

Under Revision

Price Formation In Continuous  
Double Auctions; With Implications  
For Finance

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1 PRICE FORMATION IN CONTINUOUS DOUBLE AUCTIONS; WITH 1  
2 IMPLICATIONS FOR FINANCE<sup>1</sup> 2

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5 We argue that an empirically relevant theory of price and allocation adjust- 5  
6 ments in multiple parallel Continuous Double Auctions (CDA) should take into 6  
7 account that CDAs are competitive only for small quantities. Hence, any ad- 7  
8 justment theory should be based on Local Equilibrium concepts. Here, we build 8  
9 a Local Marshallian Equilibrium theory and compare it to its Walrasian coun-

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terpart. We provide experimental support for one version of Local Marshallian Equilibrium theory, where bid revision occurs at a slower speed than price adjustment. In our experiments, we induced quasi-linear, quadratic preferences. As such, they are isomorphic to finance experiments on the Capital Asset Pricing Model (CAPM). We find that one specific portfolio is mean-variance optimal in the Local Marshallian Equilibrium, thus showing that it is possible to derive a portfolio-based asset pricing theory even for markets that are off (global) equilibrium.

KEYWORDS: Continuous Double Auction, Walrasian Equilibrium, Local Marshallian Equilibrium, Price Discovery, CAPM, Mean-variance Optimality.

## 1. INTRODUCTION

General Equilibrium Theory (see, e.g., [Aliprantis, Brown and Burkinshaw \(1989\)](#)) is a widely accepted model of behavior in competitive markets. But it remains an equilibrium theory. How the equilibrium prices are discovered, and how, if at all, trading occurs out of equilibrium, remains to be explained. Until recently, attempts to settle this question have been theoretical in nature with no real evidence available to help sort the sensible from the inane. Traditional empirical analyses of markets can shed little light on the processes because they do not have access to the fundamentals. But, with the advent and development of experimental economics, it is now possible not only to know the fundamentals, but also to control them, and so, to observe the process of price discovery and equilibration in a replicable and controlled manner. These observations, in turn, can provide guidance for the development of new and more appropriate theory.

It is time to re-examine the nature of price discovery in markets. Here, we look at trading and price dynamics in the context of simultaneous multiple markets. The market organization we focus on is the continuous double auction (CDA), partly because there are a large number of experimental studies utilizing this organization and partly because the CDA is similar to many organized markets encountered in the field; e.g., purely electronic stock markets such as Euronext.

Starting with [Smith \(1962\)](#), markets experiments have shown that the CDA *can* generate competitive equilibrium (though not always). That there exists an institution that delivers is comforting. Indeed, it contrasts with the state of theoretical analysis about price discovery, where one has yet to agree on the right paradigm. Ever since the examples in [Scarf \(1960\)](#) and [Gale \(1963\)](#), it is well known that the Walrasian price adjustment (prices adjust in the direction of excess demand) would fail in more than just a few interesting cases. As such, economists have no reason to believe that equilibrium is a

state towards which economies naturally move to. Yet economists, especially in macro-economics and finance, continue to insist on equilibrium analysis. More strikingly, they insist on interpreting historical data from the field as if they reflect markets that are always in equilibrium.

We are interested in determining how prices and allocations move off-equilibrium. Our analysis will boil down to a dialogue between theory and experiments. We will focus on exchange economies.

## 2. STANDARD GENERAL EQUILIBRIUM THEORY

In this section we very briefly review the standard general equilibrium theory for exchange environments. We do this primarily to have, in one place, notation and concepts we use throughout the rest of the paper.

### 2.1. *Exchange environments*

There are  $N$  consumers, indexed by  $i = 1, \dots, N$ . There are  $K$  commodities, indexed by  $k = 1, \dots, K$ . Let  $x^i = (x_1^i, \dots, x_K^i)$  be the consumption of  $i$  and let  $X^i = \{x^i \in \mathbb{R}^K \mid x^i \geq 0\}$  be the admissible consumption set for  $i$ . Each  $i$  owns initial endowments  $\omega^i = (\omega_1^i, \dots, \omega_K^i)$  such that  $\omega_k^i > 0$  for all  $i$  and  $k$ . Consumption will occur by combining initial endowments with net trades. Let  $d^i \in \mathbb{R}^K$  be a vector of net trades. Then  $x^i = \omega^i + d^i$ . Finally, each  $i$  has a quasi-concave utility function,  $u^i(x^i)$ . We will assume that  $u^i \in C^2$  (that is, it has continuous second derivatives) although many of our results would hold under weaker conditions. We also assume that  $\{x^i \mid u^i(x^i) \geq u^i(\omega^i)\} \subset \text{Interior}(X^i)$ .

### 2.2. *Equilibrium*

Competitive market equilibrium in an exchange economy is straightforward to describe. Let  $p_k$  be the price of commodity  $k$ . Given a vector

of prices,  $p$ , the excess demand of  $i$  is  $e^i(p, \omega^i) = \arg \max_{d^i} u^i(\omega^i + d^i)$  subject to  $pd^i = 0$  and  $w^i + d^i \in X^i$ . The aggregate excess demand, of the economy, is  $e(p, \omega) = \sum e^i(p, \omega^i)$ .

A price,  $p^*$ , and a vector of trades,  $d^* = (d^{*1}, \dots, d^{*N})$  is a market equilibrium if and only if

$$e(p^*, \omega) = 0$$

and

$$d^{*i} = e^i(p^*, \omega^i), \forall i = 1, \dots, N.$$

### 2.3. Dynamics

One needs a compelling reason to be interested in equilibrium. One is the “argument, familiar from Marshall, ... that there are forces at work in any actual economy that tend to drive an economy toward an equilibrium if it is not in equilibrium already.”<sup>1</sup> While the argument is part of conventional wisdom, we unfortunately do not understand these forces. We actually know little about the true nature of price discovery, i.e., the dynamics that lead to equilibrium.

There are two basic models that are at the foundation of most early analyses of market dynamics. One, traceable to Walras, is the tatonnement dynamics. Formally it is:<sup>2</sup>

$$\begin{aligned} \frac{dp}{dt} &= e(p, \omega) \\ d^i(t) &= \begin{cases} 0 & \text{if } p \neq p^* \\ e^i(p) & \text{if } p = p^* \end{cases} \end{aligned}$$

<sup>1</sup>Arrow and Hahn (1971), p. 263.

<sup>2</sup>There are a variety of generalizations of this structure that allow for variations in the speed of adjustment such as  $dp_k/dt = \lambda_k e_k(p, \omega)$  with  $\lambda_k > 0$ . We will not need to refer to these in this paper.

A lot is known about this dynamical system. For example, if the excess demand functions satisfy a “gross substitutes condition,” then  $p(t) \rightarrow p^*$  as  $t \rightarrow \infty$ . But there are very simple exchange environments, examples from [Gale \(1963\)](#) and [Scarf \(1960\)](#), in which such convergence does not occur.

More importantly, for what follows, the tatonnement is only a theory about prices. No adjustment from the initial endowments takes place until after the prices reach equilibrium.<sup>3</sup> Trading,  $d$ , follows prices,  $p$ .

The other basic model, non-tatonnement dynamics, is described by

$$\begin{aligned}\frac{dp}{dt} &= e(p, \omega + d(t)) \\ \frac{dd^i}{dt} &= g^i(p, \omega^i + d^i(t))\end{aligned}$$

For now, the equations,  $g^i(\cdot)$  remain unspecified<sup>4</sup> except for an important feasibility constraint on this system:

$$\sum_i \frac{dd^i}{dt} = \sum_i g^i(p, \omega^i + d^i(t)) = 0.$$

The main thing we know about the non-tatonnement dynamical system is that if the  $g^i$  are continuous and if there is voluntary exchange and no speculation then it converges to an interesting rest point. Voluntary exchange and no speculation imply that  $(\nabla u \cdot \dot{d}(t) > 0)$ . Thus, as  $t \rightarrow \infty$ ,  $d(t) \rightarrow d^*$  where  $w + d^* \in \{\text{Pareto-optimal allocations}\}$  and  $p(t) \rightarrow p^*$  where  $(p^*, 0)$  is a market equilibrium for the exchange economy with the endowment  $w^i + d^{*i}$  for each  $i$ . It need not be true that  $(p^*, d^*)$  is an equilibrium of the exchange economy with the endowment  $w$ . As opposed to the Walrasian tatonnement, here prices,  $p$ , follow trades,  $d$ .

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<sup>3</sup>This might describe, for example, the “book building” process in a call market if orders can be withdrawn. It should not be expected to describe the price formation process in a continuous trading market in which transactions occur as prices are changing.

<sup>4</sup>For specific examples of this type of dynamic, see [Arrow and Hahn \(1971\)](#), [Hahn and Negishi \(1962\)](#), [Uzawa \(1962\)](#), [Friedman \(1979\)](#), and [Friedman \(1986\)](#).

### 3. SOME SYLIZED FACTS FROM EXPERIMENTS

Before proceeding with more theory, let us look at some of the experimental evidence to see whether either of the two models in Section 2 is on the right track in explaining price discovery in markets.

#### 3.1. *The Structure of Market Experiments*

For those unfamiliar with markets experiments, we briefly describe how one proceeds.<sup>5</sup> Subjects come to the lab or access the experiment online. They are told they will participate in a market experiment. An experiment proceeds in periods. At the beginning of each period each subject is given a basket of assets,  $w^i$ . Trading then occurs, via whatever institution the experimenter is using, until the period ends. At the end of the period, subjects will have traded  $d^i$  and will have final holdings of  $x^i = w^i + d^i$ . They are paid for their final holdings according to  $u^i(x^i)$ . They know this payoff function at the beginning of the experiment. If desired, a new period begins.

