Inferior Goods, Giffen Goods, and Shochu ⋆

Shmuel Baruch[1] and Yakar Kannai[2]

[1] School of Business, University of Utah, UT 84112 USA
[2] Department of Mathematics, The Weizmann Institute of Science, Rehovot, Israel

Abstract. According to a well-known result by W. Hildenbrand [6], if all consumers possess the same demand function and the density of the expenditure distribution is decreasing, than the average income effect term is non-negative even if inferior goods are present, so that the aggregate demand must be monotone. We show that if the expenditure density is uni-modal and a certain relation between the income density and individual demand is satisfied, than the average income effect term is negative and Giffen goods are not ruled out. We show that the lowest-grade rice-based Japanese spirit (shochu) satisfies this condition. The data suggest that this commodity might be a Giffen good.

1 Introduction

Consider an individual facing a price vector $\mathbf{p} \in R^n$ and having budget I. Denote his demand for the j-th commodity by $f_j(\mathbf{p}, I)$. Recall that the j-th commodity is said to be *inferior* (at $(\mathbf{p}, I)$) if $\frac{\partial f_j(\mathbf{p},I)}{\partial I} < 0$ and *Giffen* if $\frac{\partial f_j(\mathbf{p},I)}{\partial p_j} \geq 0$. (If a good is not inferior then it is said to be *normal*. A good which is normal at all income levels cannot be Giffen — see section 2.) It is well known ([8],[9]) that no simple condition rules out Giffen goods. It is also known that inferior goods exist in abundance (at least when incomes are sufficiently high). What is somewhat puzzling is the almost non-existence of Giffen goods for aggregate market demand ("Such goods are more frequent in exam questions than in real life" ([2], p. 8)).

The first breakthrough came in [6], where it was shown that if all consumers in the economy have the same demand function $\mathbf{f}(\mathbf{p}, I)$ and if the expenditure distribution density $\rho(I)$ is decreasing, then the "law of demand" holds, i.e., the price differential of demand is a strictly negative-definite matrix. In particular, there are no Giffen goods. Actually it is proved that the average income effect term $\int f_j(\mathbf{p}, I)\frac{\partial f_j(\mathbf{p},I)}{\partial I}\rho(I)dI$ is non-negative (note that our sign convention differs from that of [6]). Underlying this result is the observation that for a sufficiently low income, every good is normal. Thus, if

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there are enough poor people in the economy that have not reached their saturation points, market demand function behaves as if goods were normal.

In the next section we show that if the expenditure distribution is unimodal and the degree in which the good fails to be normal is related appropriately to the distribution then the average income effect term is negative. (This happens if the commodity is so inferior that most poor households have reached their saturation points.) Thus we exhibit a case where at least a necessary condition for the Giffen effect is satisfied. In the search for possible candidates for Giffen goods, we came across the Japanese alcoholic beverage Shochu. This is a very low grade of "spirit" distilled from rice, and as the cheapest of all alcoholic beverages is consumed mainly by the poor. Based on data in [1] we could show that not only is shochu an inferior good, but it seems to satisfy (if we are willing to make some assumptions) the condition for a negative average mean income effect term. It appears that shochu may be a candidate for a Giffen good. The description and analysis of the data is presented in section 3.

Many reasons were suggested in the literature for the difficulty of observing a Giffen good. A stability argument put forward in [3] shows that only rarely could a Giffen good be observed. Moreover, enough dispersity of preference relations (or of demand functions) effectively rule out the Giffen effect (see [4] for an early result). Since we use time series we are forced to use only a relatively short time interval. In a longer interval many characteristics would change and the effect will no longer be discernible. In particular we make the assumption (clearly wrong in the long run) that the demand for shochu does not change much over time or with expenditure.

