

Asset Pricing and Asymmetric Reasoning*

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ABSTRACT

We present a new theory of asset pricing and portfolio choices under asymmetric reasoning, contrast the predictions with those under asymmetric information, and present experimental evidence in favor of our theory. The Efficient Markets Hypothesis and its formal foundation, the Rational Expectations Equilibrium, predict that asymmetric information is irrelevant because prices correctly aggregate all available information. We argue here that asymmetric reasoning is fundamentally different: prices may not reflect all (types of) reasoning because (some) agents who observe prices that cannot be reconciled with their reasoning drop their reasoning while not giving prices the benefit of the doubt, and hence become sufficiently *ambiguity averse* so that they no longer directly influence prices.

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We present the results from an experiment, where, through manipulation of aggregate risk, we separately test the price and choice implications of our theory. Consistent with our theory, we find that i) a significant fraction of our subjects become price-insensitive, that ii) mispricing decreases as the fraction of price-sensitive agents increases when there is no aggregate risk, and iii) price-insensitive agents tend to trade to more balanced portfolios when there is aggregate risk.

JEL Classification: G11, G12, G14

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I. Introduction

The Efficient Markets Hypothesis (EMH) posits that asymmetry of information does not matter for asset prices because the action of the market properly aggregates individual information. It might be tempting to think that asymmetry of reasoning should not matter for prices either because the action of the market would again properly aggregate individual reasoning. However, the logic of EMH – and especially the logic of formalizations of the Efficient Markets Hypothesis such as Rational Expectations Equilibrium – requires that all agents know their own information, that they know what others may know, and that they know how their information and that of others will be reflected in prices. As a consequence, agents may have different information, but they must reason symmetrically. As such, the logic of EMH cannot possibly apply to asymmetric reasoning.

This paper explores the extent to which asymmetric reasoning is reflected in asset prices and in the individual choices that support those prices. To do this we conduct a series of laboratory experiments in which agents trade assets whose payoffs depend (in a known way) on the (laboratory) state of the world. The prior probability distribution over states of the world and a public signal about this distribution are available to all investors. If all investors reasoned correctly, they would all use Bayes' Rule to infer the true posterior distribution over states of the world, and hence the true distribution of asset payoffs. However, we create an environment in which application of Bayes' Rule is difficult and some investors *do not* apply it correctly; their reasoning is *incorrect*.¹ In such an environment, the question we ask is whether and to what extent the correct/incorrect reasoning of various investors is reflected in asset prices. Do prices reflect both correct and incorrect reasoning?

Our experimental findings – supported by a simple reduced-form theoretical model – demonstrate how prices may not reflect the opinions of investors who reason incorrectly. The observed portfolio choices of investors and our theoretical model suggest an explanation: confronted with prices that are at odds with their view of the world, investors who do not reason correctly

¹That many individuals make errors in Bayesian updating – even in environments much simpler than the ones we construct – has been confirmed in numerous experiments; see for instance Kahneman and Tversky (1973); Grether (1992); El-Gamal and Grether (1995); Holt and Smith (2009).

suspect that they may be wrong, and therefore view the financial prospects as sufficiently *ambiguous*² – rather than simply *risky* – so that they choose not to be exposed to the perceived ambiguity; as a result, their incorrect reasoning is not reflected in prices. Ambiguity is sensed when one is not sure about the true probabilities; risk, on the other hand, is a situation where one knows the probabilities objectively (Ellsberg, 1961).

Ambiguity aversion matters for pricing because the behavioral consequences of ambiguity aversion may be quite different from the behavioral consequences of risk aversion. In particular, ambiguity aversion may lead some investors to avoid ambiguity altogether by choosing portfolios whose payoffs are constant across the ambiguous states (at least for a wide range of prices). Such investors make choices that are *independent* of prices. If this is the case then asset prices for securities whose payoffs are *not* constant across the ambiguous states will be determined by investors who are *not* ambiguity averse and who make choices that are *dependent* on prices. (It is important to note that risk aversion will almost never lead an investor to choose a riskless portfolio, so that the choices of investors who are simply risk averse *will* be reflected in prices.) This is precisely the experimental finding of Bossaerts et al. (2010).³

Ambiguity aversion, like risk aversion, is thought of as an immutable characteristic of individuals, perhaps even genetically pre-determined (Kuhnen and Chiao, 2009; Cronqvist and Thaler, 2004), but the perception of ambiguity may arise depending on context. In the experimental setting of Bossaerts et al. (2010), ambiguity is exogenous: investors are not told the true probability distribution over states of nature. Here, ambiguity may emerge endogenously, because we present subjects-investors with difficult updating problems. A subject who has difficulty solving the problems could have his/her confidence undermined, and may perceive ambiguity if his own “solution” were very different from the “solution” that the market suggests.

²The familiar tradition that follows Savage (1954) posits that investors behave “as if” they assign complete subjective probabilities in every situation that involves uncertainty. However, the tradition that follows Knight (1939) and Ellsberg (1961) allows for the possibility that investors distinguish situations that involve known probabilities from situations that involve unknown probabilities, for which we use the term *ambiguous*.

³A *caveat* must be understood here. An agent who does not purchase a particular security does not *directly* affect the price of that security, but might *indirectly* affect the price because his/her holding of *other* securities affects supplies; hence the prices of *all* securities might be different from what they would be if that agent were entirely absent from the market. Bossaerts et al. (2010) provides an extended discussion.

Our experimental design is inspired by the well-known “Monty Hall” problem which captures a Bayesian updating problem that confounds many people.⁴ The updating problems we present are even more complicated. For instance, in one of our experimental designs, two Arrow securities are traded; the first pays if a card drawn at the end of trading is black, the second if it is red. Initially, the deck of cards contains two black cards (one spade and one club) and two red cards (one heart and one diamond). We announce that we will suspend trading midway through a trading period, draw a card, reveal and discard it – but we *never reveal and discard a heart*. After this, trade resumes until the (exogenous) termination time, at which point one of the remaining cards is drawn, the color of this card determines assets payoffs, and payments are made. Note that this presents quite a complicated Bayesian updating problem. Indeed, it requires careful calculation (that may amuse the reader) to determine the prior probability before trading begins that the card drawn at the end of trading will be black (hint: it is strictly greater than 0.5), or the posterior probability after a card is drawn, revealed and discarded.

In this setting, we hypothesize that some investors lack confidence in their reasoning – their ability to update correctly. We posit that this lack of confidence is reinforced if such investors are confronted with prices that appear at odds with their (incorrectly) updated beliefs. Such investors may then view asset payoffs as ambiguous, rather than risky. If such investors are also ambiguity averse, this will lead them to choose an unambiguous portfolio independently of the prices of these securities. That is, they will choose a portfolio with no exposure to ambiguity no matter what the prices are. Because they are *price-insensitive*, these investors will not contribute directly to the determination of the security prices.⁵ In contrast, agents who are

⁴Monty Hall was the host of a popular weekly television show (aired in the 60’s) called “Let’s Make a Deal.” In one portion of the show, Monty would present the contestant with three doors, one of which concealed a prize. Monty would ask the contestant to pick a door; after which Monty – who *knew which door concealed the prize* – would open one of the two remaining doors, never revealing the prize. Monty would then offer the contestant the opportunity to switch to the other unopened door. Updating correctly demonstrates that the probability that the original door conceals the prize is $1/3$ – as it was initially – so that switching dramatically increases the probability of success. However, many contestants – and others – update incorrectly and believe that the probability that the original door conceals the prize is $1/2$, so that switching makes no difference. For a detailed overview of the problem and its solution, see <http://mathforum.org/dr.math/faq/faq.monty.hall.html>.

⁵An alternative theory that would lead to the same individual behavior and would also stem from the comparative ignorance argument is that of Chew and Sagi (2008). A trader who doubts her Bayesian inference would prefer sources of uncertainty that do not depend on the Bayesian inference in question. The traders with such preferences for sources will choose portfolios that pay the same across all states with uncertain probabilities. The pricing implications of this behavior would be the same as under ambiguity aversion.

confident in their ability to update behave as if they know the true probabilities over outcomes, and hence regard asset payoffs as merely risky rather than ambiguous. That is, payoffs are perceived to follow a known distribution. Risk aversion affects the choices of such investors. Choices will be sensitive to prices. For example, they will not choose riskless portfolios unless prices are consistent with risk neutrality. Because of their sensitivity to prices, these investors *do* directly contribute to the determination of the security prices.

Of course some agents who are confident in their updating ability may nevertheless be wrong; hence we do not expect prices to conform perfectly to theoretical predictions under the assumption that all investors update correctly. We shall assume that only agents with beliefs that are sufficiently close to those reflected in prices remain confident.

It is an empirical issue to what extent dissonance between prices in the (financial) marketplace and an agent's own beliefs triggers the feeling of ambiguity. With problems like Monty Hall's, this is not immediately clear, because subjects have been known to be notoriously un-receptive to the correct way of reasoning even after experience that should have revealed their mistake (Friedman, 1998). Our experiments demonstrate, however, that prices appear to make market participants doubt their own way of reasoning.

Likewise, it is an empirical issue whether subjects who reason correctly are not swayed by evidence (here: prices) that reveals different reasoning. Providing support for our assumption, Halevy (2007) finds that only 4% of agents who correctly reduce compound lotteries sense (and avoid) ambiguity in the Ellsberg experiment, whereas a full 95% of agents who incorrectly reduce compound lotteries exhibit this behavior.⁶

In order to obtain clearer predictions, we consider two environments both in the theory and in the experiments. In the first, there is no aggregate risk; in the second, there is aggregate risk. When there is no aggregate risk, and if investors all update correctly, equilibrium prices should be risk-neutral with respect to the true probabilities (independently of individual preferences). If investors do not all update correctly yet continue to react to prices then prices may be

⁶These data could be interpreted differently. Specifically, the causality could run from ambiguity aversion to inability to understand compound lotteries. We argue that subjects *became* ambiguity averse because they were given a difficult task (that of correctly assessing the risks of compound lotteries).

different from risk-neutral prices. Hence this treatment provides a convenient test of price predictions. However, it does not provide a convenient test of portfolio predictions, because it may be impossible to distinguish between investors who update correctly and investors who are extremely ambiguity averse: if prices equal expected payoffs at correct probabilities, then both choose portfolios that are devoid of risk. When there is aggregate risk, equilibrium prices depend on the way individuals update *and* on individual preferences, so we cannot readily test equilibrium price predictions. However, the presence of aggregate risk provides a useful test of *individual* behavior: we predict that investors who update correctly choose to hold some aggregate risk (provided prices are not risk neutral), while investors who are ambiguity averse refrain from holding any risk at all regardless of prices.

Our central predictions are: i) if there is no aggregate risk, prices should be closer to expected payoffs based on the correct Bayesian inference as the proportion of subjects who cannot make the correct Bayesian inferences decreases; the proportion of price-insensitive subjects provides a lower bound to the latter number; ii) subjects who cannot update correctly should hold ambiguity-neutral portfolios (in our setting, these correspond to balanced portfolios), while agents who can update should hold increasingly diverging portfolios as the aggregate risk in the economy grows. These predictions are fully born out in the data. Notably, our experimental data suggest that relatively few subjects solve the updating problems correctly and that many of these subjects treat the situation as ambiguous, rather than risky.

Our results shed light on recent experimental findings of Kluger and Wyatt (2004) who also used a design suggested by the Monty Hall problem. Kluger and Wyatt found that if at least two among the six subjects in their experimental market updated correctly, then prices agreed with theoretical predictions. The authors explain this finding as resulting from Bertrand competition among those who update correctly. It seems to us that this explanation begs the question: surely subjects who update *incorrectly* Bertrand compete as well?⁷ And if subjects who update incorrectly Bertrand compete, why wouldn't this competition lead to the

⁷Theoretically, irrational traders will be bankrupt in the long run but as shown by Kogan et al. (2006), they can survive for a very long time before their wealth is brought down to zero. Moreover, Kogan et. al. show that if they act as expected utility maximizers under their subjective beliefs, "irrational traders can maintain a persistent influence on prices even after they have lost most of their wealth," (p. 220).

wrong prices? We provide an alternative explanation: those who cannot compute the right probabilities perceive ambiguity, and, as a result, become infra-marginal.

Our results also elucidate the scope of relevance of experiments for finance. Our experiments provide a microcosm of field markets, and in particular, are populated – as are field markets – with subjects who exhibit flawed reasoning. But the microcosm is not an identical replica, because the pool of subjects from which we draw is not identical with the pool of investors in field markets. In fact, we find strong *cohort effects* in our experiments: the number of subjects who do update correctly, and, as we shall see, the extent to which observed prices conform to theoretical predictions, depend critically on the student pool from which our subjects are drawn. Because of this, our experiments provide little direct information about “mispricing” in field markets. Our experiments are relevant, however, because they provide a link between flawed reasoning and equilibrium asset pricing – through the perception of ambiguity.

The possibility of incorrect reasoning is a cornerstone idea of Behavioral Finance (which usually uses terms like “cognitive bias” instead). However, most models in Behavioral Finance rely on a representative agent framework; see for instance Barberis et al. (1998); Daniel et al. (1998, 2001); Barberis et al. (2001); Rabin and Vayanos (2009). If we view the representative agent in these models as a single agent or as a number of identical agents (specifically, with the same beliefs), then reasoning might be incorrect but, since there is a single agent, reasoning cannot be asymmetric. If we view the representative agent in these models as the aggregate of non-identical agents then we seem to assume the conclusion: that the action of the market aggregates reasoning in a particular way and that this particular way leads to price biases. We argue that such aggregation is not a foregone conclusion, and our experimental evidence confirms this.