Two standard trading institutions used in experiments (among many) are the Continuous Double Auction (CDA) and the Call Market (CM). The CDA is an open outcry process in which subjects post offers for quantities  $y_k^i$  at prices  $p_k^i$ , which can be accepted by others. When accepted an offer becomes a completed trade and it is withdrawn from the marketplace. The CDA can be thought of as a non-tatonnement dynamic. The Call Market is more similar to the tatonnement dynamic. Subjects also post bids of  $(p_k^i, y_k^i)$  but, contrary to the CDA, no transaction occurs or is accepted until the market is called. If the book is closed (i.e, subjects cannot see each others' bids), this is just a sealed bid auction. If the book is open (i.e. subjects can see each others' bids) and subjects can withdraw their bids and submit new ones, then this is a tatonnement in which the Call Market is the auctioneer.

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<sup>5</sup>We describe a complete markets experiment later on.



We focus on the CDA in this paper.

### 3.2. Findings from Market Experiments

Easley and Ledyard (1992) examined data from early CDA single commodity markets.<sup>6</sup> These markets involved a series of periods called days. Each agent's payoff function remained unchanged from day to day. Rather than looking at the average price in a day, Easley and Ledyard looked at the upper and lower bound on prices for each day. They found that the bounds responded from day to day as predicted by the Walrasian model. That is, if there were an excess demand at the upper bound of the previous day's prices then the upper bound on today's prices would be higher. They also found that prices within a day seemed to respond to marginal willingness to pay (accept), a dynamic often called Marshallian. Finally they found that initial trades responded to demands and supplies at the bounds of the previous day's prices (that is, initial trades are Walrasian) while later trades responded more to local information such as the gradients of the utility functions (that is, later trades are Marshallian). But Easley and Ledyard never extended the analysis to exchange environments with non quasi-linear preferences or with more than 2 commodities.

Anderson, e.a. (2004) examined the dynamic behavior of prices in the context of environments closely related to those in Scarf (1960). These are particularly interesting environments in that the Walrasian dynamic does not lead prices to converge to the unique market equilibrium in some of them. Their experiments also involved a series of days. A quick summary, that does not do justice to their paper, is that interday price dynamics are consistent with the Walrasian tatonnement and intraday price dynamics are not.

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<sup>6</sup>These were presented to subjects as partial equilibrium worlds. But they are equivalent to exchange environments with 2 commodities and quasi-linear preferences.

Let  $\pi(\tau)$  be the average price on day  $\tau$ , and  $\Delta(\tau) = \pi(\tau) - \pi(\tau - 1)$ . Anderson et. al. find that  $\Delta(\tau) = \lambda e(\pi(\tau - 1), \omega)$  where  $\lambda$  is a diagonal matrix with  $\lambda_{ij} = 0$  whenever  $i \neq j$  and  $\lambda_{ii} > 0$  for  $i = 1, \dots, K - 1$ . That is average prices move from day to day in a manner predicted by the tatonnement model, even though the CDA is not a tatonnement system.

On the other hand, [Anderson, e.a. \(2004\)](#) uncover no such relationship for intraday trades and prices. Let  $p(t), d(t)$  be the result of the  $t$ th transaction in a particular day. *Neither*  $p(t) - p(t - 1) = \lambda e(p(t - 1), w)$  *nor*  $p(t) - p(t - 1) = \lambda e(p(t - 1), w + d(t - 1))$  seem to fit the data. Intraday trading in CDA markets cannot be explained by the standard models. Neither model from section 2.3 is consistent with the facts above.

At this point our main question remains open: what is the mechanism by which prices and quantities are driven in a CDA?

#### 4. LOCAL GENERAL EQUILIBRIUM THEORY

Typically in CDAs, whether the laboratory or the field version, the bid-ask spread, namely, the difference between the prices of the best sell and buy orders, is small, i.e., the quotes are valid only for limited quantities. Consequently, price taking may only be a good characterization for small orders. CDAs are competitive only in smalls. This contrasts with Walrasian equilibrium theory. There, agents are price-takers *no matter how large their demands*.

In this paper, we advance a general equilibrium theory for markets where price-taking only applies to small orders. At its core is the assumption that, *to avoid adverse price movements, agents only submit small orders that are optimal locally*. Therefore, we shall call it *local general equilibrium theory*. A local general equilibrium theory presumes that the forces that determine trading and price formation derive from local conditions at the current prices and allocations. Such a theory is based on the concept of a local exchange

economy.

#### 4.1. Local Exchange Environments

A local exchange economy at time  $t$  is described by the local allocation,  $x^i(t) = w^i + d^i(t)$ , a set of feasible local trades,  $F^i(x^i(t)) = \{\eta^i\} \subset \mathfrak{R}^K$ , and the local utility function,  $[\nabla u^i(x^i(t))]\eta^i$ . Feasibility requires that  $\sum_i \eta^i = 0$ . In this local economy there is a temporary local equilibrium.

The dynamics are described by the movement through time from one local equilibrium to the next. There is a Walrasian theory and a Marshallian theory.

#### 4.2. A Local Walrasian Theory

[Champsaur and Cornet \(1990\)](#) use the concept of a local Walrasian equilibrium<sup>7</sup> to create a theory of dynamic price adjustment.

Let  $\eta^i(p) \in \operatorname{argmax} \nabla u^i(x^i) \cdot \eta^i$  subject to  $p\eta = 0$  and  $\eta \in F^i$ .  $\eta^i(p)$  is  $i$ 's local excess demand function. Note that  $F^i$  is constant on  $x^i(t)$ . A local Walrasian equilibrium at  $x$  is  $(\eta^*(x), p^*(x))$  where  $\sum \eta^i(p^*(x)) = 0$ , and  $\eta^{i*}(x) = \eta^i(p^*)$ .

The dynamics of the local Walrasian model are given by

$$(4.1) \quad p(t) = p^*(x(t))$$

$$(4.2) \quad \dot{x}^i = \eta(p^*(x(t)))$$

[Champsaur and Cornet \(1990\)](#) assume that  $\nabla u^i(x^i) \gg 0, \forall x^i$  and that  $F^i = \{\eta | \eta \geq -\delta\}$ . That is, the local economy is linear in an Edgeworth box. Their main result is the following.

**THEOREM 1** (i) for all  $t$ ,  $x(t)$  is attainable, (ii)  $du^i/dt \geq 0$ , (iii)  $p(t)\dot{x}^i = 0$ , and (iv) as  $t \rightarrow \infty$ , with strict quasi-concavity of the utility functions,

<sup>7</sup>They call this a Marginal Walrasian equilibrium.

1  $x(t)$  converges to a Pareto-optimal allocation  $x^*$  and  $p(t)$  converges to a  $p^*$  1  
 2 such that  $e(p^*, x^*) = 0$ . 2

3  
 4 It is, of course, not necessarily true that  $(x^*, p^*)$  is a (global) Walrasian 4  
 5 equilibrium for  $w$ ; that is, it is not necessarily true that  $e(p^*, w) = 0$ . 5

6 A discrete version of the Local Walrasian theory has been provided by 6  
 7 [Bonnisseau and Nguenamadji \(2009\)](#). The primary difference from the above 7  
 8 is that they use the global utility,  $u^i(x^i(t) + \eta^i)$ , in place of the local utility, 8  
 9  $\nabla u^i(x^i) \cdot \eta^i$ . With that, and the discreteness of time, they get convergence 9  
 10 to Pareto-optimal allocations in a finite number of steps. 10

#### 11 12 4.3. A Local Marshallian Theory 12

13 [Samuelson \(1947\)](#) (p. 264) describes a Marshallian dynamic of quantity 13  
 14 adjustment, as opposed to the Walrasian price adjustment, as follows. “If 14  
 15 ‘demand price’ exceeds ‘supply price,’ the quantity supplied will increase.” 15  
 16 He provides a formalization of this based on the inverses of the partial equi- 16  
 17 librium *aggregate* demand and supply curves. Unfortunately, in an exchange 17  
 18 economy there is no obvious way to generate an inverse demand function or 18  
 19 an inverse supply function without making some explicit assumptions about 19  
 20 the allocations that do not seem reasonable.<sup>8</sup> In this section, we propose a 20  
 21 dynamic process, different from Samuelson’s, that uses the Marshallian in- 21  
 22 tuition.<sup>9</sup> 22

23 It is easiest to incorporate a Marshallian approach into a general equi- 23  
 24 24

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25 <sup>8</sup>If we assume there are only two goods and quasi-linear utility functions, then  $d^i(p) =$  25  
 26  $\nabla_x u^{-1}(p) - w^i$ . We can say the aggregate demand at  $p$  is  $D(p) = \sum_i \max\{0, d^i(p)\}$  and 26  
 27 the supply is  $S(p) = -\sum_i \min\{0, d^i(p)\}$ . Given  $D(p)$  the “demand price” is  $D^{-1}(Q)$ . 27  
 28 The dynamic proposed by Samuelson is  $dQ/dt = \alpha[D^{-1}(Q) - S^{-1}(Q)]$ . Left unsaid is 28  
 29 what happens to each  $d^i$ . 29

<sup>9</sup>Earlier versions of allocation mechanisms based on this intuition can be found in  
[Ledyard \(1971\)](#) and [Ledyard \(1974\)](#).

librium model if we adopt the concept of “numeraire”. For the rest of this paper, we assume that  $u_K^i(x^i) > 0, \forall x^i$ . Let  $p = (q, 1)$  and  $d^i = (r^i, s^i) \in \mathfrak{R}_+^{K-1} \times \mathfrak{R}$ . Here  $s^i$  is  $i$ 's quantity of the numeraire commodity. We will let  $\rho_k^i = \nabla u(x_k^i)/u_K^i(x^i)$ , namely,  $i$ 's marginal willingness to pay for  $r_k$  in units of  $K$ . Let  $\rho^i = (\rho_1^i, \dots, \rho_{K-1}^i)$ .