The situation considered here (partial equilibrium) differs from that studied in [10] (general equilibrium—endogenous I). The role of I is played by the budget allotted by the household for total living expenditures. We believe that to assume that there is no contemporaneous relation between prices and income is a reasonable assumption in a society where the disposable income of most households is primarily predetermined (by wages etc.). Therefore, we follow the approach taken in Hildebrand [6] and others and assume that at a given point is time, prices do not effect incomes.

2 Mean income effect terms

Recall the Slutsky relation: Let the demand $f_i(\mathbf{p}, I)$ (for the i-th commodity) be a continuously differentiable function of $(\mathbf{p}, I) \in R^{n+1}_{++}$. Then the substitution matrix $S = \{s_{ij}\}^n_{i,j=1}$ defined by

$$s_{ij} = \frac{\partial f_i(\mathbf{p}, I)}{\partial p_j} + f_j(\mathbf{p}, I)\frac{\partial f_i(\mathbf{p}, I)}{\partial I} \ . \tag{1}$$

is symmetric, negative semidefinite, and negative definite on the orthogonal complement of $\mathbf{p}$. In particular $s_{i,i} < 0$. The term $f_j(\mathbf{p}, I)\frac{\partial f_i(\mathbf{p},I)}{\partial I}$ represents the *income* effect. For our purposes it is convenient to rewrite (1) as

$$\frac{\partial f_i(\mathbf{p},I)}{\partial p_j} = s_{ij} - f_j(\mathbf{p},I)\frac{\partial f_i(\mathbf{p},I)}{\partial I} \ . \tag{2}$$

Setting $i=j$ in (2) we see that a good may be Giffen only if it is inferior somewhere. From now on we consider only (except in Remark 2) diagonal terms ($i=j$ — own price derivatives) and drop the subindices for goods and prices. Suppose that our economy consists of a continuum of consumers who differ only by the amount I they set aside for expenditure (but they all have the same demand function f). Let $\rho(I)$ denote the density of the distribution of I (so that $\int \rho(I)dI = 1$). Then the (mean) market demand is given by $\int f(p,I)\rho(I)dI$. A necessary condition for the market demand to exhibit a Giffen behavior (i.e., that $\frac{\partial \int f(p,I)\rho(I)dI}{\partial p}$ be non-negative) is that the mean income effect term , $\int f(p,I)\frac{\partial f(p,I)}{\partial I}\rho(I)dI$, is strictly negative. Hildenbrand [6] noted that if the density $\rho(I)$ is a decreasing function of I then this term is non-negative (even if the commodity is inferior for certain values of I).

We now see what may happen if the distribution is uni-modal. We assume that the following holds:

(**UM**) : The density of the expenditure distribution is a C^1 function supported on a compact interval $[0,R]$ with a unique maximum at $x=a\in(0,R)$.

It follows that for every $y\in(0,\rho(a))$ the exist numbers b_y^l and b_y^r such that $b_y^l < a < b_y^r < R$ and $\rho(b_y^l) = \rho(b_y^r) = y$. Moreover, the numbers b_y^l and b_y^r are uniquely determined and depend continuously on y. We relate the inferior character of the commodity (with respect to the individual demand function $f(p,I)$ at a given price $p\in P$ where P is an open set of price vectors) to the expenditure distribution by assuming the following:

(**ID**) : For all $y\in(0,\rho(a))$ and $p\in P,\ f(p,b_y^l) > f(p,b_y^r)$.

A demand function and a density satisfying (**UM**) and (**ID**) (for a certain fixed p) are illustrated in Fig. 1.

We may now state the main result of this section.