In proposing ways to model disagreement between rational agents, the literature has tended to go for polar extremes. Many papers (e.g., Williams (1977); Harris and Raviv (1993); Scheinkman and Xiong (2003); Basak (2005); Cao and Ou-Yang (2009); Hong and Sraer (2011)) assume that agents stick to their beliefs regardless of disagreement. But two rational agents should realize that at least one of them is wrong if they disagree. As a result, game theory has traditionally rejected the notion that agents can “agree to disagree” (Aumann, 1976) and has

taken the opposite position, namely, that all differences in beliefs must eventually be traced to differences in information – this is the “common prior assumption” of Harsanyi (1967).⁸

Few papers have taken the middle ground between blunt “agreeing to disagree” and accepting only differences in opinion that can be attributed to information. Anderson and Sonnenschein (1985) present a model where beliefs are formed by combining models with data. If agents have access to the same data, their beliefs are allowed to deviate only to the extent that their models differ. Kurz et al. (2005) assume that all agents have access to the same countably infinite, stationary time series, so that they can only disagree about the probabilities of non-ergodic events.

As far as we know, our study is the first to point out that even justifiable disagreement cannot be taken for granted when beliefs conflict. It may be all right to disagree with another agent, but it is questionable that agents disagree with the market no matter how far off their beliefs are. Basically, the only way one can be confident about one’s beliefs is if it withstands repeated confirmation of their veracity (through, e.g., reiteration of the arguments on which they are based), which will only occur if they are correct.

Nevertheless, we do reject the idea that doubt in the face of dissonant market prices would lead agents to follow the market (whose opinions are sometimes described as the “wisdom of the crowd,” see Arrow et al. (2008)). Instead, we posit that agents “throw up their arms” and decide to avoid the situation altogether.

The remainder of this paper is organized as follows. Section II presents the theory and the empirical implications. Section III describes our experiments in detail. Section IV presents the empirical results. Finally, Section V concludes.

⁸Attempts to solve games differently (Biais and Bossaerts, 1998) have had little following.

II. Theory and Empirical Implications

In this section we present a simple asset market model that unfolds over two dates: trade takes place only at date 0; consumption takes place only at date 1. There is a single consumption good.

Let there be a continuum of agents uniformly distributed on the interval $[0, 1]$ and indexed by i , two assets R and B , or Red and Black stock, and two states of the world, r and b . At date 0 the realization of the state is not known to the agents. At date 1 agents learn the realization of the state, securities pay off, and consumption takes place. The two assets are Arrow securities: In state $j \in \{r, b\}$, asset $J \in \{R, B\}$ pays one unit of wealth, and the other asset pays no wealth.

Let π_r be the probability that state r occurs, and $\pi_b = 1 - \pi_r$ the probability that state b occurs (note that π_j is equal to the expected value of asset J). Public information is available with which to compute π_j . Agents use different ways to obtain this number based on the publicly available information. As a result, beliefs about π_j may not agree. Let π_j^i be the subjective probability that state j occurs, as calculated by agent i .

We assume that a proportion α of all agents use the correct reasoning, i.e., can compute the true probability. Without loss of generality, we assume that these are the agents with highest index i : $\pi_r^i = \pi_r$ for $i \in [1 - \alpha, 1]$. The rest of the agents $i \in [0, 1 - \alpha)$ compute the probability of state r incorrectly. Their mistake is proportional to the value of their index i , and hence, all mistaken beliefs are below the correct belief: agent i has a subjective probability

$$\pi_r^i = \underline{\pi} + \frac{\pi_r - \underline{\pi}}{1 - \alpha} i,$$

where $\underline{\pi}$ is the minimal possible belief. The belief schedule is depicted in Figure 1.

Note that we have chosen the true probability to be on the boundary of the belief space, thus creating a setup where the agents with wrong beliefs have the strongest potential to influence asset prices. We could have assumed that mistaken beliefs are *above* correct beliefs, but this does not change the conclusions qualitatively. If correct beliefs are somehow *in the*

middle, then wrong beliefs above and below it would cancel, and hence, wrong beliefs would not have as much an impact.

As to the total supplies of the assets R and B , we consider two treatments. In Treatment I, the aggregate endowment of assets R and B is the same, so there is no aggregate risk in the economy. In Treatment II, there is aggregate risk because supplies of R and B are different.

In Treatment I, at date 0 each agent is endowed with an equal number of R and of B , without loss of generality, one unit of each. One can think of each agent as the aggregation of heterogeneously endowed agents who share the same beliefs π_r^i . In Treatment II, there are more units of B than of R . In the experiments, for each unit of R , the aggregate agent holds 1.16 units of B .

Let w_i be the wealth of agent i at date 1, after the state of the world is revealed. For simplicity, assume a logarithmic form for the utility that agent i derives from final wealth, i.e., $u(w_i) = \ln(w_i)$. The form of the utility function only matters for agents who do not become ambiguity averse as a result of dissonance between their own beliefs and beliefs as reflected in market prices. Ambiguity averse agents will choose allocations irrespective of their utility function. This also means that there can be quite a bit of heterogeneity (in terms of utilities) across ambiguity averse agents.

Agents may trade their endowments at date 0. Let p_R be the market prices of asset R at date 0. Absence of arbitrage dictates that the price of asset B must be $p_B = 1 - p_R$. Let (B_i, R_i) be the holdings of Black and Red securities after agents i trades.

We first derive the equilibrium choices and prices assuming that all agents maximize expected utility with their own subjective beliefs, undisturbed by prices that may suggest that their beliefs are wrong. That is, the equilibrium assumes that agents “agree to disagree.” Subsequently, we determine what happens when agents with incorrect reasoning face prices that challenge their beliefs.

Agreeing To Disagree

In Treatment I, the initial wealth of i is $w_i^0 = p_R 1 + (1 - p_R) 1 = 1$, so her optimization problem is

$$\max_{R_i, B_i} \pi_r^i \ln(R_i) + (1 - \pi_r^i) \ln(B_i) \quad s.t. \quad p_R R_i + (1 - p_R) B_i = 1.$$

The first order conditions for optimality imply that

$$R_i = \frac{\pi_r^i}{p_R}, \quad B_i = \frac{1 - \pi_r^i}{1 - p_R}.$$

Hence for any given price vector, the relative demand of agent i for asset R (as a fraction of the total demand for assets R and B) is increasing in the subjective probability π_r^i . The vector of all subjective probabilities by all agents determines the equilibrium prices.

If all agents correctly compute the true probability of state j , i.e., if $\alpha=1$, in the absence of aggregate uncertainty all agents trade so as to attain a balanced portfolio of 1 unit of each security. This ensures equilibrium, and from the above first-order conditions, the equilibrium prices can be deduced to be: $p_R = \pi_r$ and $p_B = 1 - \pi_r$.

If instead $\alpha < 1$ and if all agents maximize expected utility based on their subjective beliefs then the equilibrium prices will aggregate the beliefs of all agents. Since the correct belief about the red state are the most optimistic, it follows that the equilibrium price of the Red security will be below the correct belief, $p_R < \pi_r$. As a result, $p_B > \pi_b$.

In Treatment II, there is aggregate risk because the Red asset R is scarcer than the Black asset B . If all agents correctly compute the true probabilities, the equilibrium price of Red will be above the probability of the red state (and hence, the expected payoff on the Red security), so that demand for Red is dampened: $p_R > \pi_r$. The resulting ratio $\frac{p_R}{p_B}$ will be higher than the odds ratio of the two states $\frac{\pi_r}{\pi_b}$. If instead $\alpha < 1$, the equilibrium prices aggregate the beliefs of all agents, causing p_R to be lower than it would be if all agents updated correctly. The exact equilibrium price that is determined by the magnitude $1 - \alpha$, the relative scarcity of Red and the risk attitudes of the agents (represented here by logarithmic utility).

Disagreement Leads To Doubt And Ambiguity Aversion

The above equilibrium assumes that, when confronted with prices that contradict their computations, agents continue to use their subjective beliefs in determining optimal demands. In what follows we relax this very assumption and hypothesize that from the agents who do not hold the correct belief π_r , only those whose beliefs are within ϵ of the market price continue to use their subjective probabilities. Each of the rest of the agents, confronted with the dissonance between the market price and her subjective probability, realizes that she may have made a mistake and computed the wrong probabilities. We conjecture that in these circumstances agents perceive ambiguity because they no longer trust their own computations, while doubting the belief reflected in prices (because they cannot manage to justify these beliefs either). As a result, rather than agreeing to disagree with the market, the agents become ambiguity averse.

We make the technical assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$. This puts an upper bound on the fraction of agents with incorrect beliefs who nevertheless agree to disagree (with the price). This assumption ensures that a strictly positive fraction of agents become price-insensitive in equilibrium. Otherwise we would be back in the above equilibrium, where *all* agents “agree to disagree.”

We model choice under ambiguity aversion using the maxmin decision rule of Gilboa and Schmeidler (1989).⁹ An agent with maxmin preferences maximizes the following expression:

$$U_i(R_i, B_i) = \min\{u(R_i), u(B_i)\}.$$

If $R_i > B_i$, then $U_i(R_i, B_i) = u(B_i)$. Similarly, if $R_i < B_i$, then $U_i(R_i, B_i) = u(R_i)$. From here it immediately follows that an agents with maxmin preferences will seek a portfolio with

⁹Alternatively, one could use the theory in Ghirardato et al. (2004) to model the behavior of agents who face ambiguity. The derived representation is α – maxmin utility function $U_i(R_i, B_i) = \alpha \min\{u(R_i), u(B_i)\} + (1 - \alpha) \max\{u(R_i), u(B_i)\}$, where the coefficient α measures the degree of ambiguity aversion (not to be confused with α in this paper, which measures the fraction of agents who correctly compute probabilities). $\alpha = 1/2$ corresponds to ambiguity neutrality, and $\alpha = 1$ is the extreme degree of ambiguity aversion as in Gilboa and Schmeidler (1989). Bossaerts et al. (2010) uses the α – maxmin for their theoretical analysis. There are other types of preferences that generate ambiguity aversion. Yet the empirical findings in this paper, and those in Bossaerts et al. (2010), clearly demonstrate that a significant fraction of subjects are price insensitive, while their choices reveal that they avoid ambiguity. As such, the data favor a model of kinked preferences, of which the (α) -maxmin is an important member.

$R_i = B_i$ under *any* prices p_R and $p_B = 1 - p_R$. Notably, this does not depend on the shape of the utility function u (as mentioned before). Ambiguity averse agents can even exhibit different preferences; their eventual choices will remain the same: they will seek balanced portfolios (equal number of R and B securities).

Importantly, demands of ambiguity averse agents do not depend on relative prices either. As a result, ambiguity averse agents are price insensitive.

In contrast, knowledgeable agents continue to submit optimal demands based on their own beliefs. With a fraction α of such agents, their demands for R add up to $q_\alpha = \int_{1-\alpha}^1 R_i = \alpha \frac{\pi_r}{p_R}$. Thus, for any $p_R < \pi_r$ the knowledgeable agents create a total excess demand equal to $\alpha(\frac{\pi_r}{p_R} - 1)$. Notice that knowledgeable agents *are* price sensitive.

There is a third category of agents, namely those with incorrect reasoning but whose beliefs are within ϵ of the market price. We assume that these agents continue to optimize given their beliefs. Because $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\alpha \leq 1$, it follows $\epsilon < \frac{\pi_r - \underline{\pi}}{2}$.

Consider Treatment I and conjecture that prices will be sufficiently close to correct beliefs (we will verify later that this is true in equilibrium) such that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$. Let \underline{i} be the agent such that $\pi_r^{\underline{i}} = p_R - \epsilon$, that is, $\underline{i} = \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \epsilon - \underline{\pi})$. Therefore, the total excess demand generated by agents with (incorrect) beliefs sufficiently close to π_r equals:

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p} - 1 \right) di.$$

In the Appendix (A.3), we show that, since $\pi_r^i = \underline{\pi} + i \frac{\pi_r - \underline{\pi}}{1-\alpha}$,

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1 \right) di = \frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon).$$

Because $(\pi_r - p_R - \epsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

In equilibrium the aggregate excess demand must be zero:

$$\frac{1 - \alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon) + \alpha\left(\frac{\pi_r}{p_R} - 1\right) = 0.$$

In the Appendix (A.4), we prove that prices satisfy the following restriction.

Proposition 1. *The equilibrium price of the Red security is*

$$p_R = \pi_r - \left(\sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{1 - \alpha} \right)^2 + \epsilon^2} - \frac{\alpha(\pi_r - \underline{\pi})}{1 - \alpha} \right).$$

Notice that equilibrium prices satisfy the conjectured restriction used to derive total excess demands of agents with incorrect beliefs within ϵ of the market price, namely, $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$. See Figure 4 for a depiction of the equilibrium.

The main comparative statics properties of equilibrium prices in Treatment I are as follows. Define $|\pi_r - p_R|$ to be the mispricing in the marketplace.

Corollary 1. *Mispricing decreases in α and increases in ϵ .*

Corollary 1 states that mispricing decreases with the fraction of agents who know how to correctly compute probabilities, and increases as more agents with incorrect beliefs “agree to disagree” even if their beliefs are more distant from the market price.

Neither the fraction α nor the critical distance ϵ are directly observable, however. We need an empirically more relevant statement, which is the following. Let S denote the fraction agents who remain price sensitive (either because they know how to correctly compute probabilities or their beliefs are within ϵ of the correct beliefs). S equals:

$$S = \alpha + (\epsilon + \pi_r - p_R) \frac{1 - \alpha}{\pi_r - \underline{\pi}}.$$

Note that S is readily measurable: it suffices to identify which agents change their choices as prices change.

Corollary 2. *Mispricing decreases as S increases.*

Both Corollaries 1 and 2 are proved in the Appendix (A.5).