Let  $b_k^i$  be the amount that  $i$  expresses to the market about their willingness to pay or accept.  $b_k^i$  is the most  $i$  is willing to pay (in units of  $K$ ) for units of  $k$  or the least they are willing to accept. We will call this a bid but it could also be  $i$ 's “reserve price” where they would be willing to take a unit of  $k$  in trade at a price lower than  $b_k^i$  if they saw such a price offered in the market.

The Marshallian intuition is that quantities move towards those who are prepared to offer the higher surplus, relative to the market. That is, if there is a market price,  $q$ , then buyers with higher  $b^i - q$  will buy faster. Formally, over a time period  $\delta$ ,  $i$ 's trades will be  $\Delta r^i = r_t^i - r_{t-\delta}^i = \alpha(b^i - q)$ , where  $\alpha$  is the rate at which surplus is translated into trade.

Faced with this prospect, how should an individual choose their bid,  $b^i$ ? Individual  $i$  wants to make  $\Delta u^i = u_t^i - u_{t-\delta}^i > 0$  large, if at all possible. Locally  $\Delta u^i$  is approximately equal to  $(\rho^i - q)\Delta r^i$ , i.e.,  $(\rho^i - q)\alpha(b^i - q)$ . Therefore  $i$  wants to choose  $b^i$  so that  $b^i - q = c^i \delta (\rho^i - q)$  where  $c^i \delta$  is chosen to control the rate at which  $i$  will trade. Since this is a linear approximation of the individual's utility increase, she will not want  $c^i \delta$  to be too large.<sup>10</sup>

With these bids and this trading dynamic, trading is feasible if and only if  $\sum \Delta r^i = 0$ . This is true if and only if  $q = \frac{\sum c^i \rho^i}{\sum c^i} = \bar{\rho}$ . We can think of  $q$  as the local Marshallian “equilibrium price.” It is the only price at which individuals will not want to change their bids, given the Marshallian trade dynamic.

<sup>10</sup>See the Appendix for one possible calculation of “too large”.

To summarize, we have

$$(4.3) \quad \Delta r^i = \alpha(b^i - q)$$

$$(4.4) \quad b^i = q + c^i \delta(\rho^i - q)$$

$$(4.5) \quad \Delta s^i = -q \Delta r^i$$

$$(4.6) \quad q = \frac{\sum c^i \rho_k^i}{\sum c^i}$$

Substitute (4.4) into (4.3) and let  $\delta \rightarrow 0$ . This leads to a continuous-time local Marshallian equilibrium theory:

$$(4.7) \quad \dot{r}_k^i = \alpha c^i (\rho_k^i - q_k)$$

$$(4.8) \quad \dot{s}^i = -q \dot{r}^i$$

$$(4.9) \quad q_k = \frac{\sum c^i \rho_k^i}{\sum c^i}$$

REMARK 1 *The above is a “reduced form” competitive theory. It assumes that traders are taking two things as given: (i) prices  $q$  and (ii) the trading rule  $\Delta r^i = r_t^i - r_{t-\delta}^i = \alpha(b^i - q)$ . If  $i$  behaves competitively, then  $i$  takes  $q$  as given and chooses  $b^i = q + c^i(\rho^i - q)$ . Summing across  $i$  on both sides of this response equation and dividing by  $N$  yields  $\bar{b} = q + (1/N) \sum c^i(\rho^i - q)$ . Therefore, in equilibrium,  $q = \bar{b} = \bar{\rho}$ .*

One way to see why this might make sense is to consider what happens in a CDA. First, transactions take place when someone’s bid is accepted. So on average the transaction price will be  $\bar{b}$ . Second, those with the most to gain, those with the largest difference in  $b^i - \bar{b}$ , will trade faster than others. Thus trade should occur, on average, according to the process we described above. That is, (4.7)-(4.9) can be loosely thought of as the expected value of a stochastic process whose absorbing states are the rest points of (4.7)-(4.9).

Another way to see whether (4.3)-(4.6) might describe something real is to consider whether it is incentive compatible. Would an optimizing agent be willing to follow these rules? It can be shown that (4.3)-(4.6) satisfies two types of incentive compatibility.

Suppose  $i$  believes (4.3) and that  $q$  is unknown. If  $i$  wants to protect herself against possible losses, i.e.  $i$  wants to ensure that  $\Delta u^i = u^i(x^i(t) + \Delta x^i(t)) - u^i(x^i(t)) \geq 0$ , then  $i$  should choose  $b^i = \rho^i$ . So  $i$  should choose  $c^i = 1/\delta$ . This type of local incentive compatibility is identical to that introduced by [Dreze and de la Vallée Poussin \(1971\)](#). It is a maximin type of defensive bidding which exhibits extreme risk aversion.

One can also imagine a less defensive approach. Suppose all  $i$  believe  $\Delta r^i = \alpha(b^i - q)$  and that  $q = (1/N) \sum b^i$ , the Marshallian equilibrium price. Further suppose they choose  $b^i$  to be a local Nash Equilibrium. That is, for every  $i$ ,

$$(4.10) \quad b^i \in \operatorname{argmax} \Delta u^i = (\rho^i - q)\alpha(b^i - q)$$

$$(4.11) \quad = \left(\rho^i - \frac{\sum_j b^j}{N}\right) \left(b^i - \frac{\sum_j b^j}{N}\right)$$

Letting  $\bar{b} = \frac{\sum b^j}{N}$ , the first order conditions for this are:  $\frac{-1}{N}(b^i - \bar{b}) + \frac{N-1}{N}(\rho^i - \bar{b}) = 0$  or  $b^i = \bar{b} + (N-1)(\rho^i - \bar{b})$ . Summing over  $i$  gives  $\bar{b} = \bar{\rho} = \frac{\sum \rho^i}{N}$ . So the local Nash equilibrium has  $b^i = \bar{\rho} + (N-1)(\rho^i - \bar{\rho})$ . Since  $q = \bar{b} = \bar{\rho}$  this means  $b^i = q + (n-1)(\rho^i - q)$ . Compare this to (4.4) to see that  $c^i = \frac{N-1}{\delta}$ . Thus, local Nash equilibria look exactly like local Marshallian equilibria.

We have the following:

**THEOREM 2** (*Convergence to Pareto Optimality*)

Let  $x(t) = [r(t), s(t)]$ . For the dynamics in (4.7)-(4.9),  $[x(t), p(t)] \rightarrow (x^*, p^*)$  where  $x^*$  is Pareto-optimal and  $e(p^*, x^*) = 0$ .

*Proof:* For each  $i$ ,  $\dot{u}^i(t) = (\nabla u^i) \eta^i = u_K^i(\rho^i, 1)(\dot{r}^i, -q\dot{r}^i) = u_K^i(\rho^i - q)\dot{r}^i = u_K^i(\rho^i - q)c^i(\rho^i - q) > 0$  unless  $\rho^i = q$ . Therefore  $d(\sum u^i)/dt > 0$  unless  $\rho^i = q$

for all  $i$ . This, and the continuity of the differential equation system allows us to use  $\sum u^i$  as a Lyapunov function and apply the standard asymptotic convergence theorems.

We can also see that the dynamics of prices is given by  $\dot{q} = d\bar{\rho}/dt = \frac{1}{\sum c^i} \sum c^i \dot{\rho}^i$  where  $\dot{\rho}^i = \sum (\partial \rho^i / \partial r_k^i) \dot{r}_k^i$ . Let  $H^i$  be the matrix with terms  $\partial \rho^i / \partial r_k^i$ .  $H^i = (\frac{1}{u_K^i}) [\nabla_{r^i r^i} u^i - \rho^i \nabla_{r^i K} u^i]$ . We can then write the dynamics of prices under the local Marshallian equilibrium model as

$$(4.12) \quad \dot{q} = \frac{1}{\sum c^i} \sum a(c^i)^2 H^i (\rho^i - q).$$

One of the interesting features of this finding is that it is consistent with the normative analysis of [Saari and Simon \(1978\)](#) in which they showed it was necessary for an equilibrating mechanism to use information about the Hessian  $\nabla_{xx} u^i$  in order to be stable.  $H^i$  does this here.

#### 4.4. Equivalence of Local Marshall and Local Walras

Under certain conditions, the local Walrasian and Marshallian theories imply exactly the same dynamics. The key is the set  $F^i$ , the local feasible consumption set in the Walrasian equilibrium model. Remember, the local theories are:

Marshall:  $\dot{r}^i = c^i(\rho^i - q)$ ,  $\dot{s}^i = -q\dot{r}^i$ , and  $q = \sum (\frac{c^i}{\sum c^j}) \rho^i$ ;

Walras:  $\dot{r}^i(q) \in \text{argmax } (u_L^i)(\rho^i - q)\dot{r}^i$  s.t.  $(\dot{r}^i, -q\dot{r}^i) \in F^i$ , and  $\sum \dot{r}^i(q) = 0$ .

##### Case 1: Local Marshall is Local Walras

Suppose we have a local Marshallian equilibrium at  $t$ ,  $(\dot{r}^*(t), q^*(t))$ . Let  $F^i = \{\eta = (\dot{r}^i, \dot{s}^i) | c^i \|\rho^i(x^*(t)) - q^*(t)\| \geq \|\dot{r}^i\|\}$ . This means in particular



that there are no local income effects. Then the local Walrasian equilibrium is  $\dot{r}^i = c^i(\rho^i(x^*(t)) - q^*(t))$  and  $q = q^*(t)$ . Note that this does require  $F(x(t))$  to depend on  $q^*(t)$  and  $x^*(t)$  which is consistent with the logic of the Appendix. But it does mean that “step size” and “equilibrium prices” are being simultaneously determined.

*Case 2: Local Walras is Local Marshall*

Suppose  $F = \{\dot{r}^i \mid \|\dot{r}^i\| \leq R\}$ , i.e. no local income effects, and we have a local Walrasian equilibrium at  $t$ ,  $(\dot{r}^*(t), q^*(t))$ . Then  $\dot{r}^{*i} = \lambda(\rho^i - q^*)$  where  $\lambda\|\rho^{*i} - q^*\| = R$ . Let  $c^i = \frac{R}{\alpha\|\rho^{*i} - q^*\|}$ . Then the local Marshallian equilibrium will be the same as the local Walrasian. Note that this does require  $c^i$  to depend on  $q^*(t)$  and  $x^*(t)$ . But that is consistent with the model in the Appendix.