Theorem. If the assumptions (**UM**) and (**ID**) hold, and all individuals have the same demand function f, then the mean income effect term is negative for each $p\in P$, i.e.,

$$\int f(p,I)\frac{\partial f(p,I)}{\partial I}\rho(I)dI < 0 \ . \tag{3}$$

Proof: Integrating by parts, we see that

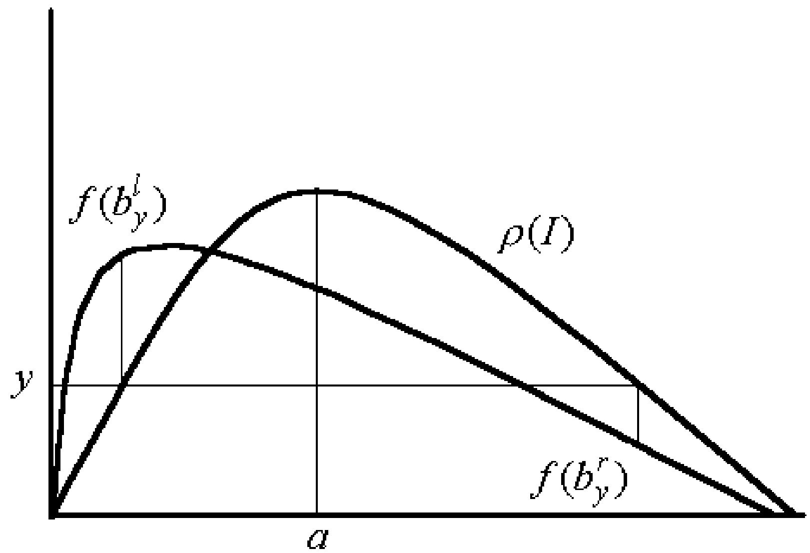

Fig. 1. Illustration of (**UM**) and (**ID**)

$$\int f(p,I)\frac{\partial f(p,I)}{\partial I}\rho(I)dI = \frac{f(p,I)^2}{2}\rho(I)\Big|_{I=0}^{I=R} - \int \frac{f(p,I)^2}{2}\rho'(I)dI \ . \tag{4}$$

By assumption, $\frac{f(p,I)^2}{2}\rho(I) = 0$ at $I = 0$ and $I = R$. Hence the mean income effect term is equal to

$$-\int_0^a \frac{f(p,I)^2}{2}\rho'(I)dI - \int_a^R \frac{f(p,I)^2}{2}\rho'(I)dI \ . \tag{5}$$

We now introduce the new integration variable $y = \rho(I)$ on each of the intervals $(0,a)$ and (a,R) (thus $I = b_y^l$ or $I = b_y^r$), and noting that $\rho(I)$ is increasing on $(0,a)$ and decreasing on (a,R), we find that

$$\int f(p,I)\frac{\partial f(p,I)}{\partial I}\rho(I)dI = -\int_0^{\rho(a)} \frac{f(p,b_y^l)^2}{2}dy + \int_0^{\rho(a)} \frac{f(p,b_y^r)^2}{2}dy < 0 \ , \tag{6}$$

where the last inequality follows from (**ID**). QED

Remark 1: We could replace the assumption that ρ is compactly supported by a suitable decay condition on the product $f(p,I)^2\rho(I)$. Similarly, it is possible to replace $(0,R)$ by another interval (contained in the non-negative half line) and the assumptions about the vanishing of f and ρ at the ends of the interval could be relaxed.

Remark 2: Recall that the (Jacobian) matrix of price derivatives $D\mathbf{f}(\mathbf{p})$ is negative semi-definite if for every vector $\mathbf{v} \in R^n$

$$\sum_{i,j=1}^{n} \frac{\partial f_i(\mathbf{p}, I)}{\partial p_j} v_i v_j \leq 0 \; . \tag{7}$$

(The left hand side may be interpreted as the own price derivative of a *package* of commodities—compare [7],p.172.) The corresponding mean income effect term is $\int \sum_{i,j=1}^{n} f_j(\mathbf{p}, I) \frac{\partial f_i(\mathbf{p},I)}{\partial I} v_i v_j \rho(I) dI$. This term is positive if $\mathbf{v}$ is proportional to $\mathbf{p}$. If $\mathbf{v} \neq \mathbf{p}$ we may generalize Theorem 1 to obtain negative mean income effect terms if for all $y \in (0, \rho(a))$, $\sum_i^n f_j(\mathbf{p}, , b_y^l) v_j > \sum_i^n f_j(\mathbf{p}, , b_y^r) v_j$ ((**ID**) holds for the package). The proof follows that of Theorem 1.