We check that our results are robust if ϵ varies across agents. Consider the simplest case, where there are two types of agents, one that become ambiguity averse only if their (subjective) probability is more than ϵ away from market prices, and another that become ambiguity averse only if their probability is more than $\delta(\leq \epsilon)$ away. As proved in the Appendix (A.6), mispricing continues to decrease with the fraction of agents α who know how to correctly compute probabilities. Moreover, mispricing decreases as the number of price-sensitive agents increases. Thus, both Corollaries 1 and 2 continue to hold when the individual price-sensitivity cutoff points are heterogeneous across agents.¹⁰

In Treatment II, equilibrium choices of price-sensitive agents (those with correct beliefs, and those with incorrect beliefs that are nevertheless within ϵ of the truth) are affected by the unbalanced supply of Red and Black securities. Price-insensitive agents (those with incorrect beliefs more than ϵ away from the truth) are ambiguity averse and choose to hold balanced portfolios (equal amounts of Red and Black securities), so only price-sensitive agents are willing to accommodate the imbalance in supply of Red and Black securities. The exact equilibrium prices depend on the relative supplies of Red and Black securities, and on the number and risk preferences of price-sensitive agents. We took the latter to be logarithmic, but any other (risk averse) preference would do in order to generate the asserted portfolio effects (the choices of price-insensitive agents do not depend on the posited utility function, as mentioned before). Whatever equilibrium prices one ends up with, the price-sensitive agents collectively absorb all the risk, acquiring more units of Black asset than of Red asset. This implies that the individual equilibrium portfolio holdings of the price-sensitive agents are (generically) unbalanced.

In Treatment II, pricing predictions are ambivalent because prices are affected not only by beliefs, but also by the imbalance in relative supplies of Red and Black securities, the exact number of price-sensitive agents, and their risk aversion. Equilibrium holding predictions

¹⁰One could enrich the theory even further and allow heterogeneity in perceived ambiguity. The idea would be to let the amount of perceived ambiguity increase with cognitive dissonance, while assuming that ambiguity aversion (once it emerges) remains the same across agents. Perceived ambiguity can be reduced by limiting the set of probabilities that the agent considers possible. In this version of the theory, all agents up to a certain level (of cognitive dissonance) would still remain price sensitive, though. Those beyond that level would become entirely price insensitive. Since the pricing and allocation predictions depend solely on dichotomous price sensitivity, the empirical implications of this richer theory would remain the same.

are, however, unequivocal: price-sensitive agents choose to invest in unbalanced portfolios, while price-insensitive agents buy balanced portfolios (with an equal number of Red and Black securities).

Thus, our theory has three main testable predictions:

Hypothesis 1. *There are price-insensitive subjects, i.e., subjects whose choices do not change as (relative) prices change.*

Hypothesis 2. *In Treatment I, mispricing $|\pi_r - p_R|$ decreases with the fraction of price-sensitive subjects, S .*

Hypothesis 3. *In Treatment II, price-insensitive subjects hold more balanced portfolios than price-sensitive subjects.*

III. Experiments

Sessions Overview

We ran nine experimental sessions in total, six corresponding to Treatment I (involving markets with no aggregate risk), and three corresponding to Treatment II (with aggregate risk). The participants were undergraduate and graduate students from the following universities: (i) Caltech (one session), (ii) UCLA (four sessions, including the three Treatment II sessions), (iii) University of Utah (two sessions), (iv) simultaneously at Caltech and University of Utah with equal participation from both subject pools (two sessions). Subjects received a sign-up reward of \$5, which was theirs to keep no matter what happened in the experiment.¹¹ The average earnings from participating in the experimental sessions was \$49 per subject, for an experiment that generally lasted 2 hours in total.

Twenty subjects participated in each session. This is sufficient for markets to be liquid enough that the bid-ask spread is at most two or three ticks (the tick size was set at 1 U.S. cent). All accounting in the experimental sessions was done in US dollars.

¹¹This sign-up compensation is compulsory at the experimental laboratories where we ran our experiments (Caltech's SSEL, UCLA's CASSEL and the University of Utah's UULEEF).

Upon arrival at the experimental laboratory, subjects sat in front of computer terminals and received a set of written instructions. The sessions began by the experimenter reading aloud the instructions, while also projecting them on a large screen. During the instruction period, if subjects had any questions, they were asked to raise their hands, and one of the experimenter would answer privately. No oral communication between subjects was allowed. They communicated their decisions via the computer terminals.

Markets

For the purpose of the experiment, we created three securities. Two of them were risky and one was risk free. The risk-free security, or *Note*, always paid \$0.50. The two risky securities were referred to as *Red Stock* and *Black Stock*. The liquidation value of Red Stock and Black Stock was either \$0.50 or \$0. Red and Black Stock were complementary securities: when Red Stock paid \$0.50, Black Stock paid nothing, and *vice versa*. Red Stock paid \$0.50 when the “last card” (to be specified below) in a simple card game was red (hearts or diamonds); Black Stock paid \$0.50 when this “last card” was black (spades or clubs). Thus, instead of being explicitly provided with the probability distributions of the securities’ payoffs, the subjects were presented with the description of card game *situations* that determined those probabilities. The experimental sessions were organized as a sequence of eight independent replications of four different situations; each situation was replicated exactly twice.

Except for the presentation of payoff probabilities, the session setup was that of a standard experimental market.¹² Trade took place through a web-based, electronic continuous open-book system called *jMarkets*.¹³ A snap shot of the trading screen is provided in Figure 2.

Within each replication, subjects were initially endowed with Red (R) and Black Stock (B), Notes, as well as cash. Subjects were given an unequal supply of the two securities; some subjects started with 12 units of R and 3 of B ; while others started with 9 units of B and no units of R (but with more cash). In the six sessions corresponding to Treatment I,

¹²See, for example, Bossaerts et al. (2010) and Meloso et al..

¹³This open-source trading platform was developed at Caltech and is freely available under the GNU license. See <http://jmarkets.ssel.caltech.edu/>. The trading interface is simple and intuitive. It avoids jargon such as “book,” “bid,” “ask,” etc. The entire trading process is point-and-click. That is, subjects do not enter numbers (quantities, prices); instead, they merely point and click to submit orders, to trade, or to cancel orders.

the aggregate endowment of R and B was equal, and hence, there was no aggregate risk in the marketplace (because the payments on Red and Black Stock were complementary). In the three sessions corresponding to Treatment II, there were more subjects with endowments tilted towards B than subjects with endowments tilted towards R , so in the aggregate there were fewer units of Red Stock than Black Stock, and hence, there was aggregate risk. Table I provides additional details of the experimental design.¹⁴

While subjects could freely trade Red Stock and Notes, they were barred from trading Black Stock. This is an important experimental design feature. In Treatment I, where there was no aggregate risk, equilibrium allocations may be indeterminate if all securities could be traded freely. First, ambiguity averse agents would like to hold balanced portfolios no matter what the prices are. There are many such balanced portfolios: one of R (Red Stock) and B (Black Stock) each, two of each; three of each; etc. Second, price-sensitive agents would want to hold balanced portfolios as well when prices equal expected payoffs (i.e., carry no risk premium), which is possible in equilibrium because there is no aggregate risk. Again, there are many portfolios that are balanced. If the market for B is closed, however, the only way to obtain balanced portfolios is to trade to positions in R that match one's holdings of B .

We consider it an important feature of our design that subjects had a reason to trade besides “agreeing to disagree” (effectively speculating that one's reasoning is better than that of others). Specifically, subjects were initially given an unequal supply of the two securities even in Treatment I. Because subjects are generally risk averse even for the relatively low levels of risk in our experiment (Holt and Laury, 2002; Bossaerts and Zame, 2006), there are gains from trading to more balanced positions. To put this differently: we would have seen trade even if all subjects agreed on how to correctly compute probabilities.

Because of the presence of cash, the Note was a redundant security. Subjects were allowed to short sell the Note if they wished. Short sales of Notes correspond to borrowing. Subjects could exploit such short sales to acquire R if they thought R was underpriced.

¹⁴The Instructions for the experiment are provided in Appendix C. More information about the experimental design can be obtained at http://leef.business.utah.edu/market_mh/frames_mh.html.

Subjects were also allowed to short sell R , in case they thought R was overpriced. To avoid bankruptcy (and in accordance with classical general equilibrium theory), our trading software constantly checked subjects' budget constraints. In particular, subjects could not submit an order such that, if it and the subject's other standing orders were to go through, the subject would generate net negative earnings in at least one state. Only orders that were within 20% of the best standing bid or ask in the marketplace were taken into account for the bankruptcy checks. Since markets were invariably thick, orders outside this 20% band were effectively non-executable, and hence, deemed irrelevant. The bankruptcy checks were effective: no agent ever ended up with negative earnings in our experiments.

The Game

Each replication of a Card Game consisted of two periods. Thus, in total there were sixteen periods (numbered from 0 to 15) per experimental session. The (end-of-replication) liquidation values of Red Stock and Black stock were determined through card games played by a computer and communicated to the subjects orally and through the News web page; see Figure 6 for a snapshot of the page (the table on the page is filled gradually as information comes in). The card games were inspired by the Monty Hall problem. One game (out of the four that we used) is as follows. The computer starts a new replication with four cards (one spades, one clubs, one diamonds, and one hearts), randomly shuffled, and face down. Cards will be eliminated during the game. The color of the *last card* to remain determines the payoffs of the two risky securities. Trade starts for one 3-minute period. Upon the conclusion of this first period, trading is halted. At that point, the computer discards one card without revealing it to the subjects. Then the computer picks one card from the three remaining cards, as follows. If the discarded card was hearts, the computer picks one card at random from the three remaining cards. If the hearts card is in the three remaining cards, the computer picks randomly from the other two (non-heart) cards. The card that was picked is then revealed to the subjects, both orally and through the News web page. Trade starts again, for another 3 minutes (period 2 of the card game replication). At the end of this second period, after markets close, the computer picks one of the two remaining cards at random. This *last card* is then revealed and determines which stock pays. If the last card is red (diamonds, hearts) then Red Stock (R)

pays \$0.50. If the last card is black, then Black Stock (B) pays \$0.50. Each subject's payoff is determined by his/her holdings at the end of the second trading period and by the color of the last card.

Four variations on this game (each replicated twice), referred to as *Card Game Situations*, were played. They differ in terms of the number of cards discarded and/or revealed after the first period, and the restriction on which cards would be revealed. This provided a rich set of equilibrium prices (in Treatment I, where prices could be identified uniquely because there was no aggregate risk) and changes of prices (or absence thereof) after the first period revelation. Table II provides details of the four card games.

Detailed information about the drawing of cards is in the set of experimental instructions provided to the subjects (see Appendix C). In addition, before each period, the experimenter reiterated the drawing rules to be applied in the coming period, to be sure subjects knew which card game applied.

The actual trading within the eight replications (two for each card game) lasted about one hour. It was preceded by a long (approximately one hour) instructional period and a practice trading session, followed by a short break (15 minutes). The purpose of the long instructional period and the trading practice session was to familiarize subjects with the setting and the trading platform. To determine to what extent subjects understood the instructions, the (oral) questionnaire included questions such as "In the game where the computer never reveals a red card after the first trading period, will you be surprised to see a black card revealed?" Or, "If the computer initially discards one card, and then shows one black card when it could have also shown diamonds, does the chance that the last card is black decrease as a result?"

During the trading periods, the News web page was projected on the screen at the times when cards were being discarded or drawn. For the rest of the time, the large screen was projecting the development of the order book as well as the transaction prices chart.

IV. Empirical Analysis

In this section we describe the data, report the number of price-insensitive subjects, assess the level of mispricing and how this correlates with the number of price-sensitive subjects, and then present a correlation study of portfolio choice and price sensitivity.

Raw Data

The data collected during the experimental sessions consists of all posted orders and cancellations for all subjects along with their transactions and the transaction prices for the Red Stock (R) and the Note. (Remember that subjects could not trade the Black Stock B .) Figure 3 displays the evolution of transaction prices for R in the six sessions for Treatment I (no aggregate risk). Time is on the horizontal axis (in seconds). Solid vertical lines delineate card game replications; dashed vertical lines indicate between-period pauses when the computer revealed one or two cards. Horizontal line segments indicate predicted price levels assuming prices equal expected payoffs computed with correct probabilities. Each star is a trade. Volume is huge: over 1,100 trades take place typically during an experimental session, or one transaction per 2.5 seconds.

Figures 3b and 3c display trading prices in experiments that represent two extremes. In Figure 3b (Utah-1), observed transaction prices appear to be rather unreactive to changes in true payoff probabilities. However, when subjects from the Caltech community are brought in (Utah-Caltech-1; half of the subjects are from Caltech, and half are from the University of Utah), prices are close to expected payoffs – reflecting correct probabilities. See Figure 3c. The comparison suggests that there might be strong cohort effects in our data.

In Utah-1 (Figure 3b), prices appear to be insensitive to variations in the card games. There were also a very large number of price-insensitive subjects (to be discussed later), indicating that the pricing we observe in that experiment may reflect an equilibrium with only ambiguity averse subjects: when there are only ambiguity averse subjects, equilibrium prices will not react to the information provided to subjects in the different card games, and *any* price level is an equilibrium. Notice that prices in Utah-1 indeed started out around the relatively arbitrary

level of \$0.45 and stayed there during the entire experiment. A notable exception is period 2 of replication 1, when it was certain that the last card would be red and hence that the Red Stock would pay, because the two revealed cards were black. Prices adjusted correctly, proving that subjects were paying attention and able to enter orders correctly, so that neither lack of understanding of the rules of the game or unfamiliarity with the trading interface can explain the information-insensitive pricing in the other periods.

The second column of Table III reports how far observed prices were on average from the true expected payoffs, stratified by experimental session and Card Game (in U.S. cents; average across transactions). The data reveal that there is a wide variability in mispricing, both across experimental sessions, Utah-1 producing the worst mispricing and Caltech-Utah-1 producing the best pricing, and across card games, with Card Game 2 producing larger mispricing than the other treatments. Formally, the median mispricing in Card Game 2 is significantly higher than that of Card Game 1 (p value of 0.047; Wilcoxon signed-rank test comparing the paired absolute mean mispricing across the two treatments), Card Game 3 ($p = 0.016$), and Card Game 4 ($p = 0.016$). The second column of Table IV reports the same for Treatment II.

