \epsilon$ if A is taken and the state is G, and $x = z - y_2 + \epsilon$ if A is taken and the state is B, with $y_1, y_2 > 0$. The noise term ϵ is a zero-mean risky payoff, parameterized by its standard deviation, σ, with support $[a, b]$, and is the private information known to E. A debt contract of face value D, gives E an expected payment $h(D, u, \sigma) \equiv \mathrm{E}[(X - D)^+ \,|\, u, \sigma]$, where X is the project payoff, of mean u and standard deviation σ. It follows that h decreases in D and increases in u, and we assume that h also increases in σ, at least over the range of D that will be relevant to the equilibrium analysis.[28] Furthermore we assume that, given the equilibrium debt level, the project means y_1 and y_2, and the smallest possible value of the standard deviation, a, E always prefers taking the project, while L always prefers not taking it, absent renegotiation. (That is, as before, we assume there exists an agency conflict of asset substitution with this project. These conditions are automatically satisfied for y_1 and y_2 close to zero and a large enough.) Finally, as in the text, we assume that p gives the probability that the state is G, and $c \equiv c_1 + c_r$ is the combined cost to time 1 information acquisition and renegotiation.

We now proceed to show that in this setting, we obtain a result analogous to Proposition 1. In particular, we show that as in Proposition 1, in equilibrium there is a cutoff level such that all types E above this cutoff level choose $r = E$, whereas all types below this cutoff level choose $r = L$. Furthermore, the characterization of this cutoff takes a similar form to that in equation (5) in the text. As before, when $p \leq \frac{1}{2}$ (and therefore $r = L$ is constrained efficient), the cutoff level will be above the highest type; that is, all types will choose $r = L$. If instead $p > \frac{1}{2}$, and therefore $r = E$

[28]This assumption follows, for instance, if we choose the family ϵ so that, when $\sigma > \sigma'$, $\epsilon(\sigma) = \epsilon(\sigma') + \eta$, where η is a zero-mean independent noise (this is the case for the normal distribution). Similarly, this assumption is also satisfies if $\epsilon(\sigma)$ is a two-point distribution — if D is not too high, then h is strictly increasing in σ.

is constrained efficient, there will be an interior cutoff level, unless p is too close to 1 and c is high. Hence, once again, L will be given strong decision rights. Additionally, the comparisons to the constrained efficient outcome and comparative statics results of Propositions 6 - 10 will all hold as before, with a suitable modification to the function $G(x)$ which measures adverse selection.

Our derivation follows along similar lines with the text. First we apply the zero-profit condition for L to find the equilibrium debt level for a contract with $r = E$ and for $r = L$, analogous to equations (1) and (2) in the text. This yields for $r = E$ and $r = L$ respectively,

$$I = \mathrm{E}\left[p\left(z + y_1 - h(D^E, z + y_1, \sigma)\right) + (1-p)\left(z - h(D^E, z - y_2, \sigma) - c\right) \,\middle|\, \sigma \in S_E\right] \tag{A.1}$$

and

$$I = D^L - pc + py_1. \tag{A.2}$$

Consequently, the expected payoff to E when $r = E$ and $r = L$ respectively, is given by,

$$U^E_{r=E} = ph(D^E, z + y_1, \sigma) + (1-p)h(D^E, z - y_2, \sigma), \tag{A.3}$$

and

$$U^E_{r=L} = z + py_1 - pc - I, \tag{A.4}$$

where D^E satisfies (A.1).

Now define $g(D, u_1, u_2, \sigma) = ph(D, u_1, \sigma) + (1-p)h(D, u_2, \sigma)$; $g(D, u_1, u_2, \sigma)$ gives the expected payoff to E taking renegotiation into account when $r = E$, debt is given by D, and when action A leads to a mean and standard deviation of X of (u_1, σ) in state G and (u_2, σ) in state B. Also note, as in the text, that the set S_E will take the form $[\hat{\sigma}, b]$, since the payoff to E when $r = E$ in (A.3) is increasing in σ, and the payoff to E when $r = L$ in (A.4) does not depend on σ. As before, we find the cutoff type $\hat{\sigma}$, by equating the expected returns to the lowest type E that chooses $r = E$ with the highest type E that chooses $r = L$.