REMARK 2 *Trying to tie the local versions of Marshall and Walras together exposes the delicate nature of the “local” arguments we are trying to make. The step sizes,  $F^i$  for Walras and  $c^i$  for Marshall appear ad hoc. It is our belief that their precise sizes are not that important, in that the dynamics will be similar in all cases. What may be different is the precise path and whether that path favors one agent over another.*

## 5. INTRODUCING A LAG

The Marshallian Local Theory of the previous section raises a practical issue. Price adjustment in CDAs often occurs at a speed far beyond adjustment of individual orders. By the time an agent has canceled old orders and submitted new orders, prices may have changed a number of times. So, let us investigate what happens if bidders are a bit slow.

### 5.1. The Model

Slow bid adjustment would mean:

$$b_t^i = q_{t-\delta} + \delta c^i(\rho_{t-\delta}^i - q_{t-\delta}).$$

Continue to assume that  $r_t^i = r_{t-\delta}^i + \alpha(b_t^i - q_t)$  and that prices adjust rapidly to the point that  $\sum \Delta r_t^i = \sum [r_t^i - r_{t-\delta}^i] = 0$ . From these we get,

$$(5.1) \quad r_t^i = r_{t-\delta}^i + \alpha \delta [c^i(\rho_{t-\delta}^i - q_{t-\delta}) - \frac{1}{N} \sum c^j(\rho_{t-\delta}^j - q_{t-\delta})]$$

$$(5.2) \quad q_t = (1/N) \sum b_t = q_{t-\delta} + \delta \left( \frac{\sum c^i}{N} \right) (\bar{\rho}_{t-\delta} - q_{t-\delta}).$$

Letting  $\delta \rightarrow 0$ , we get the continuous time system:

$$(5.3) \quad \dot{r}^i = \alpha [c^i(\rho^i - q) - \bar{c}(\bar{\rho} - q)]$$

$$(5.4) \quad \dot{q} = -\bar{c}(q - \bar{\rho})$$

Compare this to (4.7)-(4.9). First, in (4.9) prices  $q$  adjust instantaneously to the weighted average willingness to pay  $\bar{\rho}$ , while in (5.4) prices  $q$  converge exponentially to  $\bar{\rho}$ . Second, in (4.7) allocations  $r^i$  adjust, according to the Marshallian intuition, proportionally to the individual difference in the willingness to pay and the market price. In (5.3), the Marshallian adjustment is modulated by the difference between the average willingness to pay and the market price. If prices adjusted immediately this last term would vanish and we would have exactly (4.7).

**REMARK 3** *If we think of the local Walrasian model with  $F^i = \{\eta^i \mid \|\eta^i\| \leq R\}$  then the local Walrasian demand is  $c^i(\rho^i - q)$ . So one can interpret (5.4) as indicating that prices adjust proportionally to local excess demands. That is, (5.3) and (5.4) are the local equivalent of the global non-tatonnement model from Section 2.*

## 5.2. Asymptotics

If we try to proceed as in Theorem 2, we immediately run into a problem. With lags,  $du^i/dt = u_K^i(\rho^i - q)r^i = u_K^i(\rho^i - q)\alpha[c^i(\rho^i - q) - \bar{c}(\bar{\rho} - q)] = u_K^i[(\rho^i - q)\alpha c^i(\rho^i - q)] - u_K^i[(\rho^i - q)\alpha \bar{c}(\bar{\rho} - q)]$ . While the first term is positive as long as  $\rho^i \neq q$ , the second term is not necessarily so. Thus it is possible that along the dynamic path some individual utilities might decline because of the lag in the response to prices. Thus, we cannot expect convergence to occur in as orderly a manner as occurred in Theorem 2.

There is, nevertheless, a case of interest in which convergence to Pareto-optimal allocations can be proven. This is the case of quasi-linear preferences where  $u_K^i = 1$  for all  $i$ . This is true, for example, for the CAPM model of finance. There are also a lot of (experimental) data for this case.

THEOREM 3 (*Convergence to Pareto Optimality*)

Let  $x(t) = [r(t), s(t)]$ . If (i) there are no income effects, i.e.,  $u_K^i(x^i) = 1$  for all  $i$  and all  $x^i \in X$ , and (ii)  $x^i(t) > 0$  for all  $t$ , then for the dynamics in (5.3) and (5.4),  $[x(t), p(t)] \rightarrow (x^*, p^*)$  where  $x^*$  is Pareto-optimal and  $e(p^*, x^*) = 0$ .

*Proof:* We use  $\sum c^i u^i$  as a Lyapunov function. Let  $\kappa^i = c^i(\rho^i - q)$ . Then we can write  $d(\sum c^i u^i)/dt = \sum c^i \dot{u}^i = \sum c^i(\rho^i - q)\dot{r}^i = \alpha[(\sum \kappa^i \kappa^i) - (1/N)(\sum \kappa^i)(\sum \kappa^i)]$ . By the triangle inequality,  $(1/N)\sum \|\kappa^i\|^2 \geq (1/N)\|\sum \kappa^i\|^2$ . So  $\sum \|\kappa^i\|^2 \geq (1/N)\|\sum \kappa^i\|^2$  if  $\kappa^i \neq 0$  for some  $i$ . Therefore,  $d(\sum c^i u^i)/dt > 0$  unless  $\kappa^i = 0$  for all  $i$  which is true iff  $\rho^i = q$  for all  $i$ .

Condition (ii) is included above for technical reasons. If  $du^i/dt \geq 0$  along the path for all  $i$ , then (ii) wouldn't be necessary. But when  $du^i/dt < 0$  is possible for some  $i$ , we need to worry about  $x(t)$  hitting the boundary of the feasible consumption set. There are standard ways to modify (5.4) to deal with this. We do not pursue them here.

Condition (i) is included because we do not have a proof of convergence for utilities with income effects. Indeed, we believe it would be relatively easy to construct examples where such convergence will not occur. One could, of course, revise the model and impose a No Speculation condition on trades that would ensure  $du^i/dt \geq 0$ . We do not do that here largely because, as we will see below, the model as it now stands is consistent with the data.

## 6. EXPERIMENTAL EVIDENCE

Here, we return to experiments. We induced *quasi-linear, quadratic preferences*, like those underlying the Capital Asset Pricing Model (CAPM) in finance. The Local Marshallian Theory with slow bid adjustment makes intuitive predictions in that case, as we shall see.

### 6.1. *Experimental Setup*

Each experiment consisted of a number of independent replications, referred to as *periods*, of the same situation. At the beginning of a period, subjects were endowed with a number of each of 3 securities. These securities are referred to as *A*, *B* and *Notes*. Subjects were also endowed with cash, used in the trading, and perfectly substitutable for Notes. Markets in each of the securities were available, and subjects could submit orders and trade as they liked, during a pre-set amount of time. The trading interface was a fully electronic (web-based) version of a CDA, whereby untraded orders remained in the system (in other words, we implemented an open book system). After markets closed, subjects were paid depending on their final holdings of the securities, minus a fixed, pre-determined loan payment. After payment, securities were taken away and a new period started. Subjects kept the payments accumulated over the periods.

Subjects were not present in a centralized laboratory equipped with com-

puter terminals, but accessed the trading platform over the internet. Communication took place by email, phone and announcements through the main experiment web page.<sup>11</sup> In our experiments, between 30 and 42 subjects participated. The larger scale ensured that a trading environment is created that approximated the conditions of the theory: bid-ask spreads are reduced to a minimum (one tick); still, the best ask and best bid were valid only for small quantities.

End-of-period payments were determined using payoff functions that were quadratic in the holdings of A and B and linear in the Notes. Subject  $i$ , when holding  $h^i$  units of the Notes,  $C^i$  cash and the vector  $r^i$  of A and B received a payoff

$$(6.1) \quad \text{Pay}(i) = [r^i \cdot \mu] - \frac{a^i}{2}[r^i \cdot \Omega r^i] + C^i + 100h^i - L^i,$$

where  $L^i$  denotes the loan payment.

In the experiments,

$$\mu = \begin{bmatrix} 230 \\ 200 \end{bmatrix},$$

and

$$\Omega = \begin{bmatrix} 10000 & (+/-)3000 \\ (+/-)3000 & 1400 \end{bmatrix}.$$

The sign of the off-diagonal elements of the matrix  $\Omega$  varied. We changed the sign after four periods. The off-diagonal elements of  $\Omega$  were negative in periods 1 through 4 in the first experiment (28 Nov 01) and positive in periods 5 through 8. The design is reversed in the other (three) experiments:

<sup>11</sup>The interested reader can browse <http://eeps3.caltech.edu/market-020528> for an example. This web site provides the instructions for a typical experiment (the 28 May 02 experiment), the trading interface, and the announcements. To log in as observer, use ID=1, password=a.

1 the off-diagonal elements are positive in periods 1 through 4 and negative  
2 in periods 5 through 8.

3 When interpreting  $\mu$  as a vector of expected payoffs on securities A and B,  
4 and  $\Omega$  as the (positive definite, symmetric) matrix of payoff covariances, we  
5 effectively induced the mean-variance preferences at the core of the CAPM  
6 in finance.  $a^i (> 0)$  measures the risk penalty (risk aversion). The treatment  
7 (a change in off-diagonal elements of  $\Omega$ ) corresponds to a change in the  
8 covariance of the (random) payoffs on A and B.

9 Subjects are assigned one of three levels for the parameter  $a^i$ , chosen in  
10 such a way as to generate similar pricing as in the earlier (true) CAPM ex-  
11 periments reported in [Asparouhova, Bossaerts and Plott \(2000\)](#). See Table I  
12 for details.