Since in Treatment II there is aggregate risk, equilibrium prices differ from true expected payoffs even if all agents know how to compute correct probabilities because of the presence of a risk premium. When compared to the session from Treatment I with subjects from the same cohort (UCLA), average differences between transaction prices and correct expected payoffs (available from the authors) are indeed higher in all card games. This suggests that a risk premium affected prices, and hence, that subjects were risk averse. While expected in view of past experimentation with multi-security asset pricing (Bossaerts and Zame, 2006), it is nevertheless comforting to confirm risk aversion, because otherwise Hypothesis 3 (which stated that price-insensitive agents should hold more balanced portfolios than price-sensitive ones) would have been without empirical content since price-sensitive agents are assumed to be risk averse.

Hypothesis 1: Presence of Price-Insensitive Subjects

Column 3 of Table III reports the number of price-sensitive subjects in Treatment I. Price sensitivity is obtained from OLS projections of one-minute changes in a subject's holdings of Red Stock onto the difference between, (i) the mean traded price of Red Stock (during the one-minute interval), and (ii) the expected payoff of Red Stock computed using the correct probabilities.¹⁵ Agents who cannot update probabilities correctly from cards discarded and displayed are assumed to become ambiguity averse when their beliefs divert too much from market prices. Because they are ambiguity averse, they are price insensitive, which means that their choices produce a zero slope coefficient in the above regression. To be reacting rationally to price changes, all other agents should decrease holdings when prices increase, which means that the slope coefficients for these subjects ought to be negative.

The regression with which we determine price sensitivity suffers from a well-known simultaneous-equations bias, because total changes in holdings must balance out across subjects. Because slope coefficients must sum to zero, OLS estimates will be biased upward. See Appendix B for details. Because the bias is upward, we use a generous cut-off level to categorize our subjects as price sensitive. In particular, we use a cut-off of -1.65 for the t -statistic of the slope coefficient to determine that a subject is price sensitive. At the same time, we applied a conservative t -statistic level of 1.9 to categorize whether a subject is price sensitive in the other direction, namely, she increases holdings of a security when prices increase (rather perversely, as mentioned before). Subjects with t -statistics between -1.65 and 1.9 are deemed price-insensitive.

For Treatment I, Table III, third column, reveals that generally only a minority of subjects was price-sensitive and reacted to price changes in the right way (reducing holdings when prices increased). In some instances only *a single*, and in one occasion *no* subject was found to react

¹⁵We assume that agents who become ambiguity averse in a card game replication remain so throughout the entire replication. In principle, we could have enriched our analysis by considering the possibility that some learned halfway through the replication after we provided further information about the cards, in a way that could have changed their perception of ambiguity. Specifically, some subjects could have switched to or from ambiguity aversion between trading periods. Learning schemes that allow for such switches have been proposed, as in Epstein and Schneider (2007). We did not consider this refinement because we did not have sufficient data (prices; changes in portfolio allocations) to confidently determine price sensitivity, and hence, ambiguity aversion, over each trading period separately.

systematically and correctly to price changes. The flip side is that there were always price-insensitive subjects, and their number was at times substantial. This means that generally a large number of our subjects perceived ambiguity – suggesting that they did not know how to compute the right probabilities.

We do observe that a small fraction of subjects were price-sensitive in a perverse way: they tended to *increase* their holdings for increasing prices. See the fourth column of Table III. There are two possible explanations of this finding. First, these are just type II errors: the subjects at hand are really price insensitive, but sampling error causes the t -statistic to end up above 1.9. The second possibility is that we have identified subjects who are perversely price-sensitive. We could interpret their actions as reflecting *momentum trading* or *herding*: higher prices are interpreted as signaling higher future prices (momentum) or higher expected payoffs (herding). Our theory does not account for such trading behavior. Since we cannot determine which of the two possible explanations applies, we exclude subjects with t -statistics above 1.9 from the remainder of our analysis.¹⁶

The results on price sensitivity for Treatment II are reported in Table IV. The numbers (of price sensitive, both correctly and incorrectly) are similar to those for Treatment I. The relevant counterpart in Treatment I is the session UCLA, which drew participants from the same subject cohort.

Hypothesis 2: Mispricing and Number of Price Sensitive Subjects

Absent aggregate risk (i.e., in Treatment I), our theory predicts that mispricing (defined as the absolute difference between the market price and expected payoffs computed with correct probabilities) decreases with the number of price sensitive agents (Hypothesis 2).

The bottom of Table III reports two correlations (for Treatment I) when price sensitivity is measured in two ways: (i) counting all subjects whose reaction to price changes is significantly negative ($t < -1.65$); (ii) counting only those subjects whose reaction to price changes is

¹⁶In a previous version of the paper (available upon request from the authors) we presented our results in two parts: one where we include the entire subject pool and one where we exclude those with significantly positive slope coefficients ($t > 1.9$). None of our conclusions are affected qualitatively by the exclusion of perversely price-sensitive subjects.

significantly negative *and* whose average change in holdings is correct given initial holdings. Both correlations are significantly negative ($p < 0.01$), lending support to Hypothesis 2.

The bottom of Table IV reports the same correlations for Treatment II. Here, however, there is no specific reason for the correlations to be negative, because a risk premium may interfere with pricing. Mispricing as we defined it now not only captures incorrect computation of probabilities, but also risk aversion. While the correlations are negative, they are smaller, and insignificant ($p > 0.10$).

Hypothesis 3: Portfolio Choices Depend On Price Sensitivity

With Treatment II, we can test an implication of our theory for equilibrium portfolio holdings. Specifically, price-insensitive agents should be exposed to less risk than price-sensitive ones because price-insensitive agents become ambiguity averse and hence trade to balanced (risk-free) portfolios, while price-sensitive ones behave as in traditional asset pricing theory, and in equilibrium, will share the aggregate risk in the economy and choose risk-exposed portfolios.

A test of this hypothesis is important, because one alternative explanation for the empirical support for Hypothesis 2 is that price-insensitive subjects are simply noise traders, and hence, not necessarily ambiguity averse. The more noise traders in the market, the worse the conformity of observed prices with theoretical predictions. So, if true, the presence of noise traders would make the data look as if they are consistent with our theory, and Hypothesis 2 in particular. Support for Hypothesis 3, however, would speak against this explanation, because the hypothesis requires that price-insensitive agents do not make arbitrary choices, unlike noise traders.

We compute individual imbalances separately after the first and the second trading periods in each replication. Portfolio imbalance is defined to be the absolute difference between the number of Red Stock and Black Stock a subject is holding. To reduce the impact from sampling error in the classification of a subject as price (in)sensitive (based on the estimated slope

coefficient in the regression of holding changes onto price changes), we implement the following two-level estimation approach:

$$\begin{aligned}
I_i &= a + b_{between}T_i + \epsilon_i, \text{ where} \\
I_{ij}^p &= I_i + \eta_{ij}, \\
T_{ij} &= T_i + \xi_{ij}, \text{ and} \\
\eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij},
\end{aligned} \tag{1}$$

Here I_{ij}^p denotes subject i 's imbalance in Card Game j ($j = 1, \dots, 4$) after trading period p ($p=1,2$). T_{ij} denotes subject i 's t -statistic of the slope coefficient in the price-sensitivity regression for Card Game j and δ_{ij} , ξ_{ij} , η_{ij} , and ϵ_i are (assumed) normally distributed independent errors. I_i is the mean imbalance of agent i across all card game situations, and we have allowed the deviations from mean imbalance across card game situations to depend on the price-sensitivity of the subject in each of those situations, as the last of the four equations above displays. On the between-subjects level, we would expect that the less price-sensitive on average an agent is (the higher the value of T_i), the lower this agent's imbalance, i.e., $b_{between}$ should be negative. Similarly, if in a given game situation a subject displays less price sensitivity than this subject's average, then in this situation the subject should also hold a more balanced portfolio, i.e., b_{within} is expected to be negative as well.

We focus on $b_{between}$, which provides a filtered estimate of the relationship between portfolio imbalance of a subject and price sensitivity. We do not report the within-level parameter estimates (b_{within}); although with the correct (negative) signs, none were ever significantly different from zero. Throughout, we used robust maximum likelihood estimation.

As prices move away from expected payoffs computed from true probabilities, risk averse agents with correct beliefs choose more unbalanced portfolios. Therefore, our test should have more power when mispricing (defined as the difference between transaction prices and

true expected payoffs) increases. Consequently, we also present estimation results for the in-between parameters for a specification that factors in the level of mispricing, as follows:

$$\begin{aligned}
I_i &= a + b_{between}MT_i + \epsilon_i, \text{ where} \\
I_{ij}^p &= I_i + \eta_{ij}, \text{ and} \\
MT_{ij} &= MT_i + \xi_{ij}, \text{ and} \\
\eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij},
\end{aligned} \tag{2}$$

where $MT_{ij} = M_{ij}^p * T_{ij}$, and M_{ij}^p is the mean absolute mispricing in the trading period p of Card Game j of the session in which subject i participated.

The results on the between-level estimates for Treatment II are displayed in boldface in Table V. The first column within Treatment II presents the estimates when imbalance is measured after the second trading period of each replication; the second column shows the estimates when imbalance is measured after the first trading period. All estimates are highly significant. The standard errors produce t -statistics around 4, and hence, p -values below 0.001. R-squared's are also high. The predicted effect is already present by the middle of the period. By the end of the period, the magnitude of the effect increases (although not significantly). The predictions are confirmed whether we adjust for (mean) absolute mispricing (Eqn. 2) or not (Eqn. 1).

For reference, Table V also displays results for Treatment I. In this treatment, there is no aggregate risk. As a result, if pricing is correct, even subjects who know how to compute probabilities, should not hold a risky position, and hence, their choices are indistinguishable from those of ambiguity averse subjects, who avoid risk no matter whether prices are correct. Consistent with this prediction, all (between-level) coefficients are insignificant; t -statistics implied by displayed standard errors barely reach 0.5, and R^2 's are a fraction of those from Treatment II.

Consequently, our theory makes the right prediction across the two Treatments. When an effect is predicted, it is present (Treatment II, aggregate risk); when the effect is not predicted, it is not there in the data (Treatment I, no aggregate risk). Overall, the data therefore provide

strong evidence for the conjecture that price-insensitive agents behave in an ambiguity averse manner, and against the alternative that price-insensitivity merely reflects noise trading.¹⁷

We repeated our analysis using ordinary least square (OLS) regressions. Any difference of the results would stem from the “within” level noise in our data. While none of the qualitative conclusions change, all coefficients decrease in magnitude, sometimes significantly, as expected. The results can be obtained from the authors upon request.

V. Conclusions

We developed a theory of asset pricing under asymmetric reasoning. In it, we rejected the idea that agents would agree to disagree and instead posited that agents with incorrect modeling sense ambiguity when confronted with market prices that differ from those implied by their models, doubting their own reasoning while at the same time unwilling to give market prices the benefit of their doubt. In the face of ambiguity, agents become ambiguity averse and prefer to trade to portfolios that are not exposed to ambiguity, insensitive to prices.

We derived equilibrium pricing results when there is no aggregate risk: price quality (the distance of prices from correctly computed conditional expected payoffs) increases with the proportion of price sensitive agents. We contrasted choices between price-sensitive and price-insensitive agents when there *is* aggregate risk: in equilibrium, price-sensitive agents should hold risk (imbalanced positions) while price-insensitive agents should trade to balanced positions.

Experiments confirmed these predictions. When there was no aggregate risk, price quality improved significantly with the fraction of price-sensitive subjects; when there was aggregate risk, end-of-period holdings of price-sensitive subjects were more imbalanced than those of price-insensitive subjects. Importantly, when our theory predicted that the expected effects may not emerge, the results were weakened substantially: price quality and number of price

¹⁷It may be argued that price-insensitive subjects are not really ambiguity averse, but just follow a rule of thumb, investing half their wealth in each of the risky securities. The findings in Bossaerts et al. (2010) reject this interpretation, though. There, choices of price-insensitive agents do reveal ambiguity aversion, but cannot be explained by the proposed rule of thumb.

sensitive subjects correlated far less when there was aggregate risk, and the relationship between imbalance of end-of-period portfolio choices and price sensitivity became insignificant when there was no aggregate risk.

Referring to it as “comparative ignorance,” Fox and Tversky (1995) have suggested before that inability to correctly perform difficult computations may translate into a sense of ambiguity when authoritative evidence exists to the contrary. It is particularly striking that financial markets seem to exude the very authority that is necessary to convince subjects who cannot do the computations correctly that they really cannot. At the same time, and consistent with “comparative ignorance,” rather than trusting the authoritative source (here: the market), ambiguity is sensed. As a consequence, ambiguity averse agents will avoid the situation altogether, implying, in the context of financial markets, that they trade to ambiguity-free portfolios.

As such, subjects do not give prices the benefit of their doubt. Along with aversion to ambiguity, it is this feature of their reaction that produces the unusual asset pricing results explored in this paper. In contrast with a popular way to model asset pricing under differences of opinion, prices will not reflect some average of the beliefs of all the agents, but prices may actually be (about) right. This conclusion, corroborated in the data, deviates fundamentally from well-known implications of asset pricing theory under asymmetric information.

Our theory and experiments demonstrate that the role of financial markets is not limited to risk sharing and information aggregation, but extends to social cognition. That markets may facilitate social cognition was first suggested in Maciejovsky and Budescu (2007) and Meloso et al.. This paper argues, however, that the principles behind facilitation of social cognition differ substantially with those of the other two roles, and the experiments bear this out.

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Appendix

A. Mathematical Details

An agent with max min preferences maximizes the following expression:

$$U_i(R_i, B_i) = \min\{u(R_i), u(B_i)\}.$$

If $R_i > B_i$, then $U_i(R_i, B_i) = u(B_i)$. Similarly, if $R_i < B_i$, then $U_i(R_i, B_i) = u(R_i)$. From here it immediately follows that an agents with maxmin preferences will seek a portfolio with $R_i = B_i$ under any prices p_R and $p_B = 1 - p_R$.