Using the definition of g, equation (A.1) implies that,

$$\mathrm{E}\left[g(D^E, z + y_1, z - y_2, \sigma) \,\middle|\, \sigma \geq \hat{\sigma}\right] = -I + z + py_1 - (1-p)c, \tag{A.5}$$

and consequently, from equation (A.4) it follows that,

$$U^E_{r=L} = \mathrm{E}\left[g(D^E, z + y_1, z - y_2, \sigma) \,\middle|\, \sigma \geq \hat{\sigma}\right] - (2p-1)c. \tag{A.6}$$

The equilibrium cutoff value of $\hat{\sigma}$ then follows from equating this expression with $U_{r=E}^{E} = g(D^E, z + y_1, z - y_2, \sigma)$, the payoff to E when $r = E$, and solving for $\hat{\sigma}$. Similar to the definition of G in the text, define $\mathcal{G}(D, u_1, u_2, \sigma)$ by,

$$\mathcal{G}(D, u_1, u_2, \sigma) \equiv \mathrm{E}\left[g(D, u_1, u_2, \sigma) \;\middle|\; \sigma \geq \hat{\sigma}\right] - g(D, u_1, u_2, \hat{\sigma}).$$

It then follows that in parallel with equation (5) in the text, $\hat{\sigma}$ is determined by the solution to

$$\mathcal{G}(D^E, z + y_1, z - y_2, \sigma) = (2p - 1)c. \tag{A.7}$$

And now, just as with equation (5), it follows from equation (A.7) that when $p \leq \frac{1}{2}$, L once again gets control all the time, while when $p > \frac{1}{2}$, there may be an interior solution $\hat{\sigma}$ (since h increases in σ in the needed range, and therefore so does g). Indeed, all results from Proposition 1 for $\hat{x}$ now follow immediately for $\hat{\sigma}$ in this modified model, with $G(x)$ replaced by $\mathcal{G}(D, z + y_1, z - y_2, \sigma)$. And from this result, comparative statics for $\hat{\sigma}$ and comparisons with the constrained efficient outcome are analogous to those in Propositions 6-10 in the text.

To explicitly determine the cutoff type $\hat{\sigma}$, one would have to first solve for D^E and then solve equation (A.7) above. As one particularly simple example, let ϵ takes values in $\{-\sigma, \sigma\}$. If the parameters are in the right ranges, then $h(D, u, \sigma) = (u + \sigma - D)/2$. It then follows that equation (A.7) becomes

$$\mathrm{E}\left[\sigma \;\middle|\; \sigma \geq \hat{\sigma}\right] - \hat{\sigma} - 2(2p - 1)c = 0,$$

which notably, is identical to equation (5) in the text, save for σ replacing x and the constant $2c$ replacing c.

B Proofs

Proof of Proposition 1:

From the comparison of (3) and (4) it follows that, if L expects types $x \geq \hat{x}$ to choose $r = E$, then an agent of type x chooses $r = E$ if and only if

$$x - \hat{x} - G(\hat{x}) \geq -(2p - 1)c. \tag{B.1}$$

Suppose now that $G(\hat{x}) > (2p-1)c$ for all $\hat{x} \in [a,b]$. Then $\hat{x} = b$ is an equilibrium, since inequality (B.1) is never satisfied. If, on the other hand, $G(\hat{x}) < (2p-1)c$ for all $\hat{x} \in [a,b]$, then inequality (B.1) is satisfied for all x when $\hat{x}$ is set to be $\hat{x} = a$. Thus $r = E$ for all types is an equilibrium.

Finally, for any $\hat{x}$ for which $G(\hat{x}) = (2p-1)c$, inequality (B.1) reduces to $x \geq \hat{x}$, which means that types $x \geq \hat{x}$ choose $r = E$ and types $x < \hat{x}$ choose $r = L$, which is consistent with the expectations of L.

The calculation of the contractual transfer follows immediately from equations (1) and (2).