13 Each type also received a different initial allocation of A and B (nobody  
14 received any Notes to start with, i.e., Notes were in zero net supply). Sub-  
15 jects were not informed of each others' payment schedules or initial holdings,  
16 and whether these varied over the course of the experiment (they did not).  
17 This way, subjects with knowledge of general equilibrium theory could not  
18 possibly compute equilibrium prices. Specifically, subjects could not form  
19 reasonably credible expectations about where prices would tend to.

20 All accounting was done in terms of an artificial currency, the franc. At  
21 the end of the experiment, cumulative earnings were converted to dollars at  
22 a pre-announced exchange rate. On average, subjects made about \$45 for  
23 the three-hour experiment; the range of payments was \$0 to approximately  
24 \$150. These payments, however, inaccurately reflected the size of the in-  
25 centives during trading. Explicit computations of the amounts of money  
26 subjects left on the table because they did not fully optimize (and assum-  
27 ing that they could trade at end-of-period prices) revealed values over \$100  
28 per subject/period in early periods. Some subjects were savvy enough to  
29 realize part of these potential gains, but most didn't (which explains the

substantial range of payouts across subjects). As more subjects realized that there was money to be made, their actions and the ensuing price changes caused these amounts invariably to drop to approximately \$2 in later periods. Subjects seemed not to spend the extra effort needed to extract the last couple of dollars. This fact will be important to interpret some of the results.

### 6.2. The (CAPM) Equilibrium

Translating the payoff functions into the preference functions of our theory, let  $x^i = (r^i, s^i)$ , where the  $r_k$  are the quantities of A ( $k = 1$ ) and B ( $k = 2$ ) that agent  $i$  chooses, and  $s$  is money (cash plus equivalent, namely, payoffs on positions in Notes, minus the Loan payment). Then:

$$u(x^i, \theta^i) = \mu r^i - (a^i/2)(r^i)' \Omega r^i + s^i.$$

Notes will be the numeraire. In this quasi-linear world (or equivalently, in the CAPM world),

$$(6.2) \quad \rho^i = \mu - a^i \Omega r^i,$$

$$(6.3) \quad e^i(q, w^i) = (1/a^i) \Omega^{-1} (\mu - q) - w^i(t),$$

where the excess demand vector  $e^i$  now includes only the risky securities (not the numeraire asset).

The global Walrasian equilibrium price and allocations are

$$(6.4) \quad q = \mu - b \Omega \bar{w}$$

$$(6.5) \quad r^i = (1/a^i) b \bar{w}$$

where  $b = [\sum (1/a^i)]^{-1}$ , and  $\bar{w}$  denotes the per-capita average endowment,

$$\bar{w} = (1/N) \sum w^i.$$

The equilibrium in  $r$  is independent of individual  $w^i$ 's because of quasi-linearity.

In the CAPM interpretation of this economy,  $\bar{w}$  is referred to as the *market portfolio* (of risky securities). The pricing equation (6.4) captures the essence of the CAPM: it reveals that the market portfolio will be mean-variance optimal. Indeed, Roll (1977) showed that a portfolio  $z$  satisfies the following relationship for some (positive) scalar  $\beta$ :

$$(6.6) \quad q = \mu - \beta\Omega z,$$

if and only if  $z$  is mean-variance optimal. Notice that this is exactly the form of the equilibrium pricing formula in (6.4), so  $\bar{w}$  is mean-variance optimal. On the other hand, the choice equation (6.5) exhibits *portfolio separation*: individual allocations are proportional to a common portfolio, namely, the market portfolio  $\bar{w}$ .

### 6.3. Dynamics Predictions

For the version of the Marshallian Local Theory where bid adjustment is as fast as price adjustment (Section 4), the following obtains in the context of linear-quadratic preferences (CAPM preferences). From Equation (4.12), we know that  $\dot{q} = (\frac{1}{\sum c^i}) \sum \alpha(c^i)^2 H^i(\rho^i - q)$ . From (6.2),  $\rho^i - q = \mu - q - a^i\Omega r^i$ . From (6.3),  $a^i\Omega e^i = \mu - q - a^i\Omega r^i$ . Therefore,  $\dot{q} = (\frac{\alpha}{\sum c^i}) \sum (c^i)^2 H^i a^i\Omega e^i$ . But  $H^i = a^i\Omega$ . Therefore,

$$(6.7) \quad \dot{q} = (\frac{\alpha}{\sum c^i}) \sum (c^i a^i)^2 \Omega^2 e^i[q, r^i].$$

That is, price changes are related to weighted average Walrasian excess demands through the *square* of the matrix  $\Omega$ . As such, we expect price changes in one security to be related not only to the security's own excess demands, but also to the *excess demands of other securities*. The relationship is determined, among others, by the elements of  $\Omega^2$



Regarding allocations, using (4.7) and, from (6.2),  $\rho^i - q = \mu - q - a^i \Omega r^i$ , we get:

$$(6.8) \quad \dot{r}^i = \alpha c^i [\mu - q - a^i \Omega r^i].$$

Again, adjustment is driven by the matrix  $\Omega$ .

When bid adjustment is slower than price adjustment (Section 5), Equations (5.3) and (5.4) take particularly interesting forms. For price dynamics, we obtain:

$$(6.9) \quad \dot{q} = \Omega \sum (c^i a^i) e^i(q, r^i).$$

That is, price changes are related to (weighted) average Walrasian excess demands through the matrix  $\Omega$  (rather than the *square*).

Allocation dynamics take the following form:

$$(6.10) \quad \dot{r}^i = -\alpha \Omega [c^i a^i r^i - \frac{1}{N} \sum c^j a^j r^j] + \alpha (c^i - \bar{c})(\mu - q).$$

If  $c^i = \bar{c}, \forall i$ , that is all  $i$  trade with the same aggressiveness, the second term drops out:

$$(6.11) \quad \dot{r}^i = -\alpha \bar{c} \Omega [a^i r^i - \frac{1}{N} \sum a^j r^j].$$

We now have very intuitive allocation dynamics.

To see this, consider the case where all agents start with the same initial allocation. The expression in square brackets in (6.11) simplifies:

$$a^i r^i - \frac{1}{N} \sum a^j r^j = \left( a^i - \frac{1}{N} \sum a^j \right) \frac{1}{N} \sum r^j = \left( a^i - \frac{1}{N} \sum a^j \right) \bar{w}.$$

Hence,

$$\dot{r}^i = -\alpha \bar{c} \left( a^i - \frac{1}{N} \sum a^j \right) \Omega \bar{w}.$$

That is, changes in holdings are a linear transformation of the per-capita endowment (the market portfolio, in CAPM language). Except in the unlikely event that the per capita allocation is an eigenvector of  $\Omega$ , the new holdings will not be the same anymore. Agents trade away.

1           The consequences are best illustrated in a CAPM setting, where  $\Omega$  is           1  
2 the matrix of payoff covariances. Imagine that  $\Omega$  is diagonal. The diagonal           2  
3 elements of  $\Omega$  are the payoff variances. In that case, volume (the absolute           3  
4 value of the elements in  $r^i$ ) will be highest for the high-variance securities.           4  
5 That is, most adjustments take place in the high-variance securities. The           5  
6 sign of the changes in an agent's holdings of securities depends on  $a^i$  relative           6  
7 to the average ( $(1/N) \sum a^j$ ). Since these coefficients measure risk aversion in           7  
8 a CAPM setting, this means that more risk averse agents sell risky securities           8  
9 (the entries of  $r^i$  will be negative); less risk averse agents buy. Effectively,           9  
10 more risk averse agents unload risky securities, paying more attention to           10  
11 the most risky securities, because that way their local gain in utility is           11  
12 maximized. Likewise, less risk averse agents do what is locally optimal:           12  
13 increase risk exposure by buying the most risky securities first.           13

14           When  $\Omega$  is non-diagonal, the sign of the off-diagonal elements interferes           14  
15 with the above dynamics. In a CAPM setting, the off-diagonal elements           15  
16 equal the payoff covariances. Intuitively, when the off-diagonal elements are           16  
17 negative, i.e., when the payoff covariances are negative, securities are natu-           17  
18 ral hedges for each other, and the market portfolio provides diversification.           18  
19 Increasing one's risk exposure by buying mostly risky securities (or decreas-           19  
20 ing one's risk exposure by selling mostly risky securities) leads to a less           20  
21 diversified portfolio, i.e., to utility losses. Maximum local gains in utility           21  
22 are obtained by trading combinations of securities that are closer to the           22  
23 per-capita average endowment, i.e., the market portfolio. As a consequence,           23  
24 agents' portfolios of risky securities will remain closer to the market port-           24  
25 folio than if payoff covariances were zero (or positive, for that matter).           25

26           The equilibration process may not go all the way to the end. That is, equi-           26  
27 librium pricing may not be fully obtained. This may happen when agents           27  
28 do not perceive enough gains to cover the effort to trade. At that point,           28  
29 agents will not have traded back to holdings that are proportional to the           29

per-capita average endowment. In CAPM terms, portfolio separation fails.

The role of  $\Omega$  in this adjustment process is crucial. If the off-diagonal elements of  $\Omega$  are positive (payoff covariances are positive), and the equilibration process halts before fully reaching equilibrium, then violations of portfolio separation can be expected to be larger than if these off-diagonal elements are negative (payoff covariances are negative).

REMARK 4 *Were it not for the last term in (6.10),  $c^i$  and  $a^i$  would not be separately identified. So, identification will require heterogeneous aggressiveness across agents.*

#### 6.4. Experimental Findings

##### Transaction Prices

Figure 1 displays the evolution of prices of securities A (dashed line) and B (dash-dotted line). The prices of the Notes are not shown; these are invariably close to 100 francs, their no-arbitrage value. Each observation corresponds to a trade in one of the three securities. The prices of the non-trading securities is set equal to their previous trade prices. Time (in seconds) is on the horizontal axis; Price (in francs) is on the vertical axis. Vertical lines separate periods. Horizontal lines indicate equilibrium prices of A (solid line) and B (dotted line). Note that their levels change after 4 periods, reflecting the change in the off-diagonal element of  $\Omega$ , i.e., the payoff covariance.