Let the price of R be p_R . An expected utility maximizing agent i with belief π_r^i maximizes

$$U_i(R_i, B_i) = \pi_r^i u(R_i) + (1 - \pi_r^i) u(B_i).$$

The solution to this agent's optimization problem given her endowment, which by assumption is one unit of each asset, is $R_i = \frac{\pi_r^i}{p_R}$.

A.1. Excess demand of knowledgeable agents

Let q_α denote the aggregate demand of red asset by the fraction α of agents who are able to calculate the correct probabilities. Then $q_\alpha = \int_{1-\alpha}^1 R_i = \alpha \frac{\pi_r}{p_R}$. Thus, for any $p_R < \pi_r$ the knowledgeable agents create excess demand $\alpha(\frac{\pi_r}{p_R} - 1)$.

A.2. Excess demand of ambiguity averse agents

For any price p_R the agents demand risk-neutral portfolio. Because of the assumption of no aggregate endowment uncertainty for any subinterval of agents, the ambiguity averse agents create excess demand of 0.

A.3. Excess demand of price sensitive biased agents

Note that from the assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\alpha \leq 1$, it follows $\epsilon < \frac{\pi_r - \underline{\pi}}{2}$. Conjecture that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$. Let \underline{i} be the agent such that $\pi_r^{\underline{i}} = p_R - \epsilon$, that is, $\underline{i} = \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \epsilon - \underline{\pi})$. The excess demand generated by the biased agents is

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1 \right) di.$$

Since $\pi_r^i = \underline{\pi} + i \frac{\pi_r - \underline{\pi}}{(1-\alpha)}$,

$$\begin{aligned} \int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1 \right) di &= \int_{\underline{i}}^{1-\alpha} \left(\frac{\underline{\pi}}{p_R} - 1 \right) di + \int_{\underline{i}}^{1-\alpha} \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} i di = -\frac{p_R - \underline{\pi}}{p_R} \int_{\underline{i}}^{1-\alpha} 1 di + \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} \int_{\underline{i}}^{1-\alpha} i di = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} i^2 \Big|_{\underline{i}}^{1-\alpha} - \frac{p_R - \underline{\pi}}{p_R} i \Big|_{\underline{i}}^{1-\alpha} = \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} ((1-\alpha)^2 - \underline{i}^2) - \frac{p_R - \underline{\pi}}{p_R} (1-\alpha - \underline{i}) = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} (1-\alpha - \underline{i})(1-\alpha + \underline{i}) - \frac{p_R - \underline{\pi}}{p_R} (1-\alpha - \underline{i}) = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} \left(1-\alpha - \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) \left(1-\alpha + \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) - \frac{p_R - \underline{\pi}}{p_R} \left(1-\alpha - \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) = \\ &= (1-\alpha) \frac{\pi_r - \underline{\pi}}{2p_R} \left(1 - \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) \left(1 + \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) - (1-\alpha) \frac{p_R - \underline{\pi}}{p_R} \left(1 - \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) = \\ &= (1-\alpha) \frac{1}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (1-\alpha) \frac{p_R - \underline{\pi}}{p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon) = \\ &= \frac{1-\alpha}{p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon) \left(\frac{1}{2}(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (p_R - \underline{\pi}) \right) = \\ &= \frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon). \end{aligned}$$

Because $(\pi_r - p_R - \epsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

A.4. Equilibrium

In equilibrium the aggregate excess demand must be zero.

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon) + \alpha \left(\frac{\pi_r}{p_R} - 1 \right) = 0 \Leftrightarrow$$

$$\begin{aligned}\frac{1-\alpha}{2p_R(\pi_r-\underline{\pi})}(\pi_r-p_R+\epsilon)(p_R+\epsilon-\pi_r) &= \alpha\left(\frac{\pi_r}{p_R}-1\right) \Leftrightarrow \\ \frac{1-\alpha}{2(\pi_r-\underline{\pi})}(\pi_r-p_R+\epsilon)(p_R+\epsilon-\pi_r) &= \alpha(\pi_r-p_R) \Leftrightarrow\end{aligned}$$

Denote $\pi_r - p_R$ by y . Then

$$\frac{1-\alpha}{2(\pi_r-\underline{\pi})}(y+\epsilon)(\epsilon-y) = \alpha y.$$

Denote $\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})$ by K . Then

$$y^2 + 2Ky - \epsilon^2 = 0.$$

The (positive) solution to the equation is $y = \sqrt{K^2 + \epsilon^2} - K$. Note that $\lim_{\epsilon \rightarrow 0} y = 0$, i.e. the price converges to π_r as ϵ converges to zero.

The above derived equilibrium satisfies the conjecture that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$ as depicted in Figure 4.

A.5. Comparative Statics

Since $\frac{\partial K}{\partial \alpha} = \frac{(\pi_r - \underline{\pi})}{(1-\alpha)^2} > 0$ and $\frac{\partial K}{\partial \alpha} \frac{\partial y}{\partial K} = \frac{K}{\sqrt{K^2 + \epsilon^2}} - 1 < 0$, it follows that $\frac{dy}{d\alpha} = \frac{\partial K}{\partial \alpha} \frac{\partial y}{\partial K} < 0$, i.e. the difference between the price and the true probability decreases as α increases.

Let $S(\alpha, y)$ be the fraction of price-sensitive agents, as a function of the fraction of knowledgeable agents and the mispricing, $S = \alpha + (\epsilon + y) \frac{1-\alpha}{\pi_r - \underline{\pi}}$. Let $y^*(\alpha)$ be the equilibrium mispricing, as a function of α . Let $S^*(\alpha) = S(\alpha, y^*(\alpha))$ be the fraction of price sensitive agents in equilibrium, as a function of alpha. Then,

$$\begin{aligned}S^*(\alpha) &= \alpha + \left(\epsilon + \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right)^2 + \epsilon^2} - \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right) \frac{1-\alpha}{\pi_r - \underline{\pi}} = \\ &= \alpha + \frac{1-\alpha}{\pi_r - \underline{\pi}} \epsilon - \frac{1-\alpha}{\pi_r - \underline{\pi}} \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) + \frac{1-\alpha}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right)^2 + \epsilon^2}\end{aligned}$$

$$\begin{aligned}
&= \frac{1-\alpha}{\pi_r - \underline{\pi}} \epsilon + \frac{1-\alpha}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} \\
&= \frac{1-\alpha}{\pi_r - \underline{\pi}} \left(\epsilon + \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} \right).
\end{aligned}$$

For any α then we obtain $S^*(\alpha)$ and $y^*(\alpha)$. If $S^*(\alpha)$ is strictly monotonic, we can invert it and obtain $\alpha(S)$. We are interested in $y(\alpha(S))$ and $\frac{dy}{dS} = \frac{dy}{d\alpha} \frac{d\alpha}{dS}$. We know that $\frac{dy}{d\alpha} < 0$, hence we must only determine the sign of $\frac{d\alpha}{dS}$, which, under our conjecture that $S^*(\alpha)$ is strictly monotonic, coincides with the sign of $\frac{dS^*(\alpha)}{d\alpha}$. Figure 5 presents the surface of $\frac{dS^*(\alpha)}{d\alpha}$ for any α and any ϵ , given $\pi_r - \underline{\pi} = 0.5$. It illustrates that if ϵ is not too large relative to α , the derivative is positive. We show that given our assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$, $\frac{dS^*(\alpha)}{d\alpha} > 0$ for every α , so that indeed $S^*(a)$ is strictly monotonic as assumed, and it follows $\frac{dy}{dS} < 0$.

$$\frac{dS^*(\alpha)}{d\alpha} = -\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2} \left(\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2 \right)^{-1/2}.$$

We want to show that given any $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$,

$$\begin{aligned}
-\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2 \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} &> 0 \Leftrightarrow \\
\frac{\alpha(\pi_r - \underline{\pi})^2}{(1-\alpha)^2 \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0 \Leftrightarrow \\
\frac{(\pi_r - \underline{\pi})}{(1-\alpha) \sqrt{1 + \left(\frac{(1-\alpha)\epsilon}{\alpha(\pi_r - \underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0.
\end{aligned}$$

The left hand side expression is decreasing in ϵ , hence it suffices to show that the inequality holds for $\epsilon = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$.

$$\begin{aligned}
& \frac{(\pi_r - \underline{\pi})}{(1 - \alpha)\sqrt{1 + \left(\frac{(1-\alpha)\frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})}{\alpha(\pi_r - \underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1 - \alpha)}\right)^2 + \frac{\alpha^2}{(1 + \alpha)^2}(\pi_r - \underline{\pi})^2} - \frac{\alpha}{1 + \alpha}(\pi_r - \underline{\pi}) > 0 \Leftrightarrow \\
& \frac{1}{(1 - \alpha)\sqrt{1 + \frac{(1-\alpha)^2}{(1+\alpha)^2}}} - \alpha\sqrt{\frac{1}{(1 - \alpha)^2} + \frac{1}{(1 + \alpha)^2}} - \frac{\alpha}{1 + \alpha} > 0 \Leftrightarrow \\
& \frac{(1 + \alpha)}{(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2}} - \frac{\alpha}{(1 - \alpha)(1 + \alpha)}\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} - \frac{\alpha}{1 + \alpha} > 0 \Leftrightarrow \\
& (1 + \alpha)^2 - \alpha((1 + \alpha)^2 + (1 - \alpha)^2) - \alpha(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} > 0 \Leftrightarrow \\
& (1 - \alpha)(1 + \alpha)^2 - \alpha(1 - \alpha)^2 > \alpha(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} \Leftrightarrow \\
& \frac{(1 + \alpha)^2}{\alpha} - (1 - \alpha) > \sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} \Leftrightarrow \\
& \frac{(1 + \alpha)^4}{\alpha^2} + (1 - \alpha)^2 - 2\frac{(1 + \alpha)^2}{\alpha}(1 - \alpha) > (1 + \alpha)^2 + (1 - \alpha)^2 \Leftrightarrow \\
& (1 + \alpha)^2 - 2\alpha(1 - \alpha) > \alpha^2 \Leftrightarrow \\
& 1 + 2\alpha^2 > 0.
\end{aligned}$$

A.6. Heterogeneity

Here, we consider the case where some agents become ambiguity averse when their subjective belief deviates from prices with an amount δ , whereas others become ambiguity averse when beliefs deviate more than ϵ , where $\delta \leq \epsilon$. We now prove that mispricing continues to decrease in the fraction (here 2α) of agents who know how to compute probabilities (Corollary 1).

Setting

We envisage a situation where is a continuum of agents on the interval $I = [0, 1]$ and a continuum of agents on the interval $J = [0, 1]$. Any agent $i \in [\alpha, 1] \subset I$ and any $j \in [\alpha, 1] \subset J$ updates correctly and always makes portfolio choices that maximize her expected utility according to her (correct) subjective probability. Any agent $i \in [0, \alpha) \subset I$ makes portfolio choices that maximize her expected

utility according to her (incorrect) subjective probability if and only if $|\pi_r^i - p_R| < \varepsilon$, otherwise she becomes ambiguity averse and maximizes maxmin utility. Any agent $j \in [0, \alpha) \subset J$ makes portfolio choices that maximize her expected utility according her (incorrect) subjective probability if and only if $|\pi_r^j - p_R| < \delta$, otherwise she becomes ambiguity averse and optimizes maxmin utility.

An agent with max min preferences maximizes the following expression:

$$U_i(R_i, B_i) = \min \{u(R_i), u(B_i)\}.$$

If $R_i > B_i$, then $U_i(R_i, B_i) = u(B_i)$. Similarly, if $R_i < B_i$, then $U_i(R_i, B_i) = u(R_i)$. From here it immediately follows that an agents with max min preferences will seek a portfolio with $R_i = B_i$ under any prices p_R and $p_B = 1 - p_R$.

Let the price of R be p_R . An expected utility maximizing agent i with belief π_r^i maximizes

$$U_i(R_i, B_i) = \pi_r^i u(R_i) + (1 - \pi_r^i) u(B_i).$$

The solution to this agent's optimization problem given her endowment, which by assumption is one unit of each asset, is $R_i = \frac{\pi_r^i}{p_R}$.

Excess demand of knowledgeable agents

Let q_α denote the aggregate demand of red asset by the fraction α of agents who are able to calculate the correct probabilities. Then $q_\alpha = \int_{1-\alpha}^1 R_i = \alpha \frac{\pi_r}{p_R}$. Thus, for any $p_R < \pi_r$ the knowledgeable agents create excess demand $\alpha \left(\frac{\pi_r}{p_R} - 1 \right)$.

Excess demand of ambiguity averse agents

For any price p_R the agents demand risk-neutral portfolio. Because of the assumption of no aggregate endowment uncertainty for any subinterval of agents, the ambiguity averse agents create excess demand of 0.

Excess demand of price sensitive biased agents

Note that from the assumption that $\varepsilon < \frac{\alpha}{1+\alpha} (\pi_r - \underline{\pi})$ and $\alpha \leq 1$, it follows $\varepsilon < \frac{\pi_r - \underline{\pi}}{2}$.

Case A) Conjecture that $p_R + \varepsilon > \pi_r > p_R + \delta > p_R - \varepsilon > \underline{\pi}$.

The aggregate excess demand of biased agents in J is $\int_{p_R - \delta}^{p_R + \delta} \left(\frac{x}{p_R} - 1\right) dx = \left(\frac{x^2}{2p_R} - x\right)\Big|_{p_R - \delta}^{p_R + \delta} = 0$.

The aggregate excess demand of biased agents in I follows the same calculation as in the benchmark model, with the only difference that the overall size of the group is one half of what it was in that benchmark case, so the aggregate excess demand is halved.

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon).$$

Because $(\pi_r - p_R - \varepsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

Case B) Conjecture instead that $p_R + \delta > \pi_r > p_R - \varepsilon > \underline{\pi}$. Following analogous calculations to those for the I group, but substituting δ for ε at every step, the excess demand of biased agents in group J is

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \delta)(\pi_r - p_R - \delta).$$

Equilibrium

Case A) In equilibrium the aggregate excess demand must be zero.