□

Proof of Proposition 2:

Let $H_L(\hat{x})$ be the gain to L from renegotiating the contract $r = L$ in state G with asymmetric information, given that the equilibrium cut-off point is $\hat{x}$. Analogously, define $H_E(\hat{x})$ to be the gain to L from renegotiating when $r = E$. We will show below that $H_L(\hat{x})$ and $H_E(\hat{x})$ are continuous functions, that $H_L(\hat{x}) < y$ for all $\hat{x} \in (a,b]$, and that $H_E(\hat{x}) < y$ for all $\hat{x} \in [a,b)$. It follows then that both H_L and H_E are bounded uniformly away from y when $\hat{x}$ is confined to an interval $[\hat{x}_1, \hat{x}_2] \subset (a,b)$.

With full information, L extracts all the surplus from renegotiation, y, whence, for any $\hat{x} \in [\hat{x}_1, \hat{x}_2]$, L makes a net gain from acquiring information that is bounded below away from zero. That is, letting

$$g(\hat{x}_1, \hat{x}_2) = y - \max\left(\sup_{\hat{x} \in [\hat{x}_1, \hat{x}_2]} H_L(\hat{x}), \sup_{\hat{x} \in [\hat{x}_1, \hat{x}_2]} H_E(\hat{x}) \right),$$

it holds that $g(\hat{x}_1, \hat{x}_2) > 0$.

Consider now the equilibrium, given by $\hat{x}$, that obtains with $c_1 = 0$. If $\hat{x} \in (a,b)$ then, by continuity, there exist $\hat{x}_1$ and $\hat{x}_2$ with $a < \hat{x}_1 < \hat{x}_2 < b$ such that an equilibrium for c_1 small enough is given by $\hat{x} \in [\hat{x}_1, \hat{x}_2]$. Consequently, if c_1 is small enough — in particular, $c_1 \leq g(\hat{x}_1, \hat{x}_2)$ — then c_1 is worth paying for information. If $\hat{x} = a$ when $c_1 = 0$, then $\hat{x} = a$ for positive c_1, too, and the only condition required is that $c_1 < y - H_E(a)$. Analogously, if $0 = G(b) \geq (2p-1)c_r$, then the only condition required is that $c_1 < y - H_L(b)$.

Finally, let us show that, when bargaining takes the form of a TIOLI offer made by L, H_L and H_E are strictly smaller than y on $(a,b]$, respectively on $[a,b)$, and continuous. For that, we have to identify first the conditions defining L's offer.

Consider first the case ($r = L, G$). L asks a further payment u from E in exchange for the right to take the action. Since E accepts the offer if and only if $u \leq x + y$, the expected gain to L is $H_L(\hat{x}) = \max_u h_L(u, \hat{x})$, with

$$h_L(u, \hat{x}) = \mathrm{E}\left[(u - x)1_{(u \leq x+y)} \;\middle|\; x < \hat{x}\right]. \tag{B.2}$$

It is easy to see that $h_L(u, \hat{x})$ is strictly less than y for every $u \in [a, y + \hat{x}]$, and is weakly negative for u outside $[a, y + \hat{x}]$. It is also easy to see that $h_L(u, \hat{x})$ is continuous in u and $\hat{x}$, whence its maximal value, $H_L(\hat{x})$, is strictly smaller than y and continuous in $\hat{x}$.

In the other case, ($r = E, B$), L offers a payment v to E in exchange for the control right, and E accepts if and only if $v \geq x$. Then, $H_E(\hat{x}) = \max_v h_E(v, \hat{x})$, with

$$h_E(v, \hat{x}) = \mathrm{E}\left[-(x + y)1_{(v<x)} - v1_{(v \geq x)} \;\middle|\; x > \hat{x}\right] + \mathrm{E}\left[x + y \;\middle|\; x > \hat{x}\right]. \tag{B.3}$$

The rest of argument is analogous to that for H_L.

□

Proof of Proposition 3

With perfect information on both sides, the contract will be chosen so as to minimize the renegotiation costs. These equal pc_r if $r = L$ and $(1 - p)c_r$ if $r = E$, whence the result.

□

Proof of Proposition 4

The left-hand side of inequality (6) equals the total cost associated with early information acquisition and efficient renegotiation, while the right-hand side the total cost associated with late information acquisition, since the probability of renegotiating is $pF(\hat{x}) + (1 - p)(1 - F(\hat{x}))$.

□

Proof of Proposition 7

The result follows directly from Theorem 1.A.13. in Shaked and Shanthikumar (1994). For completeness, we provide a proof here.