The first observation to be made about Figure 1 is that transaction prices are almost invariably *below* equilibrium prices. Second, relative to equilibrium levels, prices generally start out lower in periods when the off-diagonal terms of  $\Omega$  are positive. In the CAPM interpretation of the experiment: asset prices start lower when the payoff covariances are positive.

## Off-Equilibrium Price Dynamics

Table II displays results from projections of changes in transaction prices of A and B onto the weighted sum of individual Walrasian excess demands. Weights are given by individuals'  $a^i$ s.<sup>12</sup> The time series for each experiment were split in two; one sub-sample covered periods with positive off-diagonal elements for  $\Omega$ ; the other covered periods with negative off-diagonal elements. Only intra-period price changes were used. Estimates of slope coefficients of aggregate excess demands are bold-faced whenever they are significant at the 1% level. Tests are one-sided; they compare the null hypothesis that the coefficient is zero against the alternative that it is positive (in the case of the projection coefficient of a security's own aggregate excess demand) or has the same sign as the off-diagonal elements of  $\Omega$  (in the case of the projection coefficient of the other security's aggregate excess demand).

The regression  $R^2$ s are small, but the  $F$  tests reveal that significance is high. The first-order autocorrelation of the error term suggests little misspecification (some are significantly negative, but one expects the data to generate a number of significant autocorrelations even if the null of no autocorrelation is right).

We observe the following.

First, a security's price change significantly and positively correlates with its weighted aggregate excess demand. Second, the signs of the cross-effects (partial correlation between a security's price change and the weighted aggregate excess demand in the *other* security) are almost always the same as that of the off-diagonal elements in  $\Omega$  (if they are not, the projection coefficient is insignificant). The estimation results are highly significant.<sup>13</sup>

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<sup>12</sup>We also ran projections with unweighted average Walrasian excess demands, but the results are qualitatively the same.

<sup>13</sup>These results replicate the findings in [Asparouhova, Bossaerts and Plott \(2000\)](#)

Table II thus suggests that the matrix of coefficients in projections of transaction price changes onto aggregate (Walrasian) excess demands has the same structure as  $\Omega$ . A closer inspection of the table suggests that this projection coefficient matrix not only reflects the signs of the corresponding elements of  $\Omega$ , but also their relative magnitude. For instance, the slope coefficient of own excess demand in the projection of the price change of security A is generally the largest; the corresponding element in  $\Omega$  happens to be largest as well.

### Allocations

According to Walrasian equilibrium theory, individual holdings of A and B should be proportional to per-capita allocations of these two securities. To measure the extent of violations, we compute the value of holdings of A as a proportion of the total value of holdings of A and B and compare the same proportion if a subject were to be holding the per-capita allocations. The absolute deviation should be zero. Table III displays the mean absolute deviations (across subjects) based on final holdings in all periods of all experiments. It is obvious that the theoretical prediction is not upheld.

Table III demonstrates, however, that the mean absolute deviations depend on the sign of the off-diagonal elements of  $\Omega$  (or the sign of the covariance between the payoffs on A and B in the CAPM interpretation). This effect emerges despite the fact that subjects started out with the same initial allocations in each period and across experiments (see Table I; the loan and [Asparouhova and Bossaerts \(2009\)](#)). There, quadratic preferences were indirectly induced, through risk. In [Asparouhova, Bossaerts and Plott \(2000\)](#), there were two risky securities; in [Asparouhova and Bossaerts \(2009\)](#), there were three. The latter setting is particularly illuminating: [Asparouhova and Bossaerts \(2009\)](#) reports that the partial correlation between changes in prices of an asset and the Walrasian excess demand of another asset reflects the magnitude and sign of the corresponding element of the payoff covariance matrix.

1 payments do differ across experiments, but in a way that is materially ir- 1  
 2 relevant). Only the sign of the off-diagonal elements of  $\Omega$  appear to have an 2  
 3 effect. Straightforward computations of standard errors (not reported) lead 3  
 4 one to conclude that the mean absolute deviations are always significantly 4  
 5 bigger in periods where the off-diagonal elements of  $\Omega$  were positive than 5  
 6 when these elements were negative. 6

7 In the CAPM interpretation, the mean absolute deviations measure vio- 7  
 8 lations of portfolio separation. Such violations have been reported in CAPM 8  
 9 experiments before – see [Bossaerts, Plott and Zame \(2007\)](#). The relation- 9  
 10 ship with the sign of the off-diagonal elements of  $\Omega$  suggests that portfolio 10  
 11 separation violations are worse when payoff covariances are positive. 11

## 12 Discussion 12

13 Let us first discuss price dynamics. The data suggest: 13  
 14 14

$$15 \quad (6.12) \quad \dot{q} = \kappa \Omega \sum a^i e^i [q, r^i], \quad 15$$

16 for some constant  $\kappa > 0$ . That is, prices changes are related to weighted 16  
 17 average Walrasian excess demands through the matrix  $\Omega$ . This is consistent 17  
 18 with the Local Marshallian Theory with slow bid adjustments [see (6.9)] 18  
 19 but *not* with the Local Marshallian Theory with fast bid adjustment [see 19  
 20 (6.7)]. 20  
 21 21

22 Second, Local Marshallian Theory with slow bid adjustments explains 22  
 23 how the final allocations depended on matrix  $\Omega$ . If the off-diagonal elements 23  
 24 are positive, and the equilibration process halts before reaching equilibrium 24  
 25 (which it did; see [Figure 1](#)), final holdings are farther from equilibrium 25  
 26 predictions. Translated in CAPM language, when payoff covariances are 26  
 27 positive, violations of portfolio separation in eventual allocations are more 27  
 28 extreme. 28  
 29 29

## 7. PREDICTIONS OF RELEVANCE TO FINANCE

Financial economists are mostly interested in *pricing models*. One class of such models, the portfolio-based models, explain the pricing of securities relative to some benchmark. In the CAPM, for instance, the prediction is that all assets are priced such that the market portfolio is mean-variance optimal, i.e., provides the maximum expected return for its risk (return variance).

All pricing models in finance derive from equilibrium restrictions. An interesting question is: can we generate similar models *off equilibrium*. Specifically, can one identify a portfolio that continuously determines the prices of all securities even while markets are off equilibrium?

We now argue that one can, by studying where prices converge to if we temporarily halt the allocation process (i.e.,  $\alpha = 0$  for a short period of time). In the CAPM setting, prices would continue to adjust according to (5.4). The stationary point of this system of differential equations is:

$$(7.1) \quad q^* = \mu - \Omega \frac{1}{\sum c^i} \sum c^i a^i r^i.$$

Notice that this equation is of the same form as the one that defines mean-variance optimal portfolios, namely, (6.6). They coincide for  $\beta = \frac{1}{\sum c^i}$  and  $z = \sum c^i a^i r^i$ . When all  $c^i$  are identical, this portfolio is the average holdings portfolio, where each agent's holdings are weighted by his or her coefficient  $a^i$ . In the CAPM interpretation of these coefficients, the holdings of more risk averse agents (agents with higher  $a^i$ ) are weighted more heavily, and *vice versa*. We will refer to the portfolio as the *risk-aversion weighted endowment portfolio*, or *RAWE* for short. The RAWE and per-capita endowments are closely related. If allocations are independent of preference coefficients  $a^i$ , then the two coincide. Such is the case, for instance, if all individual holdings are proportional to the per-capita endowment, i.e., the market portfolio, as in the CAPM equilibrium allocation.

We can go back to our experiments and study how far the RAWE portfolio was from mean-variance optimality after each transaction. We measure the distance as the difference between the Sharpe ratio (at transaction prices) of the RAWE portfolio and the maximum possible Sharpe ratio. The Sharpe ratio is defined to be the expected return and the return variance. Expected returns, variances and covariances are computed from the entries in  $\mu$  (expected payoffs),  $\Omega$  (payoff variances and covariances) and transaction prices.

In an absolute sense, it is hard to know when the distance from mean-variance optimality is “large.” To obtain a relative sense of distance, we normalize the distance by the maximum (observed) distance in an experiment. Hence, our distance measure will be between zero and one; it equals zero when a portfolio is mean-variance optimal; it equals one when the distance is maximal in the experiment at hand. To get a measure of how far the markets are at any point from Walrasian (CAPM) equilibrium, we compute the difference of the value of the market portfolio evaluated at transaction prices and its value at CAPM equilibrium. This difference too is normalized by the maximal observation in an experiment.<sup>14</sup>

The normalization and the comparison with the distance from equilibrium pricing are insightful. Figure 2 displays the evolution of the distance of the RAWE portfolio from mean-variance optimality and that of the distance from equilibrium pricing. The contrast between the two distance measures is often pronounced. The RAWE portfolio almost invariably moves quickly to the mean-variance efficient frontier, confirming our prediction. Still, prices may be far from equilibrium. The latter is more pronounced in periods when the covariance is positive (periods 5-8 in experiment 28 Nov 01; periods 1-4 in the remaining experiments).

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<sup>14</sup>Note that CAPM pricing is sufficient for the difference measure to be zero, but not necessary.



## 8. CONCLUDING COMMENTS

Previous research has shown that standard global tatonnement and non-tatonnement are not consistent with intra-period price dynamics in CDAs. Since CDAs are competitive only locally (i.e., for small quantities), we have proposed a Local Marshallian Equilibrium theory here. It is equivalent to a Local Walrasian Equilibrium theory, but our experiments shows that it cannot explain price dynamics. Instead, Local Marshallian Equilibrium with *lagged bids* is consistent with pricing data, and it explains patterns in final holdings across treatments.