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon) + \alpha\left(\frac{\pi_r}{p_R} - 1\right) = 0 \Leftrightarrow$$

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(p_R + \varepsilon - \pi_r) = \alpha\left(\frac{\pi_r}{p_R} - 1\right) \Leftrightarrow$$

$$\frac{1 - \alpha}{4(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(p_R + \varepsilon - \pi_r) = \alpha(\pi_r - p_R).$$

Denote $\pi_r - p_R$ by y . Then

$$\frac{1 - \alpha}{4(\pi_r - \underline{\pi})}(y + \varepsilon)(\varepsilon - y) = \alpha y.$$

Denote $\frac{\alpha}{1 - \alpha}(\pi_r - \underline{\pi})$ by K . Then

$$\frac{1}{4}(\varepsilon + y)(\varepsilon - y) = Ky.$$

The (positive) solution to the equation is

$$y = \sqrt{4K^2 + \varepsilon^2} - 2K.$$

Note that $\lim_{\varepsilon \rightarrow 0} y = 0$, i.e. the price converges to π_r as ε converges to zero.

The above derived equilibrium satisfies the conjecture that $p_R + \varepsilon > \pi_r > p_R + \delta > p_R - \varepsilon > \underline{\pi}$ if and only if

$$\delta \leq \sqrt{4K^2 + \varepsilon^2} - 2K.$$

Otherwise we are on case B).

Case B). In equilibrium aggregate excess demand must be zero.

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})} [(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon) + (\pi_r - p_R + \delta)(\pi_r - p_R - \delta)] + \alpha \left(\frac{\pi_r}{p_R} - 1 \right) = 0 \Leftrightarrow$$

$$\frac{1 - \alpha}{4p_R(\pi_r - \underline{\pi})} [2(\pi_r - p_R)^2 - \varepsilon^2 - \delta^2] = -\alpha \left(\frac{\pi_r}{p_R} - 1 \right) \Leftrightarrow$$

$$\frac{1 - \alpha}{4(\pi_r - \underline{\pi})} (2y^2 - \varepsilon^2 - \delta^2) = -\alpha y \Leftrightarrow$$

$$2y^2 - \varepsilon^2 - \delta^2 = -4Ky$$

$$2y^2 + 4Ky - (\varepsilon^2 + \delta^2) = 0$$

$$y = \frac{-4K + \sqrt{16K^2 + 8(\varepsilon^2 + \delta^2)}}{4}$$

$$y = \sqrt{K^2 + \frac{\varepsilon^2 + \delta^2}{2}} - K.$$

Therefore, the equilibrium with the two populations I and J is identical to the equilibrium with a unique population with parameter $\varepsilon_{avg} = \sqrt{\frac{\varepsilon^2 + \delta^2}{2}}$.

Comparative Statics

Case A) Since $\frac{\partial K}{\partial \alpha} = \frac{(\pi_r - \underline{\pi})}{(1 - \alpha)^2} > 0$ and $\frac{\partial y}{\partial K} = \frac{K}{\sqrt{4K^2 + \varepsilon^2}} - 2 < 0$, it follows that $\frac{dy}{d\alpha} = \frac{\partial y}{\partial K} \frac{\partial K}{\partial \alpha} < 0$, i.e. the difference between the price and the true probability decreases as α increases.

Case B) Since $\frac{\partial K}{\partial \alpha} = \frac{(\pi_r - \underline{\pi})}{(1 - \alpha)^2} > 0$ and $\frac{\partial y}{\partial K} = \frac{K}{\sqrt{K^2 + \varepsilon_{avg}^2}} - 1 < 0$, it follows that $\frac{dy}{d\alpha} = \frac{\partial y}{\partial K} \frac{\partial K}{\partial \alpha} < 0$, i.e. the difference between the price and the true probability decreases as α increases.

In both cases, mispricing decreases in the fraction of agents who know how to compute probabilities, confirming Corollary 1.

In regard to Corollary 2, first note that for Case B the proof for a homogeneous epsilon applies by taking the value of epsilon to be $\varepsilon_{avg} = \sqrt{\frac{\varepsilon^2 + \delta^2}{2}}$.

For Case A, on the other hand, let $S(\alpha, y)$ be the fraction of price-sensitive agents, as a function of the fraction of knowledgeable agents and the mispricing, $S = \alpha + (\delta + \frac{\varepsilon + y}{2}) \frac{1 - \alpha}{\pi_r - \underline{\pi}}$. Let $y^*(\alpha)$ be the equilibrium mispricing, as a function of α . Let $S^*(\alpha) = S(\alpha, y^*(\alpha))$ be the fraction of price sensitive agents in equilibrium, as a function of alpha. Recall $y = \sqrt{4K^2 + \varepsilon^2} - 2K$ and $K = \frac{\alpha}{1 - \alpha}(\pi_r - \underline{\pi})$. Then,

$$\begin{aligned} S^*(\alpha) &= \alpha + \left(\delta + \frac{\varepsilon}{2} + \frac{1}{2} \left(\sqrt{4K^2 + \varepsilon^2} - 2 \frac{\alpha}{1 - \alpha} \right) (\pi_r - \underline{\pi}) \right) \frac{1 - \alpha}{\pi_r - \underline{\pi}} = \\ &= \frac{1 - \alpha}{\pi_r - \underline{\pi}} \left(\delta + \frac{\varepsilon}{2} \right) + \frac{1 - \alpha}{2(\pi_r - \underline{\pi})} \sqrt{4K^2 + \varepsilon^2}. \end{aligned}$$

For any α we obtain $S^*(\alpha)$ and $y^*(\alpha)$. If $S^*(\alpha)$ is strictly monotonic, we can invert it and obtain $\alpha(S)$. We are interested in $y(\alpha(S))$ and $\frac{dy}{dS} = \frac{dy}{d\alpha} \frac{d\alpha}{dS}$. We know that $\frac{dy}{d\alpha} < 0$, hence we must only determine the sign of $\frac{d\alpha}{dS}$, which, under our conjecture that $S^*(\alpha)$ is strictly monotonic, coincides with the sign of $\frac{dS^*(\alpha)}{d\alpha}$.

$$\frac{dS^*(\alpha)}{d\alpha} = \frac{1}{2} \left(-\frac{2\delta + \varepsilon}{(\pi_r - \underline{\pi})} - \frac{\sqrt{4K^2 + \varepsilon^2}}{(\pi_r - \underline{\pi})} + \left(\frac{4\alpha(\pi_r - \underline{\pi})}{(1 - \alpha)^2 \sqrt{4K^2 + \varepsilon^2}} \right) \right).$$

We want to show that $\frac{dS^*(a)}{d\alpha} > 0$. Because $\frac{dS^*(\alpha)}{d\alpha}$ is decreasing in both ε and δ , it suffices to show that $\frac{dS^*(\alpha)}{d\alpha} > 0$ given $\varepsilon = \bar{\varepsilon} = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\delta = \bar{\delta} = \sqrt{4K^2 - \varepsilon^2} - 2K$.

$$\begin{aligned}
& \left. \frac{dS^*(\alpha)}{d\alpha} \right|_{\varepsilon=\bar{\varepsilon}, \delta=\bar{\delta}} \\
&= \frac{1}{2} \left(-\frac{2(\sqrt{4K^2 + \varepsilon^2} - 2K) + \varepsilon}{(\pi_r - \underline{\pi})} - \frac{\sqrt{4K^2 + \varepsilon^2}}{(\pi_r - \underline{\pi})} + \frac{4\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2\sqrt{4K^2 + \varepsilon^2}} \right) \\
&= \frac{1}{2} \left(-\frac{3(\sqrt{4K^2 + \varepsilon^2})}{(\pi_r - \underline{\pi})} - \frac{-4K + \varepsilon}{(\pi_r - \underline{\pi})} + \frac{4\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2\sqrt{4K^2 + \varepsilon^2}} \right) \\
&= \frac{1}{2} \left(-3\sqrt{4\left(\frac{\alpha}{1-\alpha}\right)^2 + \left(\frac{\alpha}{1+\alpha}\right)^2} + 4\frac{\alpha}{1-\alpha} - \frac{\alpha}{1+\alpha} + \frac{4\alpha}{(1-\alpha)^2\sqrt{4\left(\frac{\alpha}{1-\alpha}\right)^2 + \left(\frac{\alpha}{1+\alpha}\right)^2}} \right) \\
&= \frac{\alpha}{2(1-\alpha^2)} \left(-3\sqrt{4(1+\alpha)^2 + (1-\alpha)^2} + 5\alpha + 3 + \frac{4(1+\alpha)^2}{\alpha\sqrt{4(1+\alpha)^2 + (1-\alpha)^2}} \right).
\end{aligned}$$

Figure 7 displays the graph of $\frac{dS^*(\alpha)}{d\alpha}$ given $\varepsilon = \bar{\varepsilon}$ and $\delta = \bar{\delta}$, and demonstrates that it is always positive, i.e., the number of price sensitive agents is positively related to α .

B. Biased Slope Coefficients

To determine whether there is any simultaneous-equation bias on the estimated slope coefficients induced by overall balance in the changes in positions, we translate our setting into a more familiar framework, namely, that of a simple demand-supply setting. In particular, we are going to interpret (minus) the changes in endowments of the price-insensitive subjects as the supply in a demand-supply system with exogenous, price-insensitive supply, while the changes in endowments of the price-sensitive subjects correspond to the (price-sensitive) demands in a demand-supply system. The requirement that changes in holdings balance then corresponds to the usual restriction that demand equals supply.

We will consider only the case where price-sensitive subjects reduce their holdings when prices increase; translated into the usual demand-supply setting, this means that we assume that the slope of the demand equation is negative.

Assume there are only two subjects. One is price-sensitive, the other is price-insensitive. The former's changes in holdings corresponds to the demand \tilde{D} in the traditional demand-supply system; the latter's changes corresponds to the (exogenous) supply \tilde{S} . The usual assumptions are as follows:

$$\tilde{D} = A + BP + \epsilon,$$

with $B < 0$, and

$$\tilde{S} = \eta,$$

where ϵ is mean zero, and is independent of η . P denotes price.

We want to know the properties of the OLS estimate of B . Assume that P is determined by equating demand and supply (equivalent to balance between changes in holdings), i.e., from

$$\tilde{D} = \tilde{S}.$$

Then:

$$\text{cov}(P, \epsilon) = -\frac{1}{B} \text{var}(\epsilon) > 0.$$

Because of this, standard arguments show that the OLS estimate of B is inconsistent, with an upward bias. As such, the nominal size of the usual t -test under-estimates the true size, and one should apply a generous cut-off in order to determine whether B is significantly negative.

In our case, however, we only need to identify who is price-sensitive (i.e., whose holdings changes correspond to D in the demand-supply setting?) and who is not (whose holdings changes correspond to \tilde{S} ?). For this, we just run an OLS projection of changes in endowments on prices. The subjects with significantly negative slope coefficients are price-sensitive and hence, map into the demand \tilde{D} of the traditional demand-supply system. The argument above, however, indicated that this test is biased. Therefore, a generous cut-off should be chosen; we chose a cut-off equal to 1.6.

While we did not need this for our study, one can obtain an improved estimate of the price sensitivity once subjects are categorized as either price-sensitive or price-insensitive. Indeed, the changes in the holdings of the price-insensitive subjects can be used as instrument to re-estimate the price-sensitivity of the price-sensitive subjects. This is equivalent to using \tilde{S} as an instrument to estimate B . Indeed,

\tilde{S} ($= \eta$) and ϵ are uncorrelated, while \tilde{S} and P are correlated ($\text{cov}(\tilde{S}, P) = \text{var}(\tilde{S})/B$), so \tilde{S} is a valid instrument to estimate B in standard instrumental-variables analysis.

C. Experiment Instructions

I. THE EXPERIMENT

1. Situation The experiment consists of a sequence of trading sessions, referred to as *periods*. At the beginning of *even-numbered* periods, you will be given a fresh supply of *securities* and *cash*; in *odd-numbered periods*, you carry over securities and cash from the previous period. Markets open and you are free to trade some of your securities. You buy securities with cash and you get cash if you sell securities.

At the end of *odd-numbered* periods, the securities expire, after paying *dividends* that will be specified below. These dividends, together with your cash balance, constitute your *period earnings*. Securities do not pay dividends at the end of even-numbered periods and cash is carried over to the subsequent period, so your period earnings in even-numbered periods will be zero.

Period earnings are *cumulative* across periods. At the end of the experiment, the cumulative earnings are yours to keep, in addition to a standard sign-up reward.

During the experiment, accounting is done in real dollars.

2. The Securities You will be given two types of securities, *stocks* and *bonds*. Bonds pay a fixed dividend at the end of a period, namely, \$0.50. Stocks pay a random dividend. There are two types of stocks, referred to as Red and Black. Their payoff depends on the drawing from a deck of 4 cards, as explained later. The payoff is either \$0.50 or nothing. When Red stock pays \$0.50, Black stock pays nothing; when Red stock pays nothing, Black stock pays \$0.50.

You will be able to trade Red stock as well as bonds, but not Black stock.

You won't be able to buy Red stock or bonds unless you have the cash. You will be able to sell Red stock and bonds (and get cash) even if you do not own any. This is called short selling. If you sell, say, one Red stock, then you get to keep the sales price, but \$0.50 will be subtracted from your period earnings after the market closes and if the payoff on Red stock is \$0.50. If at the end of a period you are holding, say, -1 bonds, \$0.50 will be subtracted from your period earnings.

The trading system checks your orders against bankruptcy: you will not be able to submit orders which, if executed, are likely to generate negative period earnings.

3. How Payoffs Are Determined Each period, we start with a deck of 4 cards: one hearts (\heartsuit), one diamonds (\diamondsuit), one clubs (\clubsuit) and one spades (\spadesuit). The cards are shuffled and put in a row, face down.

Our computer takes randomly one or two cards and it discards them.