Let $t < t'$ and $F_u(w) = \frac{F(w+u)-F(u)}{1-F(u)}$ for $u \in \{t, t'\}$; F_u is the cumulative distribution function of $x - u$ conditionally on $x \geq u$, and $G(u)$ is the mean of F_u.

The fact that x has an increasing hazard rate is equivalent to $\log(1 - F(w))$ being concave, where F is its cdf. Therefore,

$$\frac{1-F(w+u)}{1-F(u)} = 1 - F_u(w)$$

decreases in u. In other words, the distribution F_t dominates the distribution $F_{t'}$ in the sense of first-order stochastic dominance (FOSD), whence it has a higher mean. Therefore, $G(t') < G(t)$, proving the result.

□

Proof of Proposition 8

Parts *(i)* and *(ii)* follow immediately from the monotonicity of G. Let us prove part *(iii)*. We'll show that, for any t, t', $F(t) = F'(t')$ implies $G(t) \geq G'(t')$, whence, by monotonicity of G and G', $G(\hat{x}) = G'(\hat{x}')$ implies that $F(\hat{x}) \geq F'(\hat{x}')$. To that end, we note that

$$G(t) \quad = \quad \frac{\int_t^\infty (1-F(x))dx}{1-F(t)}.$$

If $F(t) = F'(t') = r$, then

$$\begin{aligned}\int_t^\infty (1-F(x))\ dx - \int_{t'}^\infty \left(1-F'(x)\right)\ dx &= \int_r^1 (1-p)\ d\left(F^{-1}(p) - F'^{-1}(p)\right) \\ &\geq 0,\end{aligned}$$

where the last inequality is due to the fact that F being more dispersed than F' is equivalent with $F^{-1}(p) - F'^{-1}(p)$ being increasing in p (see Shaked and Shanthikumar (1994)). Thus $G(t) \geq G'(t')$, whence $F(\hat{x}) \geq F'(\hat{x}')$.

□

Proof of Proposition 10

For part *(i)*, note that the derivative of (7) with respect to c_I gives

$$p - (2p-1)F(\hat{x}) - (c_I + c_r)(2p-1)\frac{\partial F(\hat{x})}{\partial c_I} \quad \geq 0,$$

since $p \geq (2p-1)F(\hat{x})$ and $\frac{\partial F(\hat{x})}{\partial c_I} \leq 0$. Part *(ii)* follows from the fact that a more dispersed F translates into a higher $F(\hat{x})$.

□

C Zero Renegotiation Costs

We indicate here how the analysis is modified if the renegotiation costs are zero. Zero renegotiation costs introduces two complications into our analysis, both of which may allow for additional equilibria. In this Section, we characterize all equilibria that may follow when there are no renegotiation costs, indicate when they may exist, and demonstrate that in all cases, our primary qualitative result continues to hold: L receives control rights more often than in the constrained efficient outcome.

The first complication is that when $p > \frac{1}{2}$, for any value of c_1, the information acquisition cost in the case $(r = E, B)$ may be higher than the one imposed on L by bargaining with asymmetric information, and consequently L may choose not to acquire information. That is, Proposition 2 does not necessarily hold any more. Making the cost of becoming informed in Period 1, c_1, arbitrarily small no longer necessarily induces L to acquire information in Period 1, since $\hat{x}$ approaches b as c_1 goes to 0, and consequently, the benefit to L of acquiring information instead of bargaining while uninformed goes to 0 as well. (This follows since $r = E$ indicates that $E \in [\hat{x}, b]$, and this set converges to a point as c_1 goes to 0.) Intuitively, as c_1 goes to 0, few types choose E, and consequently, the ones that do give a good indication of their type simply through this choice, making information acquisition for L unnecessary. In contrast, if renegotiation costs are strictly positive (as in the text), as c_1 goes to 0 L will choose to acquire information in Period 1, as while the adverse selection costs will still go to 0 (since $\hat{x}$ will approach b), the costs of choosing an inefficient ex-ante allocation (given by $(2p - 1)(c_1 + c_r)$) will remain bounded away from 0.

In light of this, when $c_r = 0$, three possible equilibria may ensue: one where L acquires information (just as when $c_r > 0$); one where L does not acquire any information; and one where L randomizes between acquiring and not acquiring information. We will analyze all three cases below, and show that in all cases L will receive the control right more often than prescribed by the constrained efficient outcome.