In our experiments, we induced quasi-linear, quadratic preferences in a way that make them isomorphic with CAPM experiments. In a CAPM setting, Local Marshallian Equilibrium identifies a portfolio that remains (mean-variance) optimal throughout. This portfolio can be used as benchmark for pricing, just like the market portfolio is used as pricing benchmark in the CAPM (Walrasian) equilibrium. There is an opportunity here to dispense altogether with asset pricing theory based on global equilibrium concepts, thus providing more realism.

While the experimental findings provide solid support to our theory, they raise many new issues that need to be addressed in future research. First, can Local Marshallian Equilibrium with lagged bids predict pricing and allocation dynamics in situations with income effects (unlike in our experiments), such as, for instance, in Scarf's example ([Scarf \(1960\)](#))? Second, would Local Marshallian Equilibrium with lagged bids also apply to the dynamics of bookbuilding in Call Markets (CM)? If not, this would mean that institutions do matter; if it does, it would imply that some kind of revelation principle applies.

The theory also needs further exploring. In particular, we need a better understanding of  $c^i$ , the parameter that controls the rate at which agent  $i$  trades. Right now, this is treated as a constant, effectively making our

agents myopic, unable to form expectations about future price changes. In many contexts (including, we think, the experiments we presented here), lack of structural information about the economy (supplies of securities; other agents' preferences, etc.) may make it impossible for agents to form sensible expectations, so myopia can be defended. They are not irrational – agents optimize, if only locally. Still, as agents acquire more information about the economy, one can expect them to trade more aggressively, and hence, adjust  $c^i$ .

Information from past periods, for instance, could allow agents to better calibrate price expectations, thus generating the period-by-period learning patterns that are evident in many experimental markets. Specifically, past price information could be readily incorporated into agents' marginal willingness to pay  $\rho_k^i$ , using arguments from [Easley and Ledyard \(1992\)](#). Let  $\bar{P}_k$  be the maximum price of a trade for  $k$  in the previous day. Let  $\underline{P}_k$  be the minimum such price for  $k$ . Then let  $\rho_k^i = \bar{P}_k$  if  $\bar{P}_k \leq u_k^i/u_s^i$ . Let  $\rho_k^i = \underline{P}_k$  if  $\underline{P}_k \geq u_k^i/u_s^i$ . Otherwise, let  $\rho_k^i = u_k^i/u_s^i$  as above. Then use our local, lagged, Marshallian theory with this new willingness to pay function.

Finally, because the lag with which agents update their bids may vary from agent to agent, price and quantity dynamics will depend on who is active and who is not. Future experiments should shed light on the decision to become active.

## APPENDIX

Over the time interval  $[0, T]$ , there are  $T/\delta$  periods of length  $\delta$ . Trading at the rate  $\Delta r$  implies  $\Delta u \simeq (\rho - q)(T/\delta)(\Delta r) - (1/2)(T/\delta)^2[\Delta r H \Delta r]$ . If  $u$  is quasi-linear (like in CAPM preferences) then  $H = -\nabla_{xx}u$ , the Hessian of  $u$ . If  $u$  is not quasi-linear then  $H$  is more complicated but it is positive definite (p.d.).

If  $\Delta r = \lambda(\rho - q)$  then  $\Delta u \geq 0$  iff  $\|\rho - q\|^2 - (1/2)(\lambda T/\delta)[(\rho - q)H(\rho - q)] \geq$

0. This is true iff  $\lambda \leq \delta c^*$  where  $c^* = (2/T)\|\rho - q\|^2/[(\rho - q)H(\rho - q)]$ . Note that  $c^*$  is bounded away from 0 as  $\|\rho - q\| \rightarrow 0$ , since  $H$  is p.d. (In one dimension, the bound is  $1/H$ .) One thing this implies is the more risk averse one is (in the CAPM interpretation of quasi-linear preferences) or the longer  $T$  is relative to  $\delta$ , the lower is  $c^*$ .

Therefore a local trader will want  $\Delta r = a(b - q) = \delta c^*(\rho - q)$  or  $b = q + \delta c(\rho - q)$ .

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TABLE I

EXPERIMENTAL DESIGN DATA.

Exp. <sup>a</sup>	Subject Cat. (#) <sup>d</sup>	$a^{ib}$ ( $\times 10^{-3}$ )	Signup Reward (franc)	Endowments			Cash (franc)	Loan Repayment <sup>c</sup> (franc)	Exchange Rate \$/franc
				A	B	Notes			
28Nov01	14	2.30	125	2	8	0	400	2340	0.06
	14	0.28	125	8	2	0	400	2480	0.06
	14	0.15	125	2	8	0	400	2365	0.06
20Mar02	10	2.30	125	2	8	0	400	2320	0.06
	10	0.28	125	8	2	0	400	2470	0.06
	10	0.15	125	2	8	0	400	2370	0.06
24Apr02	14	2.30	125	2	8	0	400	2320	0.06
	13	0.28	125	8	2	0	400	2470	0.06
	13	0.15	125	2	8	0	400	2370	0.06
28 May02	13	2.30	125	2	8	0	400	2320	0.06
	12	0.28	125	8	2	0	400	2470	0.06
	12	0.15	125	2	8	0	400	2370	0.06

Footnotes to Table I.

<sup>a</sup> Date of experiment.<sup>b</sup> Coefficient  $a^i$  in the payoff function (6.1).<sup>c</sup> Coefficient  $L_n$  in the payoff function (6.1).<sup>d</sup> Number per subject type.

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TABLE II  
 PROJECTIONS OF TRANSACTION PRICE CHANGES ONTO WEIGHTED SUM OF WALRASIAN EXCESS DEMANDS

Experiment	Periods	Sign <sup>a</sup>	Security	Coefficients <sup>b</sup>		$R^2$	F-statistic <sup>c</sup>	$N^d$	$\rho^e$
				Intercept	Excess Demand				
				A	B				
28Nov01	1-4	-	A	0.1	<b>20.7</b>	<b>-6.5</b>	27.0	1138	-0.12**
				(0.0)	(2.8)	(0.9)	(<.01)		
	5-8	+	A	-0.0	-1.4	<b>1.5</b>	33.6	1138	-0.06
				(0.0)	(1.8)	(0.6)	(<.01)		
20Mar02	1-4	+	A	0.6	<b>20.0</b>	<b>5.4</b>	31.3	1224	-0.30**
				(0.1)	(2.8)	(0.9)	(<.01)		
	5-8	-	B	-0.1	0.5	<b>1.2</b>	30.3	1224	0.03
				(0.1)	(1.7)	(0.5)	(<.01)		
20Mar02	1-4	+	A	0.6	<b>18.8</b>	<b>5.4</b>	12.8	668	-0.02
				(0.1)	(3.7)	(1.1)	(<.01)		
	5-8	-	A	-0.5	-5.9	1.8	45.5	668	0.04
				(0.2)	(4.6)	(1.4)	(<.01)		
5-8	-	B	-0.3	<b>32.1</b>	<b>-8.1</b>	16.6	491	-0.07	
			(0.1)	(7.3)	(2.5)	(<.01)			
5-8	-	B	-0.0	-7.0	<b>6.2</b>	48.1	491	0.08	
			(0.1)	(5.0)	(1.7)	(<.01)			



TABLE II  
 PROJECTIONS OF TRANSACTION PRICE CHANGES ONTO WEIGHTED SUM OF WALRASIAN EXCESS DEMANDS (CONTINUED)

Experiment	Periods	Sign <sup>a</sup>	Security	Coefficients <sup>b</sup>		R <sup>2</sup>	F-statistic <sup>c</sup>	N <sup>d</sup>	$\rho^e$	
				Intercept	Excess Demand					
				A	B					
24Apr02	1-4	+	A	1.3	<b>33.6</b>	<b>10.1</b>	0.07	26.5	745	-0.03
				(0.2)	(4.6)	(1.4)		(< .01)		
	5-8	-	A	-0.3	-2.2	0.5	0.07	25.9	745	0.06
				(0.1)	(3.0)	(0.9)		(< .01)		
28May02	1-4	+	A	-0.3	<b>17.4</b>	<b>-5.2</b>	0.04	14.4	675	-0.04
				(0.1)	(5.2)	(1.9)		(< .01)		
	5-8	-	B	0.1	<b>-31.9</b>	<b>12.6</b>	0.14	52.6	675	-0.07
				(0.1)	(3.6)	(1.3)		(< .01)		
28May02	1-4	+	A	0.6	<b>18.8</b>	<b>4.9</b>	0.04	18.4	825	0.04
				(0.2)	(3.7)	(1.2)		(< .01)		
	5-8	-	A	-0.8	<b>10.5</b>	<b>6.9</b>	0.16	76.5	825	0.04
				(0.2)	(3.5)	(1.1)		(< .01)		
28May02	1-4	+	A	-0.1	<b>9.0</b>	<b>-2.9</b>	0.02	4.3	563	-0.14**
				(0.1)	(3.1)	(1.1)		(0.01)		
	5-8	-	B	-0.1	<b>-9.3</b>	<b>4.6</b>	0.08	23.6	563	0.01
				(0.1)	(3.4)	(1.2)		(< .01)		

Footnotes to Table II.

<sup>a</sup> Sign of the off-diagonal element of the matrix  $\Omega$ . The OLS coefficient matrix evidently inherits the structure of this matrix.

<sup>b</sup> OLS projections of transaction price changes onto (i) an intercept, (ii) the weighted sum of Walrasian excess demands for the two risky securities (A and B). Each individual excess demand is weighted by the coefficient  $a^i$ . Time advances whenever one of the three assets trades. Boldfaced coefficients are significant at the 1% level using a one-sided test (effect of own excess demand is positive; cross-effect has the same sign as the corresponding covariance). Standard errors in parentheses.

<sup>c</sup>  $p$ -level in parentheses.

<sup>d</sup> Number of observations.