From the remaining cards, our computer randomly picks one or two cards. If one of these cards is hearts (\heartsuit), then the computer puts it back and picks another one. Sometimes, the computer will even put back diamonds (\diamondsuit) and pick another one. The computer then reveals the card(s) it picked and we will announce this in the News Page at the end of the period (after that, another period starts with the same securities in which you can trade again). *Note that the revealed card(s) will never be hearts, and sometimes may not even be diamonds.*

Before each period, the News Page will provide all the information that you need to make the right inferences: (i) whether one or two cards are going to be discarded initially, (ii) whether one or two cards are going to be picked from the remaining cards and whether diamonds will ever be shown.

After we show the revealed cards, one or two cards remain in the deck. Our computer randomly picks a card and *this last card determines the payoff on the securities.*

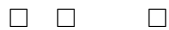
Red stock pays \$0.50 when the last card is either hearts (\heartsuit) or diamonds (\diamondsuit). In those cases, the Black stock pays nothing. This is shown in the following Payoff Table.

	Red Stock	Black Stock
If last card is \heartsuit or \diamondsuit	\$0.50	0
If last card is \clubsuit or \spadesuit	0	\$0.50

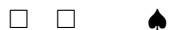
Here is an example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down, like this:

□ □ □ □

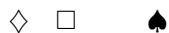
Our computer randomly discards one card (the third one in this case):



Our computer then randomly picks one card (the fourth one in this case), and reveals it, provided it is not hearts or diamonds (in this case; if it is hearts or diamonds, it replaces it with another card from the deck that is neither):



From the remaining two cards, our computer picks one at random that determines the payoffs on the stocks.



In this case, the last card picked is diamonds. As a result, you would be paid \$0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Here is another example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down:



Our computer randomly discards two cards (the second and third ones in this case):



Our computer then randomly picks one card and reveals it, provided it is not hearts (if it is hearts, it replaces it with another card from the deck):



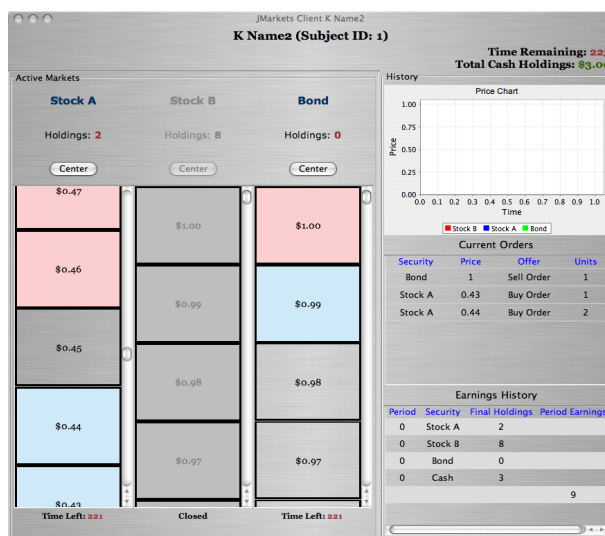
Our computer then picks the remaining card, which determines the payoffs on the stocks.



In this case, the last card picked is hearts. As a result, you would be paid \$0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Again, the announcements of the number of cards that will be discarded initially and revealed at the end of the period can be found in the News page. This page will also display the card(s) that are turned over at the end of the period, and, at the end of the subsequent period, the final card that determines the payoff on the Stocks.

II. THE MARKETS INTERFACE, jMARKETS Once you click on the Participate link to the left, you will be asked to log into the markets, and you will be connected to the jMarkets server. After everybody has logged in and the experiment is launched, a markets interface like the one below will appear



1. Active Markets

The Active Markets panel is renewed each period. In it, you'll see several scroll-down columns. Each column corresponds to a market in one of the securities. The security name is indicated on top. At the bottom, you can see whether the market is open, and if so, how long it will remain open. The time left in a period is indicated on the right hand side above the Active Markets panel.

At the top of a column, you can also find your current holdings of the corresponding security. Your current cash holdings are given on the right hand side above the Active Markets panel.

Each column consists of a number of price levels at which you and others enter offers to trade. Current offers to sell are indicated in red; offers to buy are indicated in blue. When pressing the Center

button on top of a column, you will be positioned halfway between the best offer to buy (i.e., the highest price at which somebody offers to buy) and the best offer to sell (i.e., the lowest price that anybody offers to sell at).

When you move your cursor to a particular price level box, you get specifics about the available offers. On top, at the left hand side, you'll see the number of units requested for purchase. Each time you click on it, you send an order to buy one unit yourself. On top, at the right hand side, the number of units offered for sale is given. You send an order to sell one unit each time you yourself click on it. At the bottom, you'll see how many units you offered. (Your offers are also listed under Current Orders to the right of the Active Markets panel.) Each time you hit cancel, you reduce your offer by one unit.

If you click on the price level, a small window appears that allows you to offer multiple units to buy or to sell, or to cancel offers for multiple units at once.

2. History

The History panel shows a chart of past transaction prices for each of the securities. Like the Active Markets panel, it refreshes every period. jMarkets randomly assigns colors to each of the securities. E.g., it may be that the price of the Red Stock is shown in blue. Make sure that this does not confuse you.

3. Current Orders

The Current Orders panel lists your offers. If you click on one of them, the corresponding price level box in the Active Markets panel is highlighted so that you can easily modify the offer.

4. Earnings History

The Earnings History table shows, for each period, your final holdings for each of the securities (and cash), as well as the resulting period earnings.

5. How Trade Takes Place

Whenever you enter an offer to sell at a price below or equal to that of the best available buy order, a sale takes place. You receive the price of the buy order in cash. Whenever you enter an offer to buy

at a price above or equal to that of the best available sell order, a purchase takes place. You will be charged the price of the sell order.

The system imposes strict price-time priority: buy orders at high prices will be executed first; if there are several buy orders at the same price level, the oldest orders will be executed first. Analogously, sell orders at low prices will be executed first, and if there are several sell orders at a given price level, the oldest ones will be executed first.

6. Restrictions On Offers

Before you send in an offer, jMarkets will check two things: the cash constraint, and the bankruptcy constraint.

The cash constraint concerns whether you have enough cash to buy securities. If you send in an offer to buy, you need to have enough cash. To allow you to trade fast, jMarkets has an automatic cancellation feature. When you submit a buy order that violates the cash constraint, the system will automatically attempt to cancel buy orders you may have at lower prices, until the cash constraint is satisfied and your new order can be placed.

The bankruptcy constraint concerns your ability to deliver on promises that you implicitly make by trading securities. We may not allow you to trade to holdings that generate losses in some state(s). A message appears if that is the case and your order will not go through.

Tables and Figures

Table I
Parameters of the Experimental Design

Experiment ^a	Subject	Signup	Initial Allocations ^b			
	Category (Number)	Reward (Dollar)	Red Stock (Units)	Black Stock (Units)	Notes (Units)	Cash (Dollar)
Treatment I: Sessions With No Aggregate Risk						
Caltech	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
UCLA	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Treatment II: Sessions With Aggregate Risk						
UCLA-R1	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R2	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R3	11	5	0	9	0	4
	9	5	12	3	0	1

^aIndicates affiliation of subjects. “Utah” refers to the University of Utah; “Utah-Caltech” refers to: 50% of subjects were Caltech-affiliated; the remainder were students from the University of Utah. Experiments are listed in chronological order of occurrence.

^bRenewed each period.

Table II
Card Games

Card Game Number	Implemented In Replications	Number of Cards Discarded Before Half-Time Revelation	Number of Cards Revealed Half-Time	Cards Never Revealed Half-Time
1	1, 5	1	2	hearts
2	2, 7	1	1	hearts
3	3, 6	2	1	hearts
4	4, 8	1	1	hearts, diamonds

Table III
Number of Price Sensitive Subjects and Mispricing
Treatment I (No Aggregate Risk)

Experiment	Card Game	Mean Mispricing ^a <i>M</i>	$N_{(T_b < -1.65)}$ ^b	$N_{(T_b > 1.9)}$ ^c	$N_{(T_b < -1.65, T_a \text{ correct})}$ ^d
Caltech	1	3.13	7	2	6
	2	5.54	3	2	2
	3	3.4	2	2	1
	4	1.25	4	6	2
Utah-1	1	3.5	4	1	3
	2	11.79	1	2	0
	3	11.28	0	1	0
	4	7.24	1	0	0
Caltech-Utah-1	1	3.35	5	1	1
	2	4.75	2	0	0
	3	1.62	2	0	2
	4	1.89	5	3	3
UCLA	1	4.86	2	1	1
	2	5.07	3	0	1
	3	2.53	3	3	3
	4	2.9	3	0	2
Utah-2	1	7.5	2	0	1
	2	7.95	3	3	0
	3	5.88	6	2	1
	4	4.19	2	2	2
Caltech-Utah-2	1	8.78	1	0	0
	2	2.58	2	0	2
	3	2.77	3	1	1
	4	3.38	3	0	0
<p>Corr(M, N_(T_b < -1.65)) = -0.53 (St. Error = 0.146)</p> <p>Corr(M, N_(T_b < -1.65, T_a correct)) = -0.58 (St. Error = 0.088)</p>					

^a*M* is the (absolute) difference between average transaction price and expected payoffs computed with correct probabilities, expressed in U.S. cents.

^bNumber of subjects for which $\{T_b < -1.65\}$. T_b is the t -statistic of the slope coefficient estimate in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

^cNumber of subjects for which $\{T_b > 1.9\}$.

^dNumber of subjects for which $\{T_b < -1.65\}$ and T_a has the correct sign. T_a is the t -statistic of the intercept in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

Table IV
Number of Price Sensitive Subjects and Mispricing
Treatment II (Aggregate Risk)

Experiment	Card Game	Mean Mispricing ^a <i>M</i>	$N_{(T_b < -1.65)}$ ^b	$N_{(T_b > 1.9)}$ ^c	$N_{(T_b < -1.65, T_a \text{ correct})}$ ^d
UCLA-1R	1	2.02	0	1	0
	2	10.91	0	1	0
	3	4.87	4	1	4
	4	3.64	0	1	0
UCLA-2R	1	3.91	3	2	1
	2	12.30	1	0	0
	3	8.79	5	4	2
	4	6.60	2	2	1
UCLA-3R	1	6.43	2	0	1
	2	6.06	3	2	1
	3	2.66	3	2	2
	4	3.30	1	0	0

Corr(M, $N_{(T_b < -1.65)}$) = -0.00
(St. Error = 0.316)

Corr(M, $N_{(T_b < -1.65, T_a \text{ correct})}$) = -0.14
(St. Error = 0.313)

^a*M* is the (absolute) difference between average transaction price and expected payoffs computed with correct probabilities, expressed in U.S. cents.

^bNumber of subjects for which $\{T_b < -1.65\}$. T_b is the *t*-statistic of the slope coefficient estimate in projections of one-minute changes in individual holdings of Red Stock (*R*) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

^cNumber of subjects for which $\{T_b > 1.9\}$.

^dNumber of subjects for which $\{T_b < -1.65\}$ and T_a has the correct sign. T_a is the *t*-statistic of the intercept in projections of one-minute changes in individual holdings of Red Stock (*R*) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

Table V
Price Sensitivity and Portfolio Imbalances

Independent Variable	Treatment I (No Aggregate Risk)		Treatment II (Aggregate Risk)	
	Second Trading Period	First Trading Period	Second Trading Period	First Trading Period
<i>T</i>	-11.287 ^a (22.399) ^b [0.050] ^c	-10.518 (17.244) [0.068]	-16.477 (4.396) [0.342]	-14.557 (3.579) [0.422]
<i>MT</i>	-0.730 (1.323) [0.015]	-1.986 (2.026) [0.114]	-2.898 (0.763) [0.359]	-2.499 (0.626) [0.489]

^aSlope coefficient

^bStandard error, corrected for heteroskedasticity and subject clustering.

^c R^2 .

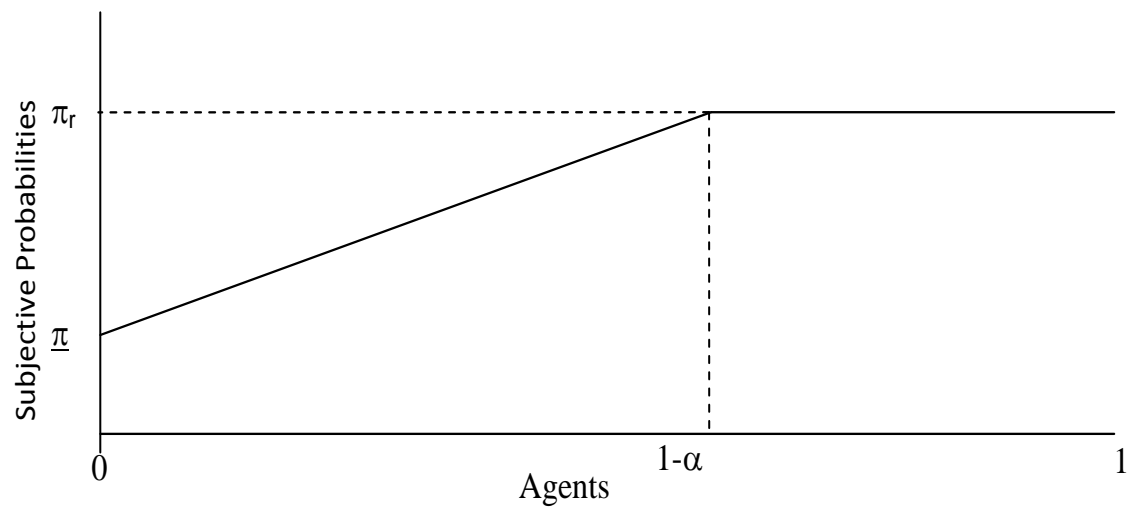


Figure 1. Beliefs. Graphical display of the relation between agent index (horizontal axis) and belief that security R will pay (vertical axis). The correct belief equals π_r . Incorrect beliefs are assumed to be below the correct belief, with a minimum equal to $\underline{\pi}$. Agents are indexed continuously from 0 to 1. A fraction α holds correct beliefs.