A second complication when there are no renegotiation costs is that when $p < \frac{1}{2}$, it no longer necessarily follows that if c_1 is small enough then in equilibrium all types E offer a contract of $r = L$. Recall that this result followed under a positive renegotiation cost because with $p < \frac{1}{2}$, renegotiation is less frequent with $r = L$, and adverse selection costs are always positive for the lowest type choosing $r = E$. Consequently, this lowest type choosing $r = E$ would always strictly prefer $r = L$ when $p < \frac{1}{2}$. Now, however, when $c_r = 0$, even though renegotiation is less frequent with $r = L$, it can be less costly for some types with $r = E$. In particular, if few enough types close to b offer $r = E$, these types can signal their types almost perfectly and thus avoid the payment of the information cost c_1, instead incurring a lower adverse-selection cost. Provided that $c_r > 0$, when c_1 is small enough, the benefit from avoiding the cost c_1 is dominated by the greater frequency that c_r must be paid under $r = E$. But when $c_r = 0$, this no longer is true, and there may be an equilibrium where some types choose $r = E$. However, we will show that in this case, as c_1 goes to 0, the measure of types that can choose $r = E$ in such an equilibrium must go to 0 as well. Consequently, we will still get a result analogous to Proposition 6 in the limit. That is, as c_1 grows small, *almost all* distortions in the assignment of the decision right from the constrained efficient outcome will be L being granted the right when the constrained efficient outcome prescribes that E should receive it, and not vice versa.[29]

Note that unlike the case of $(r = E, B)$, when $(r = L, G)$ and c_1 is small enough, information is still always acquired. This is because decreasing c_1 increases $\hat{x}$, and therefore increases the information asymmetry conditional on $x < \hat{x}$; thus the cost c_1 is reduced at the same time as the benefit from acquiring information is increased. We will assume throughout our analysis here that c_1 is small enough such that L becomes informed prior to renegotiation in state $(r = L, G)$.

We begin with the first complication, which occurs when $p > \frac{1}{2}$. As in the proof of Proposition 2, if L does not acquire information in the case $(r = E, B)$, L would instead offer v for the control rights to E, where v maximizes the right-hand side of (B.3).[30] Let $\hat{x}^i$ be the cut-off value $\hat{x}$ that would follow in equilibrium if L always acquires information at time 1 as in the text; that is, $\hat{x}$ is given by $G(\hat{x}) = (2p - 1)c_1$. And recall from the Proof to Proposition 2 that $H_E(\hat{x})$ is defined as the gain to L from renegotiating when $r = E$ and the equilibrium cutoff if information is acquired by L is $\hat{x}$. It then follows that if $c_1 \leq y - H_E(\hat{x}^i)$ (that is, the cost of acquiring information is less than the loss to L from bargaining without information), then the same equilibrium obtains as with $c_r > 0$. L chooses to become informed in state $(r = E, B)$, and consequently, the equilibrium is no different from in the text.

If instead $c_1 > y - H_E(\hat{x}^i)$, however, then L is better off not always acquiring information in the case of $(r = E, B)$. One possibility is that L never acquires information in the case $(r = E, B)$. In such a situation, the cut-off value, denoted by $\hat{x}^u$ would be determined as usual by equating the expected gain of the lowest type E that chooses $r = E$ (taking into account that L will be

[29]Intuitively, this complication arises when $c_r = 0$, because if there is an inefficient action that would otherwise be untaken in equilibrium, the types for whom this action is least inefficient may benefit by choosing this action, thereby "freely signaling" their type, and precluding costly information acquisition. While this effect is of some interest, it is rather removed from the focus of our paper.

[30]The offer v is a function of the cut-off $\hat{x}$, but we omit the dependence notationally for simplicity.

uninformed under renegotiation) with the expected gain to E of choosing $R = L$ instead.