<sup>e</sup> Autocorrelation of the error term; \* and \*\* indicate significance at the 5% and 1% level, respectively.

TABLE III  
MEAN ABSOLUTE DEVIATIONS OF INDIVIDUAL PORTFOLIO WEIGHTS FROM MARKET  
PORTFOLIO WEIGHTS

Experiment	Periods	Sign <sup>a</sup>	Period			
			1 or 5	2 or 6	3 or 7	4 or 8
28Nov01	1-4	-	0.15 <sup>b</sup> (0.02) <sup>c</sup>	0.14 (0.02)	0.12 (0.02)	0.12 (0.02)
	5-8	+	0.24 (0.03)	0.25 (0.03)	0.23 (0.03)	0.26 (0.03)
20Mar02	1-4	+	0.24 (0.03)	0.26 (0.03)	0.24 (0.03)	0.25 (0.03)
	5-8	-	0.13 (0.02)	0.11 (0.02)	0.13 (0.03)	0.11 (0.02)
24Apr02	1-4	+	0.25 (0.02)	0.26 (0.02)	0.25 (0.02)	0.25 (0.03)
	5-8	-	0.17 (0.02)	0.12 (0.01)	0.10 (0.01)	0.09 (0.01)
28May02	1-4	+	0.24 (0.03)	0.27 (0.02)	0.22 (0.02)	0.22 (0.03)
	5-8	-	0.17 (0.02)	0.15 (0.02)	0.10 (0.02)	0.10 (0.02)

Footnotes to Table III.

<sup>a</sup> Sign of the off-diagonal element of the matrix  $\Omega$ . The mean absolute deviation of final holdings from per-capita average holdings is significantly larger when this sign is positive.

<sup>b</sup> Average absolute difference between (i) the proportion individuals invest in A relative to total franc investment in securities A and B, and (ii) the corresponding weight in the per-capita holdings of A; weights are computed on the basis of end-of-period prices and holdings.

<sup>c</sup> Standard error in parentheses.

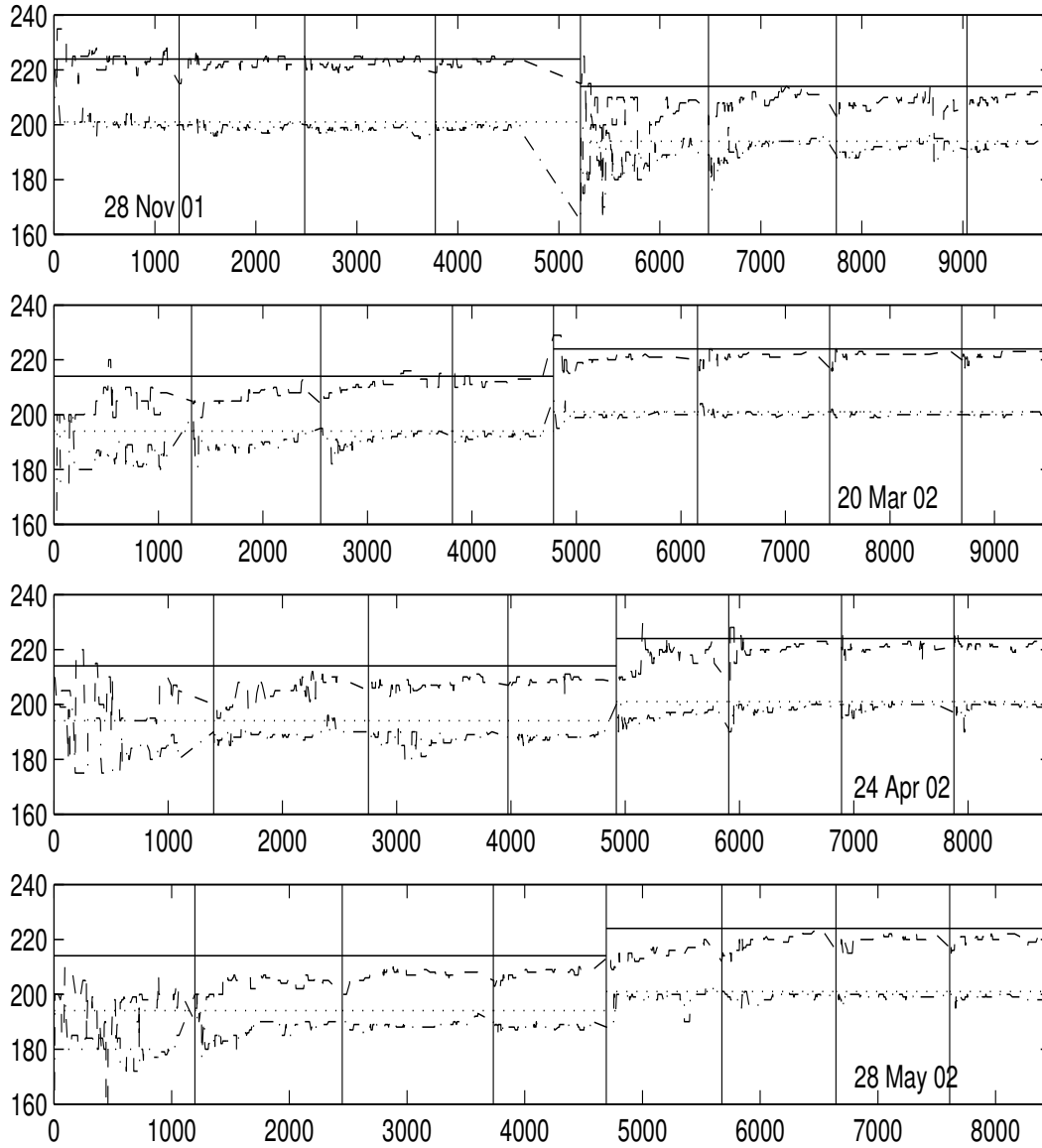


FIGURE 1.— Evolution of transaction prices of securities A [dashed line] and B [dash-dotted line]. Horizontal lines indicate equilibrium price levels [A: solid line; B: dotted line]. Time (in seconds) on horizontal axis; prices (in francs) on vertical axis. Vertical lines delineate periods.

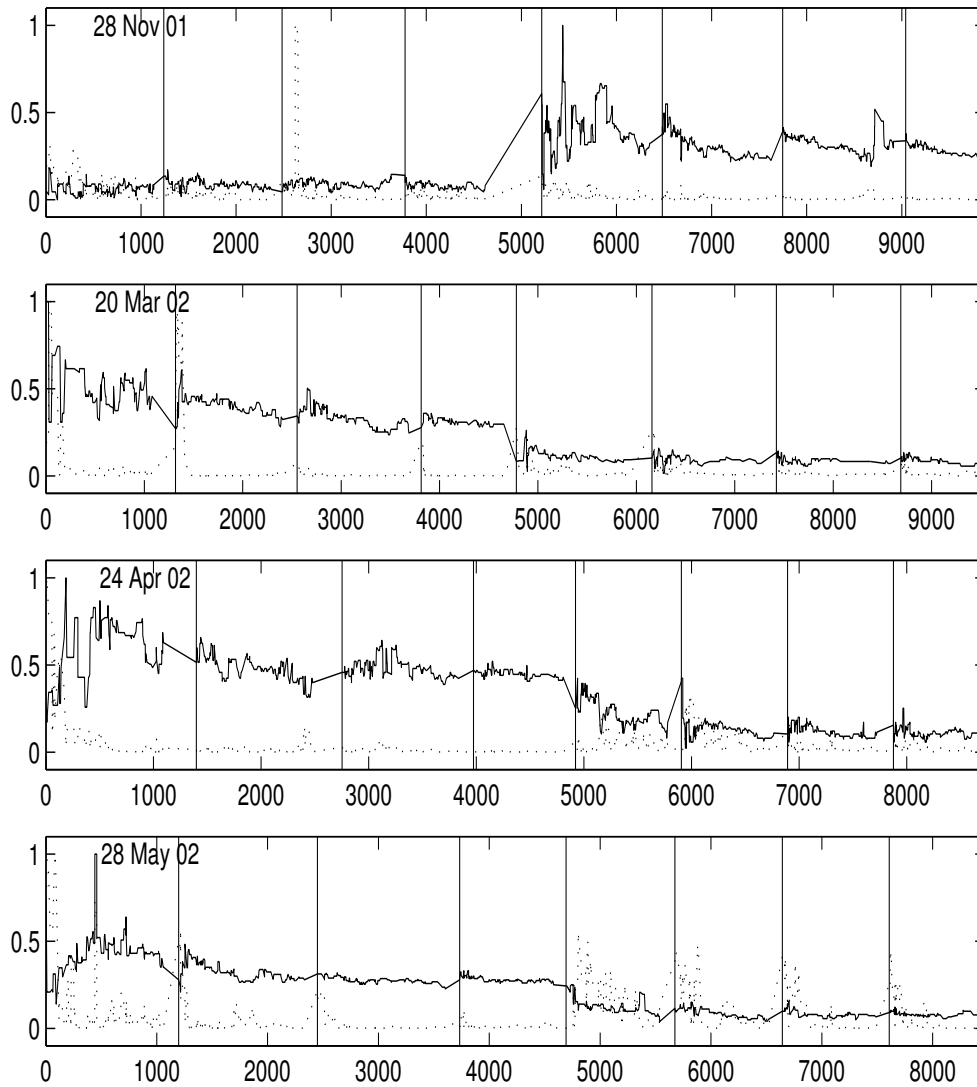


FIGURE 2.— Evolution of (i) distance of the RAWE (weighted average holding) portfolio from (mean-variance) optimality [dotted line; distance based on Sharpe ratios]; (ii) distance of prices from Walrasian equilibrium [solid line; distance based on the value of the average endowment portfolio]. Differences are scaled so that maximum difference in an experiment = 1. Time (in seconds) on horizontal axis; difference on vertical axis. Vertical lines delineate periods.