3 Shochu

The following rice-based alcoholic beverages are listed in [1] (ordered according to increasing price): shochu, second grade sake, first grade sake, and special grade sake. Shochu (distilled alcohol) is consumed more by the poor than by the rich. On the other hand, special grade sake is by far a rich person beverage. This is illustrated by Table 1, where the total expenditure for alcoholic beverages and the consumption of shochu and of special grade sake are listed (for the year 1987).

Table 1. Inferiority of Shochu

Income quintile	Living expenditure (Thousand Yen)	Alcoholic beverage expenditure (Yen)	Shochu consumption (100 ml.)	sp. gr. Sake consumption (100 ml.)
1	2113	37,822	6,910	311
2	2760	46,837	5,894	637
3	3164	51,155	5,242	415
4	3779	54,755	5,478	674
5	5040	60,003	3,946	1022

It is possible (in analogy to the classical story about potatoes) that as the price of shochu goes up, poor people who want to drink the same amount of alcohol (perhaps due to an addiction), have to give up the more expensive alcoholic drinks, and therefore consume more shochu.

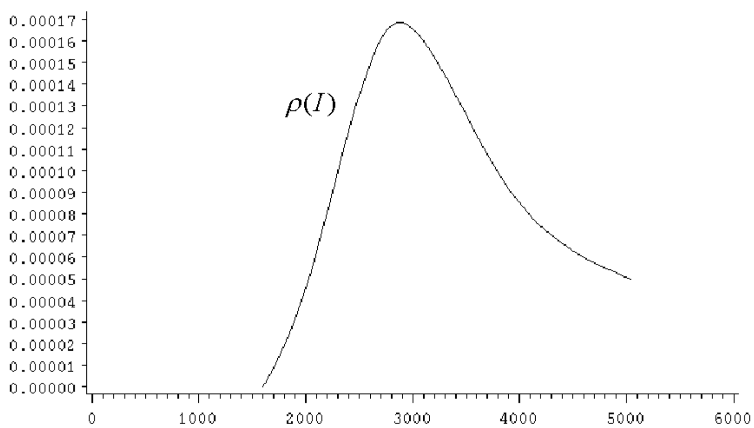

Fig. 2. Density of Living Expenditures (in Thousand Yen)

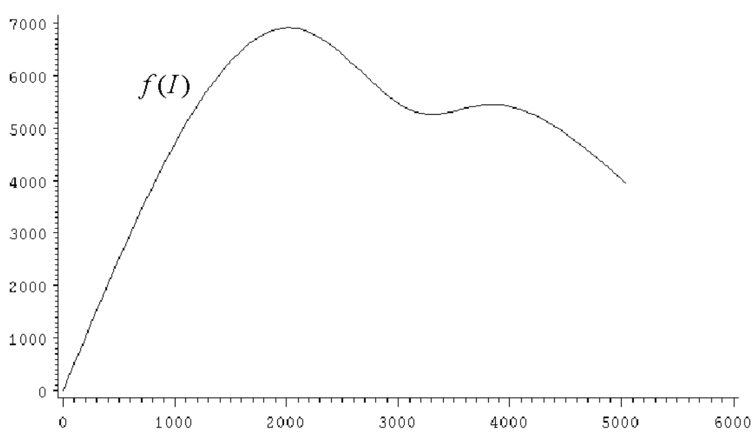

Fig. 3. The Engel Curve: The Horizontal Axis Shows Total Annual Living Expenditure in Thousand Yen; The Vertical Axis Shows Quantity of Shochu Consumed (in 100 ml.)