JMarkets Client K Name2
K Name2 (Subject ID: 1)

Time Remaining: 223
Total Cash Holdings: \$3.00

Active Markets

Stock A	Stock B	Bond
Holdings: 2	Holdings: 8	Holdings: 0
<input type="button" value="Center"/>	<input type="button" value="Center"/>	<input type="button" value="Center"/>

\$0.47	\$1.00	\$1.00
\$0.46	\$0.99	\$0.99
\$0.45	\$0.98	\$0.98
\$0.44	\$0.97	\$0.97
\$0.43		

Time Left: **221**ClosedTime Left: **221**

History

Price Chart

Legend: ■ Stock B ■ Stock A ■ Bond

Current Orders

Security	Price	Offer	Units
Bond	1	Sell Order	1
Stock A	0.43	Buy Order	1
Stock A	0.44	Buy Order	2

Earnings History

Period	Security	Final Holdings	Period Earnings
0	Stock A	2	
0	Stock B	8	
0	Bond	0	
0	Cash	3	
			9

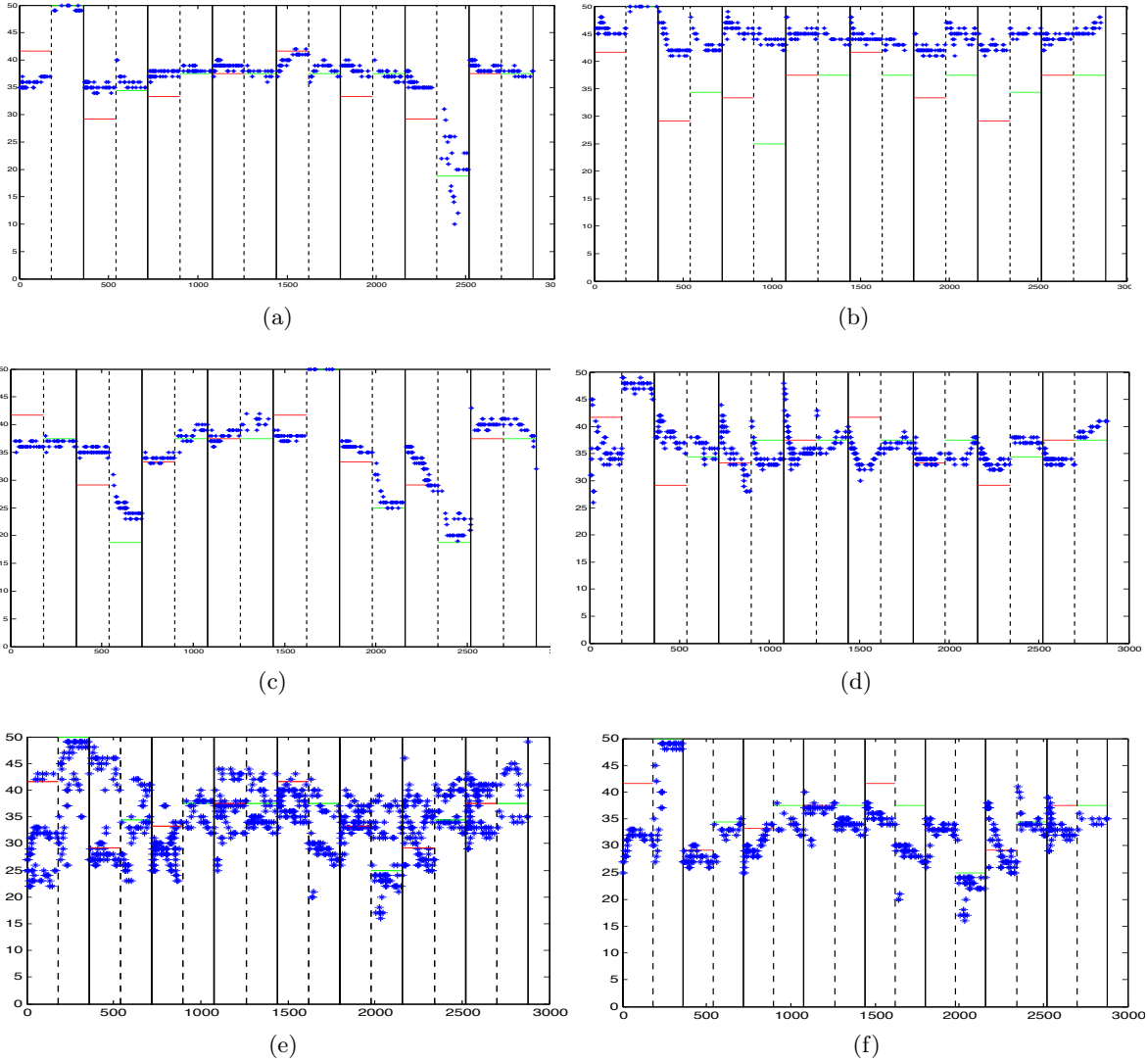


Figure 3. Evolution of Transaction Prices of Stock R in Treatment I. Blue crosses show transaction prices. Black vertical lines delineate periods. Dashed vertical lines indicate mid-period point in time when cards are partially revealed. Red horizontal line segments show true values of Stock R in the first half of the period. Green horizontal line segments indicate true values after mid-period revelation of cards. (a) Caltech; (b) Utah-1; (c) Caltech-Utah-1; (d) UCLA; (e) Utah-2; and, (f) Caltech-Utah-2.

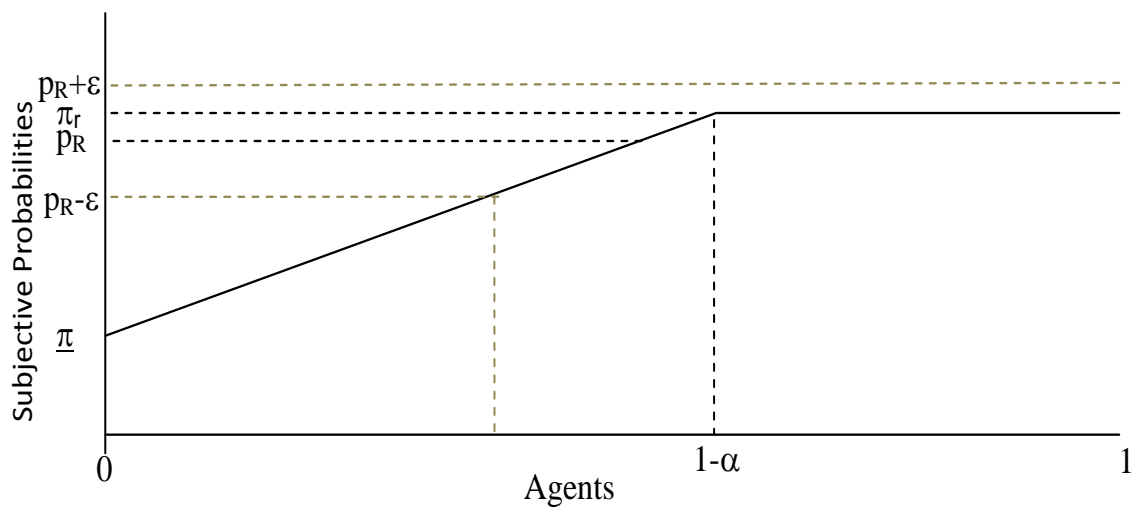


Figure 4. Equilibrium Price p_R . Agents with subjective beliefs within ϵ of the price p_R stick to their beliefs (they “agree to disagree” with the market). Those with subjective beliefs below $p_R - \epsilon$ become ambiguity averse because their beliefs are too much at odds with the market (price). The equilibrium price will be such that some agents with beliefs sufficiently close to the correct belief (π_R) continue to affect the price. As α , the proportion of agents with correct beliefs, increases, mispricing ($|p - \pi_R|$) decreases. The opposite obtains as ϵ , the maximum dissonance between subjective beliefs and market prices for which agents agree to disagree, increases.

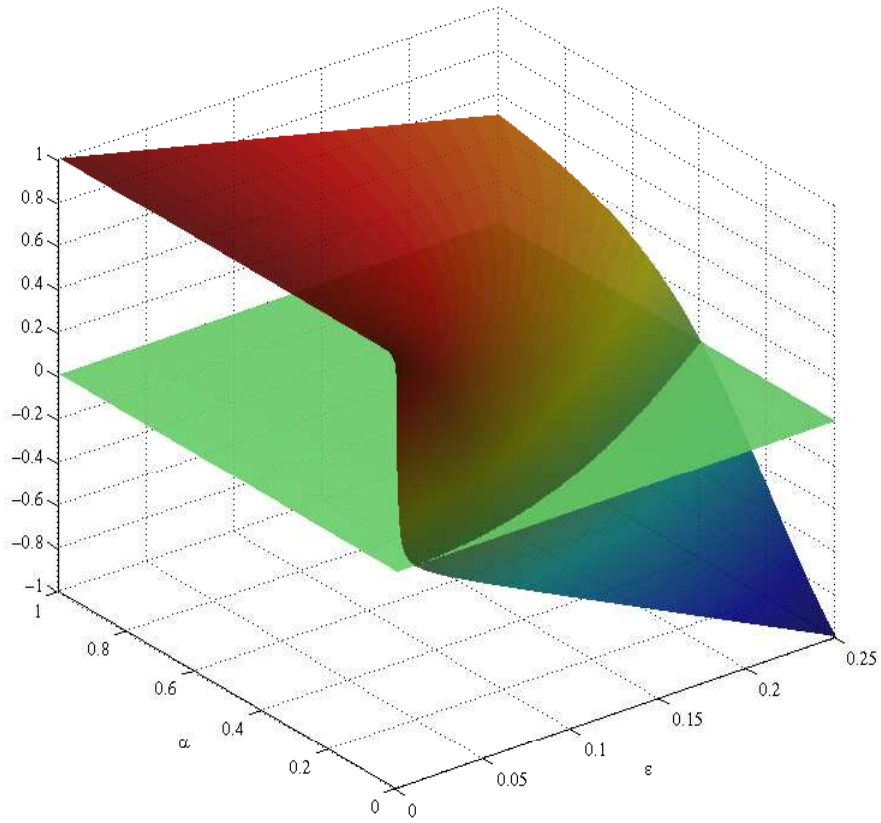


Figure 5. Change in Fraction of Price-Insensitive Agents with Changes Fraction of Agents with Correct Beliefs $\frac{\partial S}{\partial \alpha}$, as a Function of α (Fraction of Agents with Correct Beliefs) and ϵ (Maximum Dissonance between Subjective Beliefs and Market Prices for which Agents Agree to Disagree. Assumed: $\pi_r - \underline{\pi} = 0.5$.

<i>Period</i>	<i>Duration</i>	<i>Number of Cards Discarded Initially</i>	<i>Number of Cards Revealed At End</i>	<i>Cards Never Revealed At End</i>	<i>Cards Revealed</i>	<i>Last Card</i>	<i>Which Stock Pays Dividend?</i>
0	3'	1	2	♥	♠♣		
1	3'					♥	Red
2	3'	1	1	♥	♣		
3	3'					♥	Red
4	3'	2	1	♥	♣		
5	3'					♥	Red
6	3'	1	1	♥♦	♠		
7	3'					♦	Red
8	3'	1	2	♥	♦♠		
9	3'					♥	Red
10	3'	2	1	♥	♦		
11	3'					♠	Black

Figure 6. News Web Page. The page is filled gradually as cards are discarded and/or revealed (only the first six replications are included above).

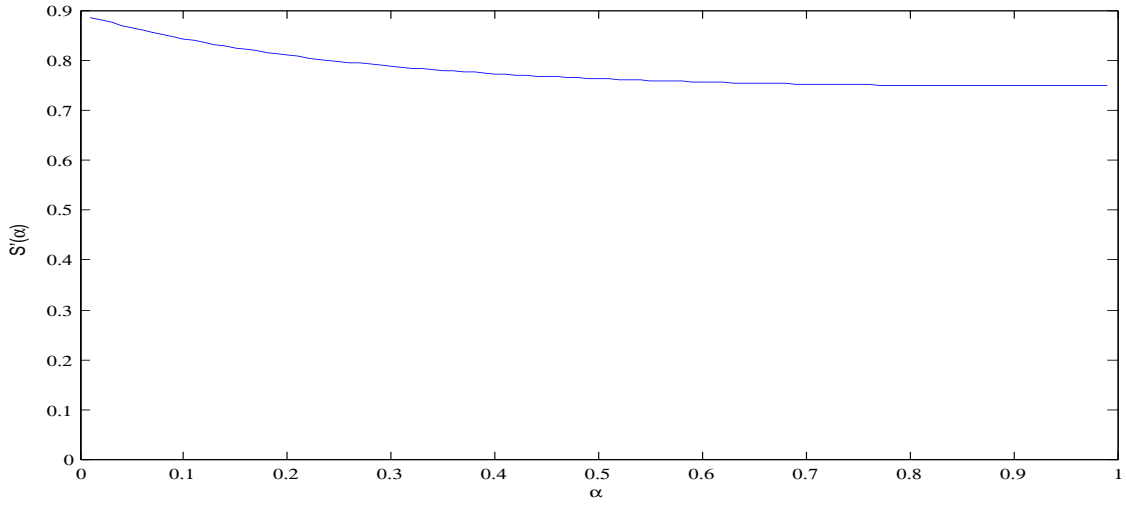


Figure 7. $\frac{dS(\alpha)}{d\alpha}$ evaluated at $\bar{\varepsilon} = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\bar{\delta} = \sqrt{4K^2 - \varepsilon^2} - 2K$.
 The figure plots the lower bound on the derivative of the fraction of price sensitive agents as a function of the fraction of agents with correct beliefs. Since this lower bound is everywhere strictly positive, the derivative is strictly positive for any α , ε and δ .