To find this, we impose the 0-profit condition for L under both $r = E$ and $r = L$ to determine D^L and D^E. This yields,

$$\begin{aligned} I - D^L &= p\left(-c_1 + y\right) \\ I - D^E &= -p\mathrm{E}\left[x \mid x > \hat{x}^u\right] - (1-p)\left(\mathrm{E}\left[y 1_{(v<x)} + \max(v,x) \mid x > \hat{x}^u\right]\right). \end{aligned}$$

The payoffs to E conditional on x in the two cases are

$$\begin{aligned} V_E^L &= -D^L \\ V_E^E &= -D^E + p(x+y) + (1-p)\max(v,x). \end{aligned}$$

Equating the two payoffs at $\hat{x}^u$ gives

$$pG(\hat{x}^u) + (1-p)\left(\mathrm{E}\left[\max(v,x) \mid x > \hat{x}^u\right] - \max(v,\hat{x}^u)\right) + (1-p)y\Pr\left(x > v \mid x > \hat{x}^u\right) = pc_1. \quad \text{(C.1)}$$

Intuitively, this condition states that the cutoff type $\hat{x}^u$ should be indifferent between incurring the adverse-selection (in connection with both the good and the bad states) and inefficient-action costs, on one hand, and paying the information cost c_1, with probability p, on the other.

The lender acquires information if and only if the condition $c_1 \leq y - H_E(\hat{x}^u)$ holds. Thus, if $c_1 \geq y - H_E(\hat{x})^u$, then there exists an equilibrium in which L does not acquire information.

Finally, in the complementary case, where $y - H_E(\hat{x}^i) < c_1 < y - H_E(\hat{x}^u)$, there exists $\hat{x}^m \in (\hat{x}^u, \hat{x}^i)$ such that $c_1 = y - H_E(\hat{x}^m)$, and L acquires information with probability q, where q is set so as to make $\hat{x}^m$ an equilibrium cutoff:

$$\begin{aligned} 0 &= q\Big(G(\hat{x}^m) - (2p-1)c_1\Big) + (1-q)\Big(pG(\hat{x}^m) - pc_1 + \\ &\quad (1-p)\big(\mathrm{E}\left[y 1_{(v<x)} + \max(v,x) \mid x > \hat{x}^m\right] - \max(v,\hat{x}^m)\big)\Big) \\ &= G(\hat{x}^m) - (2p-1)c_1 - (1-q)pc_1 + \\ &\quad (1-q)(1-p)\big(\mathrm{E}\left[y 1_{(v<x)} + (v-x)^+ \mid x > \hat{x}^m\right] - (v - \hat{x}^m)\big). \end{aligned}$$

Note that, even if the lender no longer acquires information with probability 1, our qualitative results are not changed. More precisely, if $p > \frac{1}{2}$, $\hat{x} > a$ still implies that L gets the rights too often — the measure of how much too often, $F(\hat{x})$, can be, in fact, arbitrarily close to 1, when c is small.

Consider now the case $p < \frac{1}{2}$. We know that there is no equilibrium in which some types offer $r = E$ and L acquires information in the case $(r = E, B)$, as well. One possibility is that types $[\hat{x}^u, b]$ offer $r = E$ and L does not acquire information in the case $(r = E, B)$. The equilibrium condition for type $\hat{x}^u$ is the same as above, namely

$$pG(\hat{x}^u) + (1-p)\left(\mathrm{E}\left[\max(v,x) \mid x > \hat{x}^u\right] - \max(v,\hat{x}^u\right) + (1-p)y \Pr\left(x > v \mid x > \hat{x}^u\right) = pc_1.$$

These strategies constitute an equilibrium if and only if $c_1 \geq y - H_E(\hat{x}^u)$. In the complementary case, $\hat{x}^m > \hat{x}^u$ is defined by $c_1 = y - H_E(\hat{x}^m)$ and type $\hat{x}^m$ is made indifferent between $r = E$ and $r = L$ by choosing q to solve:

$$\begin{aligned} 0 \;=\;& G(\hat{x}^m) - (2p-1)c_1 - (1-q)pc_1 + \\ & (1-q)(1-p)\big(\mathrm{E}\left[y1_{(v<x)} + (v-x)^+ \mid x > \hat{x}^m\right] - (v - \hat{x}^m)\big). \end{aligned}$$

In such an equilibrium L no longer gets the control all the time, meaning that L does not receive the control rights often enough in this case. The measure of the deviation from the optimal frequency, $1 - F(\hat{x})$, though, is made arbitrarily small by reducing c_1, which means that our qualitative result on L getting the rights too often holds approximately even for $p < \frac{1}{2}$.

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