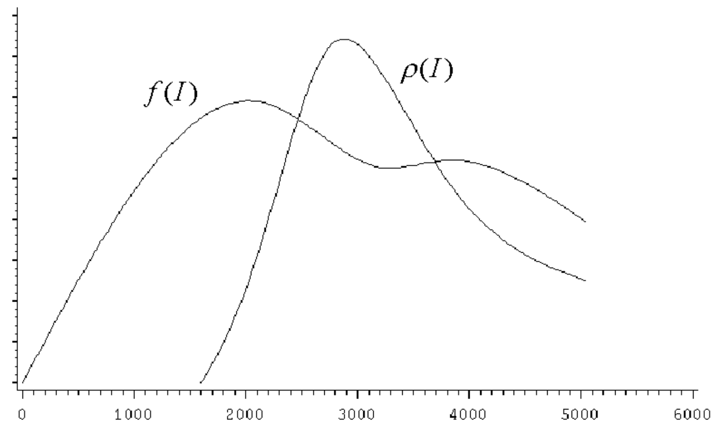

Fig. 4. Combined Engel Curve and Density of Living Expenditures

As a first step we would like to estimate the mean income effect term for shochu, or at least to determine its sign. To this end, time-series data are not appropriate, and we have to use cross-sectional data. Since the published data do not provide us with finer cross-sectional data, we use Table (1) to estimate the distribution of Income and the Engel curve (consumption as a function of income or of living expenditure) under the assumption that all individuals possess the same demand function. See figures 2 and 3 respectively. Since the conditions of the Theorem in section 2 are invariant under rescaling of the vertical axis, we provide in Figure 4 a draw of both of the curves after rescaling the vertical axis. It clearly appears that Condition (**ID**) holds and hence the mean income effect term is negative.

We now turn to examine the movements of prices and quantities consumed of shochu and special grade sake. We had data for special grade sake for the period January 1987 through March 1989. While we had data for other beverages for more extended periods, we decided to analyze shochu for the same period. When using time series one has to verify that there are no appreciable changes in the structure of the economy (such as population shifts, changes in cost of living, and so on). During the later part of 1989 (due to the illness and then the death of the emperor?) we could observe some changes. On the other hand, the period chosen is characterized by stability.

We estimate demand for a beverage by regressing the logarithms of quantities consumed on the log of the prices. We use a dummy variable dec equal to 1 if the month is December (during that month there is a large increase in alcohol consumption). To identify demand we used the one month lagged prime interest rate — it is unlikely that this variable influences demand, but it probably does have an effect on supply, and thus may be used as an identifying variable. Lagged interest (int(-1)) and dec are used as instruments (exogenous) variables as we apply a three-stage estimation method for the demand-and-supply model ([5],[11]). For the demand equation we also add an autoregressive term AR(1), and iterate 3sls ([11]).

The results are summarized in the following tables (numbers in brackets are the p values – the levels of significance – of the regression coefficients):

Table 2. First Stage

Dependent Variable	Dec	interest(-1)	R^2	DW
shochu price	-.024 (.04)	-.03 (.001)	.40	1.40
sp. gr. sake price	.058 (.005)	-.045 (.004)	.25	1.64

Table 3. Demand

Dependent Variable	Dec	own price	R^2	AR(1)	DW
shochu consumption	.62 (.000)	8.81 (.004)	.21	.59 (.02)	1.27
sp. gr. sake consumption	2.56 (.000)	-6.11 (.03)	.70	-.44 (.001)	1.96

It appears from Table 2 that the lagged interest rate may indeed be utilized as an identifying variable. The differing signs of the own price elasticities for shochu and special grade sake Table 3 suggest that while special grade sake is clearly a normal commodity, shochu may be a Giffen good. (We note that for first and second grade sake the elasticities are not statistically significant, but second grade had a positive price elasticity whereas first grade has a negative one.